THE

RATIONAL ARITHMETIC,

IN WHICH THE

SCIENCE IS FULLY DEVELOPED,

THE

ART CLEARLY EXPLAINED,

AND BOTH COMBINED IN NUMEROUS ILLUSTRATIONS;

ADAPTED TO LEARNERS

OF EVERY CAPACITY.

THE WHOLE ENFORCED BY A GREAT VARIETY OF

INTERESTING AND PRACTICAL PROBLEMS.

TO WHICH IS APPENDED,

A KEY,

CONTAINING THE ANSWERS TO THE PROBLEMS.

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PREFACE.

Every public school may be divided, in respect to the study of Arithmetic, into three classes. The first and smallest class either possess by nature, or have happily acquired, a taste, and, consequently, a talent for the study. For them there is no imperious necessity of adding another to the numerous treatises already in use; for, although they will meet with much difficulty, through indefinite and confused modes of expression and incomplete demonstrations, in arriving at the philosophy of the subject, yet, in spite of these obstacles, they will eventually comprehend the important principles of Arithmetic, and, what is remarkable, adopt the same modes of expression which have so much opposed their own progress.

To this class the Rational Arithmetic, though not indispensable, will be of very essential service. Had all learners been of this class, however, the author would have been spared the labor and expense he has devoted to this work.

But there is a class of medium ability, including about one half, who may be saved incalculable labor and vexation, by using this book, while pursuing this difficult study.

It is expected, however, that the third class will most truly appreciate this work. They include about one third, and consist of those unfortunate scholars whose minds act too slowly for the patience of teachers, and are too obtuse to derive much advantage from the text-books so ill-adapted to their wants.

It is for these two classes, the last in particular, that the Rational Arithmetic is prepared; to their wants it is thought to be well adapted; and it is expected that they will hereafter assume a more just relative standing with their schoolmates; not waiting, as heretofore, for the results of business life to prove them possessed of minds, less active indeed, yet not inferior in strength and capacity of improvement.

The author knows no other written Arithmetic that is adapted to *learners;* they seem to him rather books of reference for those who already understand the subject, and are able to perceive the princi-

ples without explanation. Some, indeed, have *attempted* to meet the wants of learners, by introducing the several subjects by a page or two of puerile questions which are seldom noticed either by teachers or scholars. Others found the principles upon the imaginary answers to be given to such questions by those acknowledged to be ignorant. Such must be a very uncertain foundation, especially so when, as sometimes happens, these leading questions are so misformed as to lead astray. Instance the following: "In 11, how *much more* does the 1 in the tens' place stand for than the 1 in the unit's place? In 880, *how much more* does the 8 in the hundreds' place stand for than the 8 in the tens' place? It is just so in all cases; therefore, *A figure at the left of another stands for ten times as much as it would in the place of that other figure.*" The simple learner will, probably, understand these questions literally, and give for answers, so far as he is able, 9 more, 720 more, &c., between which and the principle purporting to be derived from them, there is no direct connection. Had these questions been thus, How *many times as much* ——— as ———? instead of "How *much more* ——— than ———?" they would have led the intelligent mind directly to the principle. In presence of the teacher to correct false answers, to sum up and enforce the conclusion, such questions *properly* asked are well enough; but otherwise, they are extremely vexatious and discouraging, and most scholars will pass over them without adding to their knowledge.

In the Rational Arithmetic it has been the object to prepare matter for *the intelligent study of the learner by himself*, that in due time he may, in a well conducted recitation, exhibit with credit and pleasure, both to himself and teacher, his *thorough knowledge of the lesson.* Such results are far different from ordinary experience. Teachers who have desired to *ground their pupils in the principles* of the science, while the text-books have failed to afford the necessary instruction, have, by *oral instruction*, and *black-board illustrations*, endeavored, with only partial success, to effect this object, at present, so indispensable. Such instruction, though efficient with the more intelligent and active minds, proves insufficient for a large portion of every school. It is expected that the Rational Arithmetic will come to the relief of such teachers, enabling them with less labor to secure much happier results.

The *peculiarities of the Rational Arithmetic* are: 1st, A *philosophical arrangement*, and *systematic treatment* of the several subjects. Multiplication and Division, being only particular cases of Addition and Subtraction, respectively, follow their heads in the natural order, in the fundamental principles; but in fractions and compound numbers, from the greater convenience, they resume the common order again. The subjects under the head of Percentage, being applications of the principle of Proportion, very properly follow Proportion. The study of Arithmetic being now so extensive, it is no longer necessary to place Interest nearer the beginning of the book than its

proper place, to insure a knowledge of it. Indeed, all the subjects are so arranged that each is explained on principles previously taught.

2d. *A complete development of the fundamental principles.* Numeration, in particular, both integral and fractional, the foundation of the whole superstructure, has received especial attention.

3d. *A full description and thorough explanation* of the various applications of the fundamental principles.

4th. *A constant regard to the abridgment of labor*, by viewing numbers through their factors, and relations, canceling common factors when both multiplication and division are involved in the same process; and always operating upon fractions in a manner to secure the simplest terms in results.

5th. *The giving of a reason for everything stated*, and in such style that the repetition of the language will induce in the learners the understanding of the reason which it embodies.

6th. *Numerous references to other parts of the book* for information bearing upon the subject in hand.

7th. *An exclusion of all such indefinite expressions* as "5 times greater," "seven times too large," "seven times too small," "increases it ten times," "5 times too great," "100 times larger or smaller;" *and all such provoking expressions* as "it is obvious," "it is plain," "evidently," &c., whose office is only to occupy the place of an inconvenient reason. These peculiar excellences, it is thought, warrant the title assumed for the book.

It is recommended to teachers, although the author knows no written arithmetic so easily understood, that the younger pupils, previously to taking up this book, shall have well studied Colburn's First Lessons, or some other intellectual arithmetic; but when it is taken up, that they accommodate their speed to thoroughness; that they take special notice of the numerous references to other parts of the book, where their memories may be refreshed with necessary information upon the present subject.

Teachers will, of course, as far as circumstances admit, classify their pupils, assign lessons, and hear recitations in this, as in other studies. Although each is left to his own experience and tact in conducting recitations, yet we may urge the importance of *securing in some way a thorough analysis of everything, the giving of a reason for each step in the solution of problems, showing its bearing upon, or tendency towards the final result.* In the author's experience, it has proved well to require the pupils to bring to the recitation, not only the results, but the written process of their work; also to exhibit their skill in the solution of problems upon a black-board sufficiently ample to accommodate the whole class. The problems may be those of the ordinary lesson, or such as may be suggested on the occasion; and the recitation should be such as shall exhibit the scholar's knowledge of the principles involved in the process.

The impossibility of preventing the access of the scholars to the

"Key published for the use of teachers *only*," the immense injury done to the moral sense by the futile attempt, the securing of greater faithfulness on the part of teachers, and, on the other hand, the accommodation of the better scholars, who dislike to have the answers obtruded upon their notice before they shall have given the problem a fair trial, with other reasons, have induced the author to append the Key to the Arithmetic, where it may be conveniently, and *innocently* accessible to all. But should any prefer the book and key separate, by signifying their desire to the publisher, it may be gratified.

To the numerous friends who advised this undertaking, who have encouraged its progress, and are ready to receive it, and give it "a start in the world," the author takes this occasion to express his gratitude. He also acknowledges his obligations to the numerous authors whose works he has consulted; yet few will discover anything heretofore published, except a few select problems, as the Rational Arithmetic is chiefly derived from an experience of more than ten years' teaching of the mathematics, half of which has been devoted exclusively to arithmetic.

LOWELL, *October* 1846.

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ARITHMETIC.

I. NUMERATION.

1. Arithmetic Defined.

Arithmetic is the science of numbers, and the art of computing by them.

As a science, arithmetic explains the nature and properties of numbers; and demonstrates the principles and rules for the practice of the art.

As an art, arithmetic explains the methods of working by numbers for the solution of numerical problems.

2. Formation of Numbers.

A single thing of any kind is called a unit, or One.

The larger numbers are *formed* by the successive addition of units. Thus, if to one, another unit of the same kind be added, the collection forms the number, Two.

The collection of two and one forms the number, Three.
The collection of three and one forms the number, Four.
The collection of four and one forms the number, Five.
The collection of five and one forms the number, Six.
The collection of six and one forms the number, Seven.
The collection of seven and one forms the number, Eight.
The collection of eight and one forms the number, Nine.

In like manner, the addition of one unit to any number, forms the next larger number.

3. Arabic Figures.

Among the various methods of *expressing* numbers, the Arabic is superior; and is now in general use. According to this method, all numbers can be expressed by different combinations of one, or more, of ten figures.

The figures are, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

The first nine figures are also called *digits*. And each digit, expressing one of the first nine numbers, has the same name as the number which it expresses.

Thus: One is expressed by this figure, 1, called One.
Two is expressed by this figure, 2, called Two.
Three is expressed by this figure, 3, called Three.
Four is expressed by this figure, 4, called Four.
Five is expressed by this figure, 5, called Five.
Six is expressed by this figure, 6, called Six.
Seven is expressed by this figure, 7, called Seven.
Eight is expressed by this figure, 8, called Eight.
Nine is expressed by this figure, 9, called Nine.

The other figure, 0, called Cipher, unlike the digits, does not express a number, nor have any value; but yet, as we shall see, it is not without its use.

4. Expression of Tens, or Units of the Second Order.

There is no appropriate figure to express the next number, called ten, or any of the larger numbers; but these same digits are made to express other numbers by occupying different *places* in relation to each other. When they stand alone, or in the *first place*, each expresses a certain number of units of the *first order*. But ten units of this order are considered collectively as forming one unit of the *second order;* and the digits are made to express units of the second order, called tens, by occupying the *second place* from the right hand. Thus:

10 is one *ten*, called Ten.
20 is two *tens*, called Twenty.
30 is three *tens*, called Thirty.
40 is four *tens*, called Forty.
50 is five *tens*, called Fifty.
60 is six *tens*, called Sixty.
70 is seven *tens*, called Seventy.
80 is eight *tens*, called Eighty.
90 is nine *tens*, called Ninety.

Here the digits express ten times as much, or numbers ten times as large, as when they stand in the first place, or alone, *because they occupy the second place*, and not because there is any value in the cipher. The cipher merely occupies the first place, in order that there may *be* a second place for the digit to occupy. So always, the cipher is used to occupy places where nothing of value is needed; but which must be occupied, in order that the digits required for the expression of the number, may stand in their proper places.

5. Expression of Numbers from Ten to One Hundred.

The numbers between the tens, that is, between ten and twenty, twenty and thirty, &c., are expressed by making every digit, in succession, occupy the first place, together with each digit in the second place. Thus:

10 is one unit of the second order, called Ten.
11 is ten and one, called Eleven.
12 is ten and two, called Twelve.
13 is ten and three, called Thirteen.
14 is ten and four, called Fourteen.
15 is ten and five, called Fifteen.
16 is ten and six, called Sixteen.
17 is ten and seven, called Seventeen.
18 is ten and eight, called Eighteen.
19 is ten and nine, called Nineteen.
20 is two tens, or two units of the second order, called Twenty
21 is two tens and one, called Twenty-one.
22 is two tens and two, called Twenty-two.
23 is two tens and three, called Twenty-three.
24 is two tens and four, called Twenty-four.
25 is two tens and five, called Twenty-five.
26 is two tens and six, called Twenty-six.
27 is two tens and seven, called Twenty-seven.
28 is two tens and eight, called Twenty-eight.
29 is two tens and nine, called Twenty-nine.
30 is three tens, or units of the second order, called Thirty.
31 is three tens and one, called Thirty-one.
32 &c., is three tens and two, called Thirty-two.
41 &c., is four tens and one, called Forty-one.
51 &c., is five tens and one, called Fifty-one.
61 &c., is six tens and one, called Sixty-one.
71 &c., is seven tens and one, called Seventy-one.
81 &c., is eight tens and one, called Eighty-one.
91 &c., is nine tens and one, called Ninety-one.
99 is nine tens and nine, called Ninety-nine.

6. Expression of Hundreds, or Units of the Third Order.

Ninety-nine is the largest number that can be expressed by two figures. The next larger number is ten tens, or ten units of the second order, which are considered collectively as forming one unit of the third order, called one hundred.

This unit is also expressed by the first digit; but, it being a unit of the *third order*, the digit is put in the *third place*. And the other digits, by occupying the third place, are made to express units of the third order, or hundreds. Thus:

100 is One *hundred*.
200 is Two *hundred*.
300 is Three *hundred*.
400 is Four *hundred*.
500 is Five *hundred*.
600 is Six *hundred*.
700 is Seven *hundred*.
800 is Eight *hundred*.
900 is Nine *hundred*.

7. Expression of Numbers from One Hundred to One Thousand.

The numbers between the hundreds are expressed, by making all the numbers less than one hundred, in succession, occupy their own places at the right of each digit in the third place. Thus:

101 is one hundred and one.
210 is two hundred and ten.
311 is three hundred and eleven.
425 is four hundred and twenty-five.
543 is five hundred and forty-three.
608 is six hundred and eight.
717 is seven hundred and seventeen.
876 is eight hundred and seventy-six.
999 is nine hundred and ninety-nine.

8. Expression of Thousands, or Units of the Fourth Order.

Nine hundred and ninety-nine is the largest number that can be expressed by three figures. The next larger number is ten hundreds, or ten units of the third order, which are considered collectively as forming one unit of the fourth order, called *one thousand*.

To express thousands, or units of the *fourth order*, the digits are put in the *fourth place*.

9. Expression of Numbers from One Thousand to Ten Thousand.

Any number between the thousands is expressed by using such digits as are needed in their proper places at the right of the thousands. Thus:

1000 is one thousand.
2001 is two thousand and one.
3020 is three thousand and twenty.
4500 is four thousand and five hundred.
5055 is five thousand and fifty-five.
6107 is six thousand one hundred and seven.
7819 is seven thousand eight hundred and nineteen.
8011 is eight thousand and eleven.
9999 is nine thousand nine hundred and ninety-nine.

10. EXPRESSION OF TEN-THOUSANDS AND UNITS OF OTHER ORDERS.

Ten units of the *fourth order*, form one unit of the *fifth order*, called a *ten-thousand.* And the ten-thousands must occupy the *fifth place.*

In the same manner, higher orders of units are formed, to an unlimited extent; *ten units* of any order forming *one unit* of the next *higher* order, to be expressed in the next higher *place;* while the lower places are used for the expression of units of lower orders.

Whence it follows that one unit of any order equals ten units of the next lower order; this law prevailing, even below units of the first order, to an unlimited extent, as will be shown, (**163.**) Hence the law of the local value of figures is, that any digit, by each removal to the next higher place, is made to express ten times as much, and by each removal to the next lower place, one tenth as much as it would before such removal.

N. B. The term unit means a unit of the first, or lowest order, unless otherwise specified.

11. TABLE EXHIBITING THE FORMATION, NAME, AND EXPRESSION OF ONE UNIT OF EACH OF THE FIRST TEN ORDERS.

A single thing of any kind	forms one Unit,	1.
Ten units of the same kind	form one Ten,	10.
Ten tens	form one Hundred,	100.
Ten hundreds	form one Thousand,	1000.
Ten thousands	form one Ten-thousand,	10000.
Ten ten-thousands	form one Hundred-thous.	100000.
Ten hundred-thousands	form one Million,	1000000.
Ten millions	form one Ten-million,	10000000.
Ten ten-millions	form one Hundred-million,	100000000.
Ten hundred-millions	form one Billion,	1000000000.

These units of ten different orders may be expressed in one number. Thus:

Billion.	Hundred-million.	Ten-million.	Million.	Hundred-thousand.	Ten-thousand.	Thousand.	Hundred.	Ten.	Unit.
1	1	1	1	1	1	1	1	1	1

since each unit now occupies the place appropriated to units of its own order. This number is read, One Billion, one hundred and eleven Million, one hundred and eleven Thousand, one hundred and eleven.

12. Illustration of the Absolute and Relative value of the Unit of different Orders.

Suppose there should be a country having ten states, each state having ten cities, each city having ten schools, each school having ten classes, each class having ten scholars, and each scholar having ten cents to pay for a writing-book. How many cents would it take to buy a book for a scholar?—books for a class?—for a school?—for a city?—for a state?—for the country? It would take

for one scholar,	ten times	1 ct.,	equal to	10 cts.
for one class,	ten times	10 cts.,	equal to	100 cts.
for one school,	ten times	100 cts.,	equal to	1000 cts.
for one city,	ten times	1000 cts.,	equal to	10000 cts.
for one state,	ten times	10000 cts.,	equal to	100000 cts.
for the country,	ten times	100000 cts.,	equal to	1000000 cts.

Other answers.—It would take, one cent being a unit of the first order,

for one scholar, a Ten-cent-piece,
which is a unit of the second order;
—for one class, a Dollar-bill,
which is a unit of the third order;
—for one school, a Ten-dollar-bill,
which is a unit of the fourth order;
—for one city, a Hundred-dollar-bill,
which is a unit of the fifth order;
—for one state, a Thousand-dollar-bill,
which is a unit of the sixth order;
—for the country, a Ten-thousand-dollar-bill,
which is a unit of the seventh order.

Suppose one man should make all these writing-books, and, having done them up in one package, should sell it to the president of the country; of this package the president should make ten equal packages, and sell one of them to the governor of each state; each governor should make of his package ten equal packages, and sell one of them to the mayor of each city; each mayor should make of his package ten equal packages, and sell one of them to the teacher of each school; each teacher should make of his package ten equal packages, and sell one of them to the head scholar of each class, and each head scholar, on opening his package, should find just ten books, nine of which he should sell to his class, and keep the other himself.

One book being a unit of the first order, a unit of what order would be each head scholar's package?—each teacher's package?—each mayor's package?—each governor's package?—the president's package?

How many books in the president's package?—in each governor's package?—mayor's package?—teacher's package?—head scholar's package?

Suppose each one should pay the cents for his book, or package, to the person from whom he had received it. Then each scholar would pay his head scholar nearly a handful of cents; each head scholar would pay his teacher nearly a "double handful;" each teacher would pay his mayor nearly three pints; each mayor would pay his governor nearly two pecks; each governor would pay the president nearly five bushels; and the president would pay the book-binder nearly fifty bushels, and just a *million* of cents.

It would take the book-binder's son more than two months to count them, if, instead of going to school, he should count, at the rate of one cent every second, for three hours every half day, except Saturday afternoons and Sundays.

13. Manner of reading Numbers.

In any number, the first three figures express so many hundreds tens and units of *Units;* the second three, so many hundreds tens and units of *Thousands;* the third three, so many hundreds tens and units of *Millions;* and each succeeding three, so many hundreds tens and units of *Billions, Trillions, Quadrillions, Quintillions, Sextillions, Septillions. Octillions, Nonillions, Decillions,* &c., respectively.

Hence, to *read* numbers, count off the figures from the right, into periods of three figures each, and beginning at the left, read each period separately, as so many hundreds tens and units, naming each period as it is read, except the right hand period, which is understood to be units without its name being called. Thus:

Thirty one *Quintillion.*	five hundred and sixty two *Quadrillion,*	eight hundred and ninety six *Trillion,*	one hundred and twenty five *Billion,*	nine hundred and forty four *Million,*	three hundred and sixty one *Thousand,*	two hundred and ninety nine.
31	562	896	125	944	361	299.

Nine *Quintillion,*	six hundred and one *Quadrillion,*	ten *Trillion,*	nine hundred and eighty two *Billion,*	five *Million,* and		six hundred.
9	601	010	982	005	000	600.

14. Exercises in reading Numbers.

Read the following numbers,

1,	19.	10,	20907.	19,	120643790008074.
2,	70.	11,	417016.	20,	9064798030020.
3,	98.	12,	5008840.	21,	3479000019.
4,	502.	13,	40910008.	22,	80060400700091.
5,	610.	14,	136000200.	23,	811123365.
6,	847.	15,	6500004.	24,	347000016011.
7,	1005.	16,	1147865479.	25,	333311112222.
8,	5049.	17,	416000.	26,	8800006600004400 0.
9,	9153.	18,	900001317601.	27,	5500000010020001 0.

15. Manner of writing Numbers.

To *write* numbers in figures, first write the left hand period, which may require one, two, or three figures, then, in succession, write the other periods, allowing three places for each period.

Write in figures nine quintillion, six hundred and one quadrillion, ten trillion, nine hundred eighty-two billion, five million, and six hundred, 9,601,010,982,005,000,600.

In this number, 9 only must occupy the period of quintillions, 601 the period of quadrillions, 010 the period of trillions, 982 the period of billions, 005 the period of millions, 000 the period of thousands, and 600 the period of units.

16. Exercises in writing Numbers.

In like manner write the following numbers.

1. One hundred and three.
2. Three hundred and one.

3. One thousand, and ten.
4. Two thousand, one hundred and seven.
5. Twenty thousand, and thirty.
6. Fifty thousand, seven hundred and five.
7. Three hundred thousand, and fifty.
8. Seven hundred and seven thousand, seven hundred and twenty.
9. One million, three hundred and seventy.
10. Five million, six hundred thousand, and seventy-three.
11. Five hundred ninety million, forty-seven thousand, and eight.
12. Three billion, six hundred seventy million, three hundred and two.
13. Forty-five billion, seven million, seventy thousand, and seven.
14. Fifty trillion, six hundred fifty-seven million, and five hundred.
15. Six trillion, seven hundred and three billion, twenty million, and twelve.
16. Seventy-seven million, ten thousand, and nineteen.
17. Eight billion, five hundred and thirty thousand.
18. Forty-nine billion, three hundred and sixty.
19. Eighty six quadrillion, ten billion, one hundred million, and sixty.

II. ADDITION.

17. Addition Defined and Illustrated.

Addition is the uniting of two, or more, numbers to form another number equal to their sum. Thus:

1. If you should place 5 cents in a pile, and on that pile put 3 more cents; how many cents would there be in the pile?

You are taught, (**2**,) that numbers are formed by successive additions of one unit; but here you are required to form a number by the addition of 3 units. This can be done by adding the 3 units, 1 at a time; thus, 5 cents and 1 cent are 6 cents, and 1 cent are 7 cents, and 1 cent are 8 cents, which is the whole number of cents in the pile.

2. A man paid a 5 dollar-bill for a pair of boots, and a

3 dollar-bill for a pair of shoes; how many dollars did he pay away?

To answer this question, you must add 3 dollars to 5 dollars as we added 3 cents to 5 cents in the 1st example. But if you remember what the sum of 5 and 3 is, you need not add the 3, one at a time, but all at once, saying 5 dollars and 3 dollars are 8 dollars; therefore, he paid away 8 dollars.

18. EXPLANATION AND USE OF SIGNS.

For convenience and brevity, *signs* are often employed in arithmetic. Thus: $=$ *Two horizontal lines* are the sign for *equality*. It implies that what precedes the sign equals what follows it; as 100 cents $=$ 1 dollar; read, 100 cents equal 1 dollar.

$+$ *The right cross* is the sign for *addition*. It implies that the number which follows the sign is to be added to what precedes it, as $5+3=8$; read 5 plus 3 equal 8.

Plus is the Latin word for *more*, and means here the same as if you should say 5 *more* 3, or 5 and 3 *more* equal 8.

19. ADDITION TABLE.

In order to perform addition with facility, you will, before attempting further progress, correctly ascertain, and thoroughly commit to memory, the *sum* of each combination of two numbers in the following table.

2 & 2 are	7 & 3, or 3 & 7 are	8 & 5, or 5 & 8 are
3 & 2, or 2 & 3 are	8 & 3, or 3 & 8 are	9 & 5, or 5 & 9 are
4 & 2, or 2 & 4 are	9 & 3, or 3 & 9 are	6 & 6 are
5 & 2, or 2 & 5 are	4 & 4 are	7 & 6, or 6 & 7 are
6 & 2, or 2 & 6 are	5 & 4, or 4 & 5 are	8 & 6, or 6 & 8 are
7 & 2, or 2 & 7 are	6 & 4, or 4 & 6 are	9 & 6, or 6 & 9 are
8 & 2, or 2 & 8 are	7 & 4, or 4 & 7 are	7 & 7 are
9 & 2, or 2 & 9 are	8 & 4, or 4 & 8 are	8 & 7, or 7 & 8 are
3 & 3 are	9 & 4, or 4 & 9 are	9 & 7, or 7 & 9 are
4 & 3, or 3 & 4 are	5 & 5 are	8 & 8 are
5 & 3, or 3 & 5 are	6 & 5, or 5 & 6 are	9 & 8, or 8 & 9 are
6 & 3, or 3 & 6 are	7 & 5, or 5 & 7 are	9 & 9 are

20. EXPLANATION OF THE WRITTEN PROCESS OF ADDITION.

1. A man paid 25 dollars for a cow, and 3 dollars for a sheep; how many dollars did they cost?

Under the 25 write the 3, so that it shall stand in a column with the 5; and, since both the 5 and 3 express units of the *first order*, (4,) add them together, and write 8, their sum, directly under them, in the *first place*, and, the 2 expressing units of the *second order*, write it beside the 8, in the *second place*, which gives 28 dollars for the answer.

```
25
 3
--
28
```

2. A man having sold the produce of his farm, received 48 dollars for potatoes, 25 dollars for wheat, 32 dollars for rye, 28 dollars for corn, and 54 dollars for hay; how many dollars did he receive?

Arrange the numbers together, so that the units of each order shall stand in a column; then ascertain the sum in the lowest column; thus, 8 and 5 are 13, and 2 are 15, and 8 are 23, and 4 are 27 units of the *first order;* but since ten units of any order make one unit of the next higher order, (10,) these 27 units of the first order will make 2 units of the second order and 7 units of the first order; hence write the 7 in the first place, and add the 2 with those of the same kind in the second column; thus, 2 and 4 are 6, and 2 are 8, and 3 are 11, and 2 are 13, and 5 are 18 units of the second order, making 8 units of the second order, and 1 unit of the third order; therefore, write the 8 in the second place, and the 1 in the third place, which gives 187 dollars for the answer.

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 48
 25
 32
 28
 54
---
187
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21. PROOF OF ADDITION.

To prove the correctness of any operation in addition, *repeat* the operation, combining the figures of each column in the *opposite* order. If the two results agree, probably both are correct.

22. MODEL OF A RECITATION.

What is the sum of the following numbers, 35468, 503. 2300, 95 and 90072?

Arrange these numbers together, so that the units of each order shall stand in a column; 2 and 5 are 7, and 3 are 10, and 8 are 18 *units*, equal to 8 units, which write in the units' place, and 1 ten, which add with the tens; 1 and 7 are 8, and 9 are 17, and 6 are 23 *tens*, equal to 3 tens, which write in the tens' place, and 2 hundreds, which add with the other hundreds; 2 and 3 are 5, and 5 are 10, and 4 are 14 *hundreds*, equal to 4 hun-

```
 35468
   503
  2300
    95
 90072
------
128438
```

dreds, which write, and 1 thousand, which add with the other thousands; 1 and 2 are 3, and 5 are 8 *thousands*, which write; 9 and 3 are 12 *ten-thousands*, equal to 2 ten-thousands, which write, and 1 hundred-thousand, which write, giving 128438, the sum required.

Hence, OBSERVE; *that in addition, the units of each order, beginning with the lowest, are added separately, and reduced,* (**208**,) *as far as may be, to, and added with units of the next higher order, writing in each place only the excess over exact units of the next higher denomination.*

23. EXERCISES IN ADDITION.

In like manner, solve and explain the following problems.

1. Mr. Sampson sold 6 loads of potatoes, measuring, severally, 36, 34, 38, 28, 29, and 33 bushels; how many bushels did he sell?

2. Mr. Mason bought 5 hogs, weighing, severally, 375, 358, 416, 410, and 400 pounds; how many pounds did all weigh?

3. Mr. Thomson's wagon weighed 2097 pounds, and the load of hay on the wagon, 1988 pounds; what was the weight of both?

4. Mr. Wilson sold 6 fat oxen, weighing, severally, 907, 1216, 1189, 1075, 899, and 934 pounds; what was the weight of all?

5. How many strokes does a clock, which strikes the hours, strike in 12 hours.

6. How many days in a year, there being in January 31 days; in February 28; in March 31; in April 30; in May 31; in June 30; in July 31; in August 31; in September 30; in October 31; in November 30; and in December 31?

7. Mr. Johnson bought a farm with the buildings and stock upon it, paying 5000 dollars for the land, 2500 for the house, 975 for the barn, 507 for the other buildings, and 1650 for the stock and farming tools; how many dollars did all these things cost him?

8. Mr. Jackson paid 4150 dollars for land, 2000 dollars for a house, 725 dollars for a barn, 609 dollars for other buildings, and 1200 dollars for stock and tools; what was the whole cost?

9. Mr. Jameson paid 10000 dollars for a factory, 5967 for land, 8096 for cotton, 4870 for labor, and 908 for teaming; how many dollars do these sums amount to?

10. What is the sum of all the numbers that you speak in counting one hundred?

11. How many square miles in the New England States, there being in Maine 35000; in New Hampshire 9491; in Vermont 8000; in Massachusetts 7800; in Rhode Island 1225; and in Connecticut 4764?

12. How many square miles in the Middle States, there being in New York 46085; in New Jersey 8320; in Pennsylvania 47000; and in Delaware 2100?

13. How many square miles in the Southern States, there being in Maryland 9356; in Virginia 70000; in North Carolina 50000; in South Carolina 33000; in Georgia 62000; in Alabama 51770; in Mississippi 48000; and in Louisiana 48320?

14. How many square miles in the Western States, there being in Tennessee 45000; in Kentucky 40000; in Ohio 44000; in Indiana 36400; in Illinois 55000; in Michigan 60000; in Missouri 64000; and in Arkansas 55000?

15. How many square miles in the 26 states, mentioned in the last four problems?

16. If the time from the creation of the world to the deluge was 1656 years, thence to the building of Solomon's temple 1344 years, thence to the birth of Christ, 1004 years; how old is the world in the year of our Lord 1846?

17. How long since the deluge?

18. How old was the world at the birth of Christ?

19. How long since the building of Rome, which was 753 years before Christ?

20. How long since Lycurgus established his laws at Lacedæmon, which was 131 years before the building of Rome?

21. How many miles from Augusta in Maine, to New Orleans in Louisiana, it being from Augusta to Portland 53 miles, thence to Boston 118 miles, to Hartford 160, to New York 123, to Philadelphia 90, to Baltimore 100, to Washington 38, to Richmond 123, to Raleigh 165, to Charleston 265, to Savannah 113, to Talahassee 331, to Mobile 320, and to New Orleans 160?

22. How far from Natches in Mississippi, to Boston in Massachusetts, it being to Tuscaloosa 350 miles, to Nashville 230, to Louisville 210, to Cincinnati 110, to Wheeling 230, to Pittsburg 115, to Buffalo 160, to Albany 360, and to Boston 150?

III. MULTIPLICATION.

24. Multiplication Defined and Illustrated.

Multiplication is the producing of a number equal to as many times one given number as there are units in another given number. Thus:

In 1 bushel are 32 quarts. How many quarts in 8 bushels? Since there are 32 quarts in one bushel, in 8 bushels there are 8 times 32 quarts, the amount of which may be ascertained by addition. But when the amount of several times the *same* number is to be ascertained, it can be done by a *shorter process*. Instead of writing the 32 quarts 8 times, as in the 1st operation, write it only once, as in the 2d operation, and under it write 8, to show how many times the 32 should be taken. Then 8 times 2 units, (which is the same in amount as the eight 2's of the units' column in the 1st operation,) are 16 units, equal (**10**) to 6 units, which write in the units' place, and 1 ten, which reserve to join with the other tens. 8 times 3 tens, (which is the same in amount as the eight 3's of the tens' column in the 1st operation,) are 24 tens, and the 1 ten, which was obtained from the units, are 25 tens, equal to 5 tens, which write in the tens' place, and 2 hundreds, which write in the hundreds' place; and the amount, 256 quarts, is obtained as before.

1st Operation.	2d Operation.
32	32
32	8
32	——
32	256 quarts,
32	
32	
32	
32	
——	
256 quarts,	

25. Definitions of Terms, and the Sign for Multiplication.

A product is a number produced by multiplication.

A multiplicand is a number to be multiplied.

A multiplier is a number showing how many times a multiplicand is taken to form a product.

Thus, the 32 in the above example, is the multiplicand, 8 the multiplier, and 256 is the product.

The multiplicand and multiplier are also called **producers**, or *factors* of their product.

Thus, 32 and 8 are *factors* of 256.

× *The oblique cross* is the sign for *multiplication*. It

implies that the number which precedes the sign, is to be multiplied by the number which follows it. Thus, $32 \times 8 = 256$, which is read, 32 multiplied by 8 equals 256, or 8 times 32 equals 256.

26. MULTIPLICATION TABLE.

In order to perform multiplication with facility, you will, before attempting further progress, correctly ascertain, and thoroughly commit to memory, the *product* of each combination of two factors in the following table.

2 times 2 equal	7×3, or $3 \times 7 =$	8×5, or $5 \times 8 =$
3×2, or $2 \times 3 =$	8×3, or $3 \times 8 =$	9×5, or $5 \times 9 =$
4×2, or $2 \times 4 =$	9×3, or $3 \times 9 =$	6 times 6 equal
5×2, or $2 \times 5 =$	4 times 4 equal	7×6, or $6 \times 7 =$
6×2, or $2 \times 6 =$	5×4, or $4 \times 5 =$	8×6, or $6 \times 8 =$
7×2, or $2 \times 7 =$	6×4, or $4 \times 6 =$	9×6, or $6 \times 9 =$
8×2, or $2 \times 8 =$	7×4, or $4 \times 7 =$	7 times 7 equal
9×2, or $2 \times 9 =$	8×4, or $4 \times 8 =$	8×7, or $7 \times 8 =$
3 times 3 equal	9×4, or $4 \times 9 =$	9×7, or $7 \times 9 =$
4×3, or $3 \times 4 =$	5 times 5 equal	8 times 8 equal
5×3, or $3 \times 5 =$	6×5, or $5 \times 6 =$	9×8, or $8 \times 9 =$
6×3, or $3 \times 6 =$	7×5, or $5 \times 7 =$	9 times 9 equal

27. MODEL OF A RECITATION.

1. In one hogshead are 63 gallons. How many gallons in 9 hogsheads?

Since there are 63 gallons in 1 hogshead, in 9 hogsheads there are 9 times 63 gallons, which is obtained by multiplying 63 by 9. Thus, 9 times 3 units are 27 units, equal to 7 units, which write, and 2 tens, which add with the tens; 9 times 6 tens, and the 2 tens obtained from the units, are 56 tens, equal to 6 tens, which write, and 5 hundreds, which write also, giving 567 gallons for the answer.

$$\begin{array}{r} 63 \text{ gallons,} \\ 9 \phantom{\text{ gallons,}} \\ \hline 567 \text{ gallons,} \end{array}$$

28. EXERCISES IN MULTIPLYING WHEN THE MULTIPLIER CONSISTS OF BUT ONE FIGURE.

In like manner, solve and explain the following problems.

1. How many gallons in 3 hogsheads?
2. How many gallons in 7 hogsheads?
3. In 1 hour are 60 minutes. How many minutes in 5 hours?

4. How many minutes in 8 hours?

5. If 100 cents equal 1 dollar, how many cents are equal to 6 dollars?

6. How many cents in 4 dollars?

7. How many cents in 9 dollars?

8. In 1 mile are 320 rods. How many rods in 7 miles?

9. How many rods in 5 miles?

10. If 2240 pounds of cotton load 1 car, how many pounds will load a train of 8 cars?

11. How many pounds will load 7 cars?

12. If it require 40000 inhabitants to send 1 representative to Congress, how many inhabitants in a state which sends nine representatives?

13. If 30000 persons in a year die of drunkenness, how many will die drunkards in the next 5 years, unless people become more temperate?

14. If each of these drunkards makes seven persons unhappy, how many will thus be made unhappy in the next 5 years by their drunkenness?

15. If sound move 1142 feet in a second, how far off is the thunder, when 6 seconds elapse between seeing the lightning and hearing the thunder?

16. If the salary of the president be 25000 dollars a year, how much has been paid to each of the presidents?

17. What is the product of 2796 multiplied by 4?

18. Multiply 675 by 5.

19. How many are 1789×7?

29. Composite and Prime Numbers.

1. How many trees in an orchard, which has 15 rows of 18 trees each?

Since there are 18 trees in 1 row, in 15 rows there will be 15 times 18 trees; which is obtained by multiplying 18 by 15. This multiplier, consisting of two figures, presents a difficulty which, however, you can obviate by obtaining a part of the product at a time; thus:—Since in 15 rows there are 3 times 5 rows, multiply 18 by 5 for the trees in 5 rows, and this product by 3 for the trees in 3 times 5, or 15 rows, which will give the answer required.

18 trees in 1 row.
 5
90 trees in 5 rows.
 3
270 trees in 3 times 5, or 15 rows.

A number which is composed of two, or more factors, as $15 = 5 \times 3$, or $42 = 7 \times 3 \times 2$, &c., is called a *composite number*.

A *prime number* is a number which has no factors, except itself and unity; as 1, 3, 5, 7, 11, 13, 17, 19, 23, 29, &c.

Hence OBSERVE, *that, when the multiplier is a composite number, the product may be obtained, as in the last example, by separating the multiplier into two, or more factors, and multiplying first by one factor, then* that product *by another factor, and so on, until all the factors have been used. The last product will be the product required.*

30. MODEL OF A RECITATION.

If 1 gallon of molasses costs 42 cents, what will be the cost of 1 hogshead at the same rate?

Since 1 gallon costs 42 cents, 1 hogshead, which is 63 gallons, will cost 63 times 42 cents. This is obtained by separating the multiplier into its factors, $9 \times 7 = 63$, and multiplying first by 9, to obtain the cost of 9 gallons, and this product by 7, to obtain the cost of 7 times 9 gallons, or 63 gallons; which is 2646 cents, the answer required.

```
  42 cents, the cost of 1 gallon.
   9
 ----
 378 cents, the cost of 9 gallons.
   7
 ----
2646 cents, the cost of 63 gallons.
```

31. EXERCISES IN MULTIPLYING BY COMPOSITE NUMBERS.

In like manner, solve and explain the following problems.

1. How many gallons in 35 hogsheads?
2. How many gallons in 45 hogsheads?
3. How many rods in 21 miles, there being 320 in one mile?
4. How many minutes in 18 hours?
5. How many cents in 42 dollars?
6. How many rods in 63 miles?
7. What would 28 bales of cotton come to, at 75 dollars a bale?
8. What would 16 chests of tea cost, at 87 dollars a chest?
9. What would be the cost of a drove of 56 horses, at 84 dollars a piece?

32. MODEL OF A RECITATION.

How many are 24 times 27?

Since $6 \times 4 = 24$, first obtain 6 times the multiplicand, which is 162, then 4 times this product, making 648, which is 4 times 6 times, or 24 times the multiplicand, as required.

$$\begin{array}{rl} 27 & \\ 6 & \\ \hline 162 & = 6 \text{ times } 27. \\ 4 & \\ \hline 648 & = 4 \text{ times } 6 \text{ times, or } 24 \text{ times } 27. \end{array}$$

33. Exercises in Multiplying when both Factors are Abstract Numbers.

In like manner, solve and explain the following problems.

1. What is 81 times 47?
2. What is 48 times 70?
3. How much is 79×54?
4. Multiply 123 by 72.
5. Multiply 4004 by 64.
6. Multiply 50000 by 35.
7. Multiply 908070 by 45.
8. Multiply 18273645 by 36.

34. Model of a Recitation.

1. What would 10 cows cost, at 25 dollars each?

Since 1 cow costs 25 dollars, 10 cows cost 10 times 25 dollars; the amount of which is ascertained by annexing a cipher to the multiplicand, making 250 dollars; for now the figures of the multiplicand, occupying places one degree higher, express 10 times their former value, (**4.**)

2. If 128 dollars be paid to each of 1000 men, how many dollars would they all receive?

Since 1 man would receive 128 dollars, 1000 men would receive 1000 times 128 dollars; the amount of which is ascertained by annexing three ciphers to the multiplicand, making 128000 dollars; for thus, the figures of the multiplicand are made to occupy places three degrees higher, and, consequently, (**4, 10,**) express 1000 times their former value.

35. Exercises in Multiplying by one Unit of any Order.

In like manner, solve and explain the following problems.

1. What will 10 yards of cloth cost, at 5 dollars a yard?
2. What must I pay for 100 sheep, at 7 dollars apiece?
3. What would be the cost of a rail-road 100 miles in length, at 5796 dollars a mile?
4. What would be the price of 10000 feet of boards at 2 cents a foot?

5. What is the stage fare for 1000 miles, at 5 cents a mile?

6. Multiply 161 by 10.
7. What is 100 times 1728?
8. How many are 18×1000?
9. What is the product of 125 and 1000?
10. Multiply 200 by 100.
11. Multiply 5000 by 100000.
12. How many are 1020×1000?
13. Multiply 1000 by 1000.

36. Model of a Recitation.

A farmer raised 84 bushels of potatoes on each of 40 acres; what was the whole number of bushels?

Since 84 bushels were raised on 1 acre, on 40 acres there were 40 times 84 bushels raised. The multiplier being a composite number, (**29**,) whose factors are 4 and 10, multiply first by 4, to ascertain the bushels on 4 acres, then multiply that product by 10, which is done by annexing a cipher, (**4**,) to ascertain the bushels raised on 10 times 4, or 40 acres, which gives 3360 bushels, as required.

```
  84 bushels on 1 acre.
  40
 ----
3360 bushels on 40 acres.
```

37. Exercises in Multiplying by any Number of Units of the same Order.

In like manner, solve and explain the following problems.

1. What will 30 barrels of flour come to, at 7 dollars a barrel?

2. If it take 20 men 6 days to do a job, how long would it take 1 man to do it?

3. If 320 rods make a mile, how many rods in 500 miles?

4. How long would it take 1 man to do what 40 men could do in 2 days?

5. How much is 300 times 125?
6. What is the product of 72 and 900?
7. Multiply 1836 by 6000.
8. How much is 700×700?
9. Multiply 2500 by 2500.

38. Model of a Recitation.

1. In 1 quart are 2 pints. How many pints in 67 quarts?

Since there are 2 pints in 1 quart, in 67 quarts there will be 67 times 2 pints; the amount of which may be ascertained by multiplying 2 pints by 67. But the multiplier, 67, being a prime number, (**29**,) presents a difficulty. This, however, you can obviate by *taking a different view of the question.* Thus, since there are 2 pints in 1 quart, *there will be* 2 *times*

67 2 —— 134 pints.	*as many pints as quarts*, the amount of which may be ascertained by multiplying 67 by 2, making 134 pints, which is the answer required.

39. Exercises in Changing the Order of the Factors for Multiplication.

In like manner, solve and explain the following problems.

1. How many pints in 29 quarts?
2. What is the price of a bushel of nuts, at 6 cents a quart?
3. What must I give for 15 lemons, at 4 cents apiece?
4. If a man plant 6 grains of corn in a hill, how many grains will it take to plant a field having 75 rows of 100 hills each?
5. What would an ox, weighing 873 pounds, come to, at 10 cents a pound?
6. If 27 men receive 100 dollars apiece, how much do they all receive?
7. If 100 cents make a dollar, how many cents in 47 dollars?
8. If 1000 mills make a dollar, how many mills in 71 dollars?
9. How many cents in 53 dollars?
10. How many mills in 53 dollars?
11. If a man earn 10 dollars a week, how much would he earn in a year, which is 52 weeks?
12. If 2 men thresh 20 bushels of rye in a day, how much would they thresh in 23 days?
13. How many soldiers in a brigade, which consists of 32 companies of 60 soldiers each?
14. How many pounds of beef in 13 barrels of 200 pounds each?
15. How many squares on a chequer-board, there being 8 rows of squares, and 8 squares in each row?
16. If you draw straight lines across your slate, both ways, so as to make 8 rows of squares one way, and 12 rows the other way, how many squares would there be?
17. How many trees in an orchard which has 41 rows of 40 trees each?

40. Proof of Multiplication.

You may, perhaps, infer, (**26, 38**,) that *the product of two factors is the same, whichsoever be made the multiplier.*

This is true; and to make it still more evident, you will carefully attend, to the following demonstrations.

$$\begin{array}{c} 3 = 1 + 1 + 1 \\ 7 \qquad\qquad 7 \\ \hline 7 \text{ times } 3 = 21 = 7 + 7 + 7 = 3 \text{ times } 7. \end{array}$$

Explanation: 7 times 3 is the same as 7 times each unit in 3. The units in 3 are 1 + 1 + 1, which multiplied by 7 give 7 + 7 + 7 = 3 times 7.

Generally—the product of two factors is *as many times* the multiplicand as there are units in the multiplier, (24,) and in multiplying we multiply *each unit in the multiplicand*. But multiplying *one unit*, gives the *multiplier*. Consequently, multiplying *each unit* in the multiplicand, *will give as many times the multiplier as there are units in the multiplicand*.

Hence, *to prove* the correctness of an operation in multiplication, *make the multiplicand the multiplier, and repeat the operation*. If the results agree, probably both are correct.

41. Exercises in Proving Multiplication.

1. Prove that 5 times 3 is equal to 3 times 5.
2. Prove that 6 times 5 must be equal to 5 times 6.
3. Demonstrate the equality of 6×3 and 3×6.
4. How many hills in a potato-field having 20 rows lengthwise, and 16 rows breadthwise?
5. How many hills in a cornfield having 50 hills one way, and 25 hills the other way?
6. Demonstrate that the *product of any two factors* will not be changed by changing the *order* of the factors.

42. General Explanation of Multiplication.

1. Mr. Farmer gave 67 dollars an acre for a farm of 222 acres. What did his farm cost?

Since 1 acre cost 67 dollars, 222 acres must have cost 222 times 67 dollars. Neither of these numbers can be separated into convenient factors. But observe that the multiplier, 222 = 200 + 20 + 2. Hence, you may multiply by these parts of the multiplier, separately, and then add the three products. This will give as many times the multiplicand as there are units in the multiplier, (24,) and consequently, the right answer to the question. Thus:

67 dolls. cost of 1 acre. 200 13400 dolls. cost of 200 acres.	67 dolls. cost of 1 acre. 20 1340 dolls. cost of 20 acres.
	67 dolls. cost of 1 acre. 2 134 dolls. cost of 2 acres.

134 dolls. cost of 2 acres.
1340 dolls. cost of 20 acres.
13400 dolls. cost of 200 acres.

14874 dolls. cost of 222 acres, the answer required.

This operation may be very much abridged. Thus:

```
  67
 222
 ---
 134
134
134
-----
14874
```

Having written the *whole* multiplier under the multiplicand, multiply first by the 2 units, then by the 2 tens, or 20, and then by the 2 hundreds, or 200, arranging the products together, *so that the units of the same orders may stand in the same columns.* Multiply by the 2 units, as usual. The factors of 20 being 2 and 10, multiply by the 2, and make this product 10 times as large, (**29**,) by writing it one degree to the left, (**4**.) The factors of 200 being 2 and 100, multiply by 2, and make this product 100 times as large, by writing it two degrees to the left, (**34**.) The sum of these products will be the answer required.

2. Multiply 20003 by 1007.

```
   20003
    1007
  ------
  140021
 20003
--------
20143021
```

First take the multiplicand 7 times, then 1 thousand times. To multiply by the 1000, write once the multiplicand, three degrees to the left, (**34**.) The three ciphers need not be annexed; for, without them, each figure of this product will be in the column of units of its own order, and therefore will be added in the right place.

43. Model of a Recitation.

How much is 30508 times 403070?

```
     403070
      30508
 ----------
    3224560
  2015350
1209210
-----------
12296859560
```

Here the multiplicand is to be taken 8 times, 500 times, and 30000 times. Multiply by the 8. The factors of 500 being 5 and 100, multiply by 5, and make this product 100 times as large, by writing it up two degrees. The factors of 30000 being 3 and 10000, multiply by 3, and make this product 10000 times as large, by writing it up four degrees. The

sum of these partial products, thus arranged, will be the product required.

44. OBSERVATION.

OBSERVE, *that in these operations* (**42, 43**) *the multiplicand is multiplied by each digit in the multiplier, that the first figure in each partial product is of the same denomination as the multiplying figure, and that the sum of the partial products is the product required.*

45. EXERCISES IN MULTIPLICATION.

In like manner, solve and explain the following problems.

1. A man travelled 26 days, at the rate of 47 miles a day. How far did he travel?
2. If a chaise wheel turn round 346 times in 1 mile, how many times will it revolve in the 25 miles from Boston to Lowell?
3. How much money would be required to pay 37 men 75 dollars apiece?
4. What must I pay for 29 fat oxen, at 43 dollars apiece?
5. What will 97 tons of iron come to, at 57 dollars a ton?
6. If a vessel sail 158 miles a day, how far would it sail in the month of April?
7. If 786 yards of cloth are made, daily, in a factory which runs 313 days a year, what is made annually in that factory?
8. How much wheat can be raised on 95 acres, at 38 bushels an acre?
9. Multiply 1728 by 144.
10. How much is 4004 times 999?
11. What is the product of 6075 and 67?
12. How much is 160012 $\times$ 333?
13. Multiply 1836 by 1010.
14. Multiply 1111 by 2222.
15. Multiply 2222 by 1111.
16. Multiply 3000024 by 309.
17. Multiply 309 by 3000024.

46. MODEL OF A RECITATION.

What is the product of 32000 and 2300?

```
32000
2300
-----
 96
64
-------
73600000
```

The factors of 2300 being 23 and 100, multiply by 23, placing it under the digits of the multiplicand, and multiplying without regard to the ciphers on the right. This gives 736. But, since 23 times 32 units of *any order* will be 736 units of the *same order*, as surely as 23 times 32 things of *any kind* will give 736 things of the *same kind;* 23 times 32 *thousands* will be 736 *thousands.* Therefore annex three ciphers, (**34**,) that it may have the thousands' place; then annex two ciphers more, (**30**,) to multiply by the other factor in the multiplier; which gives the product required.

47. Observation.

Observe, *that, by this process,* (**46**,) *as many ciphers will be annexed to the product of the digits as there are on the right of both factors.*

48. Exercises in Multiplying, when the Factors express Units of the higher Orders.

In like manner, solve and explain the following problems.

1. How far is it from Boston to Liverpool, if a vessel sail from Boston at the rate of 150 miles a day, and arrive at Liverpool in 20 days?
2. How far from the earth to the sun, if it take light 480 seconds to come from the sun, at 200000 miles a second?
3. What is the capital of Boston Bank, there being 12000 shares, at 50 dollars a share?
4. What is the capital of Massachusetts Bank, there being 3200 shares, at 250 dollars a share?
5. State Bank has 30000 shares, at 60 dollars each. What is its capital?
6. If Massachusetts' house of representatives has 500 members, and a session lasts 90 days; how much money would it take to pay 2 dollars a day to each member?
7. How many weekly newspapers will it require to furnish 30000 subscribers one year?
8. What would be the cost of a railroad, 40 miles in length, at 40000 dollars a mile?
9. Multiply 740 by 6050.
10. Multiply 6050 by 740.

.. How many are 3400 times 390?
12. How many are 390 times 3400?
13. What is the product of 140 multiplied by 140?
14. Multiply 1600 by itself.
15. 55500 × 4400 is how much?
16. 1910 × 170 are how many?
17. If the multiplicand be 160000, and the multiplier 2400, what will be the product?
18. 121212 × 8080 = ?

49. General Exercises in Addition and Multiplication.

1. How many months was Andrew Jackson president?
2. How many months was John Quincy Adams president?
3. How many pounds of pork on 150 wagons, each loaded with 6 barrels, with 200 pounds in a barrel?
4. If a house have 20 windows, of 24 panes each, how many panes in all the windows?
5. What number is 9000 times 165?
6. What number contains 144 twelve times?
7. What number contains one thousand and fifteen 607 times?
8. What would be the sum of 457 set down ten thousand times, and added up?
9. What is the cost of a road 40 miles long, of which one half cost 1750 dollars a mile, and the other half, 1800 dollars a mile?
10. If a quantity of provisions would last 500 men 30 days, how long would it last 1 man?
11. How many men would consume in 1 day what would last 500 men 30 days?
12. If a bushel of wheat afford 70 ten-cent loaves, how many cent loaves may be obtained from it?
13. How many yards of cloth, 1 quarter wide, are equal to 27 yards 5 quarters wide?
14. How long would it take a man, working 1 hour a day, to do what he could in 26 days, working 12 hours a day?
15. If a boy attend school constantly 3 terms of 12 weeks, and 1 term of 11 weeks; how many hours is he in school, at 33 hours a week?
16. How many strokes will the city clock strike in the month of June?
17. If it take 594 bricks to pave 1 rod of side-walk, how many would it take to pave a walk a mile long?

18. What are a man's annual expenses, who pays 3 dollars a week for board, 6 dollars a month for clothes, 10 dollars a quarter for travelling expenses, 1 dollar a week for benevolent purposes, and for other items 75 dollars ?

19. What is a man's income, who receives a salary of 15 dollars a week, and 10 dollars a month interest money ?

20. What is the value of a drove of cattle, consisting of 12 oxen at 55 dollars apiece, 15 cows at 30 dollars apiece, 18 heifers at 16 dollars apiece, and 14 yearlings at 10 dollars apiece ?

21. What is the amount of the following bill ?

Boston, April 25, 1846.

Mr. John Merchant,

Bought of Charles Wholesale,

27	yards of Black Broadcloth,	at	$6	a yard,
25	" Blue	"	" 7	"
18	" Drab Cassimere,	"	3	"
24	Vest Patterns,	"	2	a pattern,

Received payment,

CHARLES WHOLESALE.

22. What is the foot of the following bill ?

Hanks, Harris & Co. Boston, April 27, 1846.

Bought of Burt & Townsend,

1200	pairs Boys' Shoes,	@ $1 per pair,	. .
400	" Men's "	@ 2 "	. .
600	" " Boots,	@ 3 "	. .

23. What is the foot of the following account ?

Mr. Isaac Speculator,

1846.		To Jonathan Farmer,	Dr.
Jan.	31.	To 17 Cords Wood, @ $7 per cord,	
Aug.	1.	" 9 Tons Hay, @ 15 " ton,	
Oct.	12.	" 10 Loads Potatoes, @ 8 " load,	
"	15.	" 18 Barrels Apples, @ 2 " barrel,	

24. How many scholars can a school-room accommodate, in which are 4 divisions of seats, 11 rows of seats in each division, and 6 seats in a row ?

25. How many are $4 \times 11 \times 6$?

26. How many seats in a church, in which the body pews are in 4 rows of 18 pews each, the wall pews in 2 rows of 24 pews each, and the gallery pews in 12 rows of 4 pews each, there being 6 seats in each pew?

27. How many shingles will cover the roof of a house, each of the two sides being 32 feet long and 16 feet wide; if it take 3 shingles to extend a foot in each direction?

28. What is the product of $32 \times 3 \times 16 \times 3 \times 2$?

29. If the earth move in its orbit 68000 miles an hour, how far does it move in 24 hours?

30. How far in its orbit does the earth move in the month of February?

31. How far does the earth move in the 4 months which have 30 days each?

32. How far does the earth move in the 7 months which have 31 days each?

33. How many miles does the earth move in a year, as shown in the last three problems?

34. If the moon is 240000 miles distant, and the sun is 400 times as far off, what is the distance of the sun?

35. What number is that whose factors are 3, 5, 7?

36. What is the product of the first ten prime numbers?

37. What sum of money must be divided among 27 men, so that each man may receive 115 dollars?

38. Two men depart, in opposite directions, from the same place, one at the rate of 27, and the other 31 miles a day. How far are they apart in a week?

39. Two men depart, in the same direction, from the same place; but one travels 10 miles a day farther than the other. How far apart are they in a week?

40. The product of *two equal factors* being called the *second power*, or square of that repeated factor, what is the second power of 12? Ans. $12 \times 12 = 144$.

41. What is the second power of 15?

42. What is the second power of 30?

43. What is the square of 50?

44. What is the square of 100?

45. The product of *three equal factors* being called the *third power*, or cube of that repeated factor, what is the cube of 12? Ans. $12 \times 12 \times 12 = 1728$.

46. What is the cube of 15?
47. What is the third power of 9?
48. What is third power of 25?
49. The product of *four equal factors* being called the *fourth power* of that repeated factor, what is the fourth power of 3? Answer, $3 \times 3 \times 3 \times 3 = 81$.
50. What is the fourth power of 5?
51. Any number being the first power of itself, what are the first ten powers of 2?
52. What are the first ten powers of 10?
53. Multiply 144 by the third power of 10.
54. Multiply 18 by the fifth power of 10.
55. Multiply 500 by the second power of 10.

IV. SUBTRACTION.

50. The Principles of Subtraction Illustrated.

In Numeration (2) you were taught that the addition of one unit to any number, formed the next larger number.

Hence, it follows that taking one unit from any number, leaves the next smaller number.

In Addition you were taught that two or more numbers, consisting of any number of units, could be united into one larger number, equal to their sum.

Hence, it follows that *any number can be separated into two or more smaller numbers, the sum of which equals the original number.*

The father of John and Henry promised to give them 10 cents; but, as John was the older boy, he should have 7 cents, and Henry might have the remainder of them.

Henry, in trying to make his part as many as possible, studied out these curious questions.

1. *How many will remain*, when John has taken 7 from the 10 cents?
2. *How many more* are the whole 10, than John's 7 cents?
3. *How many less* than the whole 10, are John's 7 cents?
4. *How many must be added* to John's 7, to make the whole 10 cents?
5. *How many must I take from* the whole 10, to leave John's 7 cents?

6. *What is the difference* between John's part, and the whole 10 cents?

7. *What is the difference* between the whole 10 cents, and John's part?

8. If 10 cents are separated into two parts, one of which is 7, *what is the other part?*

But he found that, to answer all his questions, he had only *to take* 7 *from* 10, and that, in every case, only 3 *cents remained* for his part.

51. Definitions of Terms, and the Sign for Subtraction.

Subtraction is the *taking from* a number.

Minuend is a given number *to be diminished* by subtraction.

Subtrahend is a given number *to be subtracted.*

By subtraction the minuend is separated into two parts, one of which equals the subtrahend.

To ascertain the other part, is the purpose of the operation. This is done by taking the subtrahend from the minuend. The number which is left is the part required, and is called the *Remainder*. It is the *Difference* between the minuend and subtrahend.

Observe, in the questions above, (**50**,) that 10 is the given number to be separated into two parts, and, therefore, is the *Minuend;* that 7 is the given part of the minuend, and, therefore, is the *Subtrahend;* that 3 is the other part, or *Remainder* of the minuend, and, that the two parts of the minuend, $7 + 3 = 10$, the whole minuend.

Observe also, in these questions, the different *uses* of subtraction.

— *One horizontal line* is the *sign for subtraction.* It implies that the number which follows the sign, is to be taken from what precedes it, thus: $10 - 7 = 3$, which is read, 10 *minus* 7 equals 3. *Minus* is the Latin word for *less*, and, here, means the same as if you should say 10 *less* 7, or 7 *less than* 10 equals 3. 10 is the minuend, 7 the subtrahend; and 3 is the difference.

52. Subtraction Table.

In order to perform subtraction with facility, you will, before attempting further progress, correctly ascertain, and

thoroughly commit to memory, the *difference* between the two numbers of each combination in the following table.

2 — 2 =	3 — 3 =	4 — 4 =	5 — 5 =
3 — 2 =	4 — 3 =	5 — 4 =	6 — 5 =
4 — 2 =	5 — 3 =	6 — 4 =	7 — 5 =
5 — 2 =	6 — 3 =	7 — 4 =	8 — 5 =
6 — 2 =	7 — 3 =	8 — 4 =	9 — 5 =
7 — 2 =	8 — 3 =	9 — 4 =	10 — 5 =
8 — 2 =	9 — 3 =	10 — 4 =	11 — 5 =
9 — 2 =	10 — 3 =	11 — 4 =	12 — 5 =
10 — 2 =	11 — 3 =	12 — 4 =	13 — 5 =
11 — 2 =	12 — 3 =	13 — 4 =	14 — 5 =

6 — 6 =	7 — 7 =	8 — 8 =	9 — 9 =
7 — 6 =	8 — 7 =	9 — 8 =	10 — 9 =
8 — 6 =	9 — 7 =	10 — 8 =	11 — 9 =
9 — 6 =	10 — 7 =	11 — 8 =	12 — 9 =
10 — 6 =	11 — 7 =	12 — 8 =	13 — 9 =
11 — 6 =	12 — 7 =	13 — 8 =	14 — 9 =
12 — 6 =	13 — 7 =	14 — 8 =	15 — 9 =
13 — 6 =	14 — 7 =	15 — 8 =	16 — 9 =
14 — 6 =	15 — 7 =	16 — 8 =	17 — 9 =
15 — 6 =	16 — 7 =	17 — 8 =	18 — 9 =

53. MODEL OF A RECITATION.

1. A man bought a farm for 2325 dollars, and sold it for 2548 dollars. How many dollars did he gain?

He gained the difference between what he gave, and what he received for his farm.

Here, 2548 is the minuend, (as the *larger* of the two given numbers, when there is any difference between them, *is always the minuend*,) and 2325 is the subtrahend. It will be most convenient to take the units of each order from units of the same order, beginning with the lowest. Therefore, write the subtrahend under the minuend, placing the units of each order under those of the same order. Take 5 units from 8 units, and 3 units remain, which write in the units' place; 2 tens from 4 tens, 2 tens remain, which write in the tens' place; 3 hundreds from 5 hundreds, 2 hundreds remain, which write in the hundreds'

```
2548 Minuend.
2325 Subtrahend.
————
 223 Remainder.
```

place; and 2 thousands from 2 thousands, nothing remains. Consequently, 223 dollars is the answer required.

54. PROOF OF SUBTRACTION.

To *prove* the correctness of this, or any operation in subtraction, add together the remainder and subtrahend. If this sum agree with the minuend, probably the operation is correct; for the remainder and subtrahend, being the two parts into which the minuend is separated, the reünion of these parts *ought* to reproduce the minuend.

55. EXERCISES IN SUBTRACTING WHEN NO FIGURE OF THE SUBTRAHEND EXCEEDS THE CORRESPONDING FIGURE OF THE MINUEND.

In like manner, solve and explain the following problems.

1. Charles having 25 cents, gave 12 of them for a book. How many cents had he left?
2. Charles paid 25 cents for a book and slate, 13 cents was the price of the slate, what was the price of the book?
3. John said he was 25 years younger than his father, who was 37 years old. How old was the boy?
4. A merchant 35 years old, had traded 14 years. How old was he when he commenced business?
5. In a school of 84 scholars, only 33 are girls. How many boys in that school?
6. A man sold a chaise and harness for 198 dollars; but the price of the chaise was 163 dollars. What was the price of the harness?
7. A house and the land on which it stood cost 2350 dollars; but the house cost all but 350 dollars. What was the cost of the house?
8. If I deposite in a bank 1675 dollars, and afterwards draw out 1000, how much have I then remaining in the bank?
9. Mr. Walker's farm is worth 3000 dollars, and Mr. Dole's farm is worth 2000 dollars; if they exchange farms, what should Mr. Dole pay Mr. Walker?
10. What is the difference between 5643 and 643?
11. How much more is 12345 than 2040?
12. How much less is 1620 than 1840?
13. Subtract 203040 from 516273.

56. MODEL OF A RECITATION.

1. A man paid 85 dollars for a watch; but was obliged to sell it for 67 dollars. What was his loss?

He lost the difference between what he gave, and what he received for his watch.

$$\begin{array}{r} 85 \\ 67 \\ \hline 18 \end{array}$$

Arrange the numbers and proceed as before directed. 7 units, however, cannot be taken from 5 units. But, since (**10**) 1 unit of any order, equals 10 units of the next lower order, reduce (**208**) one of the 8 tens to units, making 10 units, which, united with the 5 units, make 15 units, from which take the 7 units, 8 units remain, which write in the units' place, and take the 6 tens, *not* from 8 tens, for one of them has been reduced to units and disposed of; but take 6 tens from 7 tens, 1 ten remains, which write in the tens' place. Hence, 18 dollars is the answer required.

2. What is the difference between 9342 and 5739?

$$\begin{array}{r} 9342 \\ 5739 \\ \hline 3603 \end{array}$$

Reduce one of the 4 tens to units, making ten units, which, united with the 2 units, make 12 units, from which take the 9 units, 3 units remain; take the 3 tens of the subtrahend from the other 3 tens of the minuend, nothing remains; therefore, write a cipher in the tens' place; reduce one of the 9 thousands to hundreds, making 10 hundreds, which, united with the 3 hundreds, make 13 hundreds, from which take the 7 hundreds, 6 hundreds remain; take the 5 thousands from the other 8 thousands, 3 thousands remain. Hence, the whole difference is 3603.

57. EXERCISES IN SUBTRACTING, WHEN SOME FIGURES OF THE SUBTRAHEND EXCEED THE CORRESPONDING FIGURES OF THE MINUEND.

In like manner, solve and explain the following problems.

1. A man gave 5 dollars for a hat, and 20 dollars for a coat. How much less did his hat cost than his coat?
2. Dr. Franklin died A. D. 1790, and was 84 years old. In what year was he born?
3. George Washington was born A. D. 1732, and died in 1799. How old was he when he died?
4. The Puritans landed at Plymouth in 1620. How many years since?

5. How long since Columbus discovered America in 1492?
6. How many years since the declaration of Independence by the United States in 1776?
7. The Rocky Mountains are 12500, and the Andes 21440 feet high; how much higher are the Andes than the Rocky Mountains?
8. The Mississippi river is 3600 miles long, and the Missouri river is 4500 miles long; how much longer is the latter than the former?
9. In Massachusetts are 7800 square miles, and in New Hampshire 9491; how much more land in New Hampshire than in Massachusetts?
10. How much larger is New York, which contains 46085 square miles, than Massachusetts, which has 7800 square miles?
11. Subtract 147 from 222.
12. From 671 take 584.
13. How much is 746—475?
14. What must be added to 999, to make 1492?
15. What must be subtracted from 1840, to leave 1776?

58. Model of a Recitation.

1. A man obtained at a bank, 300 dollars, but at the same time, he paid back 18 dollars for interest; how many dollars had he left?

He had left the difference between what he received and what he paid back, which is ascertained by subtracting 18 from 300.

Here there are no units from which to take the 8 units, neither is there any ten to reduce to units; therefore, reduce one of the 3 hundreds to tens, (**56**,) making 10 tens; leaving 9 of these tens, reduce the other to units, making 10 units, from which take the 8 units; 2 units remain. Take the 1 ten in the subtrahend, from those 9 tens that you left unused; 8 tens remain. There is nothing to take from the other 2 hundreds; therefore, write them in the hundreds' place. Hence, 282 dollars is the answer required.

$$\begin{array}{r} 300 \\ 18 \\ \hline 282 \end{array}$$

2. Subtract 30206, from 5000000.

5000000 30206 ——— 4969794	Reduce one of the 5 millions to hundred-thousands, making 10; one of which, (leaving 9,) reduce to ten-thousands, making 10; one of which, (leaving 9,) reduce to thousands, making 10; one of which, (leaving 9,) reduce to hundreds, making 10; one of which, (leaving 9,) reduce to tens, making 10; one of which, (leaving 9,) reduce

to units, making 10 units, from which subtract the 6 units; 4 units remain. Subtract the other figures of the subtrahend from the 9s that were left; saying, cipher from 9 tens leaves 9 tens; 2 hundreds from 9 hundreds leaves 7 hundreds; cipher from 9 thousands leaves 9 thousands; 3 ten-thousands from 9 ten-thousands leaves 6 ten-thousands; blank from 9 hundred-thousands leaves 9 hundred-thousands; and blank from 4 millions leaves 4 millions. Hence, the whole remainder is 4969794.

59. OBSERVATION.

OBSERVE, *in these operations, that the units of each order in the subtrahend, beginning with the lowest, are subtracted from the units of the same order, in the minuend, when possible; otherwise, one of the units expressed by the next higher digit in the minuend, is mentally reduced (leaving 9s in the intervening places) to the order of the deficient figure, and united with it, when the subtraction is made from what then remains in the several places of the minuend.*

60. GENERAL EXERCISES IN SUBTRACTION.

In like manner, solve and explain the following problems.

1. The top of a flag-staff, 25 feet long, which was fastened to the top of a liberty-pole, was 104 feet high; how high was the liberty-pole?

2. If 17 feet should be broken from the top of a tree, 100 feet high, how high would be the stump?

3. The bell on a church is 75 feet from the ground, but the vane is 102 feet from the ground; how many feet from the bell to the vane?

4. If the Creation was 4004 years B. C., and the Deluge 2348 years B. C., how many years from the Creation to the Deluge?

5. How many years from the Creation, 4004 years B. C. was Saul made the first king over Israel, in 1095, B. C.?

6. In 1820, New Orleans had 27176 inhabitants; in 1825, 35000 inhabitants; what was the increase in five years?

7. In A. D. 1825, New Orleans had 35000 inhabitants; in 1830, 46310; what was the increase in five years?

8. In A. D. 1830, New Orleans had 46310 inhabitants; in 1835, 60000; what was the increase in these five years?

9. In A. D. 1835, New Orleans had 60000 inhabitants; and Charleston, S. C., had 34500; how many more inhabitants in New Orleans, than in Charleston, S. C., in 1835?

10. In A. D. 1820, Philadelphia had 119325 inhabitants; in 1825, 140000; what was the increase in these five years?

11. In A. D. 1825, Philadelphia had 140000 inhabitants; in 1830, 167811; what was the increase in these five years?

12. In A. D. 1830, Philadelphia had 167811 inhabitants; in 1835, 200000; what was the increase in these five years?

13. In A. D. 1820, Boston had 43298 inhabitants; in 1825, 58277; what was the increase in these five years?

14. In A. D. 1825, Boston had 58277 inhabitants; in 1830, 61381; what was the increase in these five years?

15. In A. D. 1830, Boston had 61381 inhabitants; in 1835, 78613; what was the increase in these five years?

16. In A. D. 1820, New York city had 123706 inhabitants; in 1830, 203007; what was the increase in these ten years?

17. In A. D. 1835, New York city had 269873 inhabitants; Boston had 78613; how many more inhabitants had New York than Boston?

18. How much farther through the middle of the sun than through the middle of the earth; the former being 883217 miles, and the latter being 7916 miles?

19. What is the difference between the diameters of the Earth and Jupiter; the former being 7916 miles, and the latter 89170 miles?

20. How much faster does the Earth move than Jupiter; the former moving 68000 miles an hour, the latter 30000 miles an hour?

21. How much is 1000 — 999?

22. How much more is 380064 than 87065?

23. How much smaller is 8756 than 37005078?

24. How much must you add to 7643, to make 16487?

25. How much must you subtract from 2483, to leave 527?

26. What is the difference between 487068 and 24703?

27. If you divide 3880 dollars between two men, giving one 1907 dollars; how much will you give the other?

28. Subtract 2222 from 3111.

29. Subtract 9 from 1000.

30. Seven millions, minus seventeen, is how much?

V. DIVISION.

61. THE PRINCIPLES OF DIVISION ILLUSTRATED.

1. A butcher having 35 sheep, began Monday morning, and killed 5 every morning as long as they lasted; how many days did they last?

Since he killed 5 sheep each day, they would last as many days as there are times 5 sheep in 35 sheep.

After he had killed 5, Monday, 30 remained; Tuesday, 25 remained; Wednesday, 20 remained; Thursday, 15 remained; Friday, 10 remained; Saturday, 5 remained; and, Sunday, he killed the last 5; and none remained. Hence they lasted 7 days.

```
35
 5 Monday.
—
30
 5 Tuesday.
—
25
 5 Wednesday.
—
20
 5 Thursday.
—
15
 5 Friday.
—
10
 5 Saturday.
—
 5
 5 Sunday.
—
 0
```

But when it is to be ascertained how many times a given number can be subtracted from another given number, that is, how many times a subtrahend is *contained* in a minuend, it can be done by a *shorter* process than subtracting *once* the subtrahend at a time.

```
5)35(7 days.
  35
  —
  00
```

Write 35, the minuend; draw a line on each side, to distinguish it from the other numbers to be written with it, and at the left hand, write 5, the subtrahend. Now, *think* how many 5s there are in 35, and place the number at the right hand. To ascertain whether you thought the *right* number, subtract so many times 5 *all at once*. If there is *nothing* left, your number is *right*; for, if there are exactly 7 fives in 35, then the sum of 7 times 5, subtracted from 35, *should* leave nothing.

2. A teacher having 48 scholars studying arithmetic, separated them into classes of 12 scholars each; how many *classes* did he make?

Since he put 12 scholars into each class, he would make as many classes as there are times 12 scholars in 48 scholars.

```
12)48(4 classes.
   48
   —
   00
```

Write the 48; draw a line on each side; and write the 12 at the left hand. Now, how many 12s do you *think* there are in 48? Four 12s. Very well! Place the 4 at the right hand, and ascertain whether 4 such classes take exactly all of the 48 scholars.

3. A butcher killed 35 sheep in 7 days; how many would that be each day?

Killing *one* each day would require 7 sheep; therefore, he would kill as many each day, as he had times 7 sheep.

```
7)35(5 sheep a day.
  35
  —
  00
```

Write the 35, draw the lines, and write the 7 at the left hand, *think* how many 7s there are in 35, and place the number at the right hand. This number is the answer required, if 7 multiplied by it make exactly 35.

4. A teacher having 48 scholars studying arithmetic, separated them into 4 equal classes; how many could he put into each class?

Putting *one* into each class would require 4 scholars; therefore, he could put as many into each class, as he had times 4 scholars.

```
4)48(12 scholars a class.
  48
  —
  00
```

Arrange the two given numbers, *think* how many 4s there are in 48, and place the number at the right hand for the answer required. Then ascertain whether 4 multiplied by this number, take exactly all the scholars.

62. OBSERVATION.

OBSERVE, *in the first and second examples,* (**61,**) *that the purpose is to divide a number into equal parts of a* GIVEN SIZE, *to ascertain the* NUMBER *of such parts; but in the third and fourth, that the purpose is to divide a number into a*

GIVEN NUMBER *of equal parts, to ascertain the* SIZE *of such parts.*

Observe, also, that each of these purposes is effected by ascertaining HOW MANY TIMES ONE GIVEN NUMBER IS CONTAINED IN ANOTHER.

63. DEFINITION OF TERMS, AND THE SIGN FOR DIVISION.

Division is the separating of a number into equal parts of a *given size*, or into a *given number* of equal parts.

Dividend is a number *to be divided* into equal parts of a given size, or into a given number of equal parts.

Divisor is a number which expresses either the *size*, or *number* of the equal parts *to be made* of the dividend.

Quotient is the required number which must express either the *number*, or *size* of the equal parts *made* of the dividend.

$\div$ *A horizontal line between two dots*, is the sign for *division*. It implies, that what precedes the sign is to be divided by the number which follows it.

Thus; $35 \div 5 = 7$, which is read, 35 divided by 5 equals 7; or, 5 in 35, 7 times. Here 35 is the dividend, 5 the divisor, and 7 the quotient.

64. EXERCISES FOR ILLUSTRATING THE PRINCIPLES OF DIVISION.

Solve and explain the following problems on the left, like the first and second; and those on the right, like the third and fourth of the preceding examples, (**61.**)

1. If a man have 15 apples, to how many boys could he give 3 apples apiece?

2. A man gave 15 apples equally to 5 boys; how many would that be for each boy?

3. How many oranges, at 6 cents apiece, can you buy for 24 cents?

4. If you should pay 24 cents for 4 oranges, how much would they cost apiece?

5. How many apples, at 3 cents apiece, can you buy for 18 cents?

6. If 6 apples cost 18 cents, what is the cost of each apple?

7. How many barrels of flour, at 8 dollars a barrel, could you buy for 40 dollars?

8. If you should pay 40 dollars for 5 barrels of flour, what would be the price of each barrel?

9. If 6 shillings make a dollar, how many dollars in 42 shillings?

10. How many shillings in a dollar, if 42 shillings make 7 dollars?

11. If beef cost 9 cents a pound, how much could be bought for 54 cents?

12. What would be the cost of 1 pound of beef, if 6 pounds cost 54 cents?

13. Mr. Jones bought sugar at 7 cents a pound, expending 49 cents; how many pounds did he get?

14. If Mr. Jones should expend 49 cents for 7 pounds of sugar, how much would that be a pound?

15. How many pews would accommodate 63 persons, if 7 persons could sit in one pew?

16. If 63 persons would fill 9 pews, how many persons would be accommodated in one pew?

17. If 8 ninepences make one dollar, how many dollars in 72 ninepences?

18. If 72 ninepences make 9 dollars, how many ninepences make one dollar?

19. How many classes, of 10 scholars each, in a school of 80 scholars?

20. If 80 scholars be put into 8 equal classes, how large would be the classes?

21. How many sections could be made in a company of 64 soldiers, if 8 soldiers make a section?

22. Make 8 equal sections of 64 soldiers, and tell me how many soldiers you put into a section?

23. How many 3s are there in 12?

24. Four times what number makes 12?

25. How many 3s can be subtracted from 15?

26. Five times what number will amount to 15?

27. How many times can 3 be subtracted from 18?

28. What number taken 6 times will equal 18?

29. How many times is 3 contained in 21?

30. What number taken 7 times makes 21?

31. How many times 4 equal 20?

32. Five times what num ber equals 20?

33. Into how many parts of 4 each, can 24 be separated?

34. If 24 be separated into 6 equal parts, how many in each part?

35. Into *how many parts* of 10 each, can 30 be separated?

36. If 30 be separated into 3 equal parts, *how large is each part?*

37. Into *how many parts* of 5 each can 35 be divided?

38. If 35 be divided into 7 equal parts, *how large is each part?*

39. Into *how many parts* of 5 each can 45 be divided?

40. If 45 be divided into 9 equal parts, *how large is each part?*

41. What number must 6 be multiplied by to make 48?

42. What number multiplied by 8 will make 48?

43. What number must 7 be multiplied by to make 56?

44. What number multiplied by 8 will make 56?

45. Divide 63 into equal parts of 7 each. How many are the parts?

46. Divide 63 into 9 equal parts. How large are the parts?

65. DIVISION TABLE.

In order to perform Division with facility, you will, before attempting further progress, correctly ascertain, and thoroughly commit to memory the *quotient* of each combination of two numbers in the following table.

2 ÷ 2 =	3 ÷ 3 =	4 ÷ 4 =	5 ÷ 5 =
4 ÷ 2 =	6 ÷ 3 =	8 ÷ 4 =	10 ÷ 5 =
6 ÷ 2 =	9 ÷ 3 =	12 ÷ 4 =	15 ÷ 5 =
8 ÷ 2 =	12 ÷ 3 =	16 ÷ 4 =	20 ÷ 5 =
10 ÷ 2 =	15 ÷ 3 =	20 ÷ 4 =	25 ÷ 5 =
12 ÷ 2 =	18 ÷ 3 =	24 ÷ 4 =	30 ÷ 5 =
14 ÷ 2 =	21 ÷ 3 =	28 ÷ 4 =	35 ÷ 5 =
16 ÷ 2 =	24 ÷ 3 =	32 ÷ 4 =	40 ÷ 5 =
18 ÷ 2 =	27 ÷ 3 =	36 ÷ 4 =	45 ÷ 5 =

6 ÷ 6 =	7 ÷ 7 =	8 ÷ 8 =	9 ÷ 9 =
12 ÷ 6 =	14 ÷ 7 =	16 ÷ 8 =	18 ÷ 9 =
18 ÷ 6 =	21 ÷ 7 =	24 ÷ 8 =	27 ÷ 9 =
24 ÷ 6 =	28 ÷ 7 =	32 ÷ 8 =	36 ÷ 9 =
30 ÷ 6 =	35 ÷ 7 =	40 ÷ 8 =	45 ÷ 9 =
36 ÷ 6 =	42 ÷ 7 =	48 ÷ 8 =	54 ÷ 9 =
42 ÷ 6 =	49 ÷ 7 =	56 ÷ 8 =	63 ÷ 9 =
48 ÷ 6 =	56 ÷ 7 =	64 ÷ 8 =	72 ÷ 9 =
54 ÷ 6 =	63 ÷ 7 =	72 ÷ 8 =	81 ÷ 9 =

66. MODEL OF A RECITATION.

1. How many quarts are there in 600 pints?

Since there are 2 pints in a quart, there will be as many quarts as there are times 2 pints in 600 pints.

2 is contained 3 times in 6 units of the *first* order, but, in 6 units of the *third* order, which are 100 times as large, (**6**,) it must be contained 100 times as often, which is 300 times. 300 quarts, at 2 pints each, take 600 pints, which subtracted from 600 pints, nothing remains. Hence, 300 quarts is the answer required.

```
2 ) 600 ( 300 quarts.
    600
    ---
```

67. EXERCISES IN DIVIDING UNITS OF ANY ONE ORDER.

In like manner, solve and explain the following problems.

1. If in a certain school-room 2 scholars sit at a desk, how many desks will accommodate 200 scholars?
2. If in a certain school there are 80 scholars, and 2 teachers, how many scholars are there for each teacher?
3. If a man pay 3 dollars apiece for hats, how many hats can he buy for 90 dollars?
4. If Mr. Farmer sell 2 cows for 40 dollars, how much is that apiece?
5. At 4 dollars a yard for cloth, how many yards can be bought for 80 dollars?
6. If 800 dollars a year be paid to 4 female teachers, how much is that apiece?
7. At the rate of 5 miles an hour, how long would it take to travel 500 miles?
8. If 6 shares in a bank cost 600 dollars, how much is that a share?
9. At an average of 7 persons to a family, how many families in a town of 7000 persons?
10. If 90000 dollars be the cost of 3 miles of rail-road, what is the cost per mile?

68. MODEL OF A RECITATION.

1. A hatter made in a year 560 hats, and packed them for market in boxes holding 8 hats apiece. How many boxes would he need?

Since each box would hold 8 hats, he would need as many boxes as there are times 8 hats in 560 hats. But, one unit

of any order making ten units of the next lower order, (10) the 5 hundreds are equal to 50 tens, which with the 6 tens, make 56 tens; 8 is contained 7 times in 56 units of the *first* order, but in 56 units of the *second* order, which are 10 times as large, (4,) it must be contained 10 times as often, which is 70 times; 70 boxes at 8 hats each, would take 560 hats, which subtracted from 560 hats, nothing remains. Hence, 70 boxes is the answer required.

```
8 ) 560 ( 70 boxes.
    560
    ———
```

69. Exercises in Reducing Units of a High to a Lower Order for Division.

In like manner, solve and explain the following problems.

1. How many pairs of boots could be bought for 150 dollars at 3 dollars a pair?
2. If 350 dollars be paid for 5 horses, how much is that apiece?
3. How many hours will it take to travel 350 miles at 7 miles per hour?
4. If a stage travel 120 miles in 12 hours, how far is that an hour?
5. How many times is 5 contained in 450?
6. Into how many parts of 9 each can 6300 be divided?
7. If 3200 be divided into 8 equal parts, how large are the parts?
8. Divide 2500 into 5 parts; how large is each part?
9. What number must 7 be multiplied by to produce 4900?
10. What number multiplied by 3 will produce 27000?
11. Divide 100 by 4.
12. Divide 1000 by 8.
13. If 1800 be the dividend and 9 the divisor, what will be the quotient?

70. Explanation of the Written Process of Division.

1. How many yards are there in 9636 feet?

Since there are 3 feet in a yard, there will be as many yards as there are times 3 feet in 9636 feet.

3 is contained 3 times in 9 units of the *first* order, but in 9 units of the *fourth* order, it must be contained 1000 times as often, (**10**,) that is, 3000 times; 3000 yards at 3 feet each, take 9000 feet, which subtracted from 9636 feet, leave 636 feet; 3 is contained in 6 *units* 2 times; therefore, in 6 *hundreds*, it is contained 200 times; 200 yards at 3 feet each take 600 feet, which subtracted from 636 feet leave 36 feet; 3 is contained in 3 *units* 1 time, therefore, in 3 *tens* it is contained 10 times; 10 yards at 3 feet each take 30 feet, which subtracted from 36 feet leave 6 feet, in which 3 is contained 2 times; 2 yards at 3 feet each take 6 feet, which subtracted from 6 feet, nothing remains. Hence, 3000 yards + 200 yards + 10 yards + 2 yards = 3212 yards, is the answer required.

```
3 ) 9636 ( 3000 yards.
    9000    200 yards.
    ----     10 yards.
     636      2 yards.
     600   ----
     ---   3212 yards.
      36
      30
      --
       6
       6
      --
```

This operation may be abridged by omitting some unnecessary figures. Instead of the ciphers belonging to the first number in the quotient, write the digits of the other numbers as they are obtained, which will finally leave each figure in its own place.

```
3 ) 9636 ( 3212 yards.
    9---
    ----
     6--
     6--
     ---
      3-
      3-
      --
       6
       6
      --
```

The product of the divisor and the first quotient figure is 9 thousand; omitting the ciphers, it will be sufficient to write the 9 in the thousands' place, and subtract it from the thousands; then bring down the 6 hundreds only, for consideration; 200 times the divisor is 6 hundreds, which being subtracted from the hundreds, bring down the 3 tens; 10 times the divisor is 3 tens, which being subtracted from the tens, bring down the 6 units; 2 times the divisor is 6 units, which being subtracted from the units, nothing more

of the dividend remains. Hence, 3912 yards is the answer required, as before.

71. MODEL OF A RECITATION.

Divide 2848 by 4, or find how many times 4 is contained in 2848.

4 is contained 7 times in 28 units, but in 28 hundreds it is contained 100 times as often, (**68**,) or 7 hundred times; 7 hundred times 4 are 28 hundred, which subtract from the hundreds, and bring down the 4 tens; 4 is contained 1 time in 4 units, but in 4 tens it is contained 10 times as often, or 1 ten times; 10 times 4 are 4 tens, which subtract from the tens and bring down the 8 units; 4 is contained 2 times in 8 units; 2 times 4 are 8, which subtracted, nothing remains; consequently, 712 is the result required.

```
4 ) 2848 ( 712
    28
    ——
      4
      4
      —
       8
       8
       —
```

72. EXERCISES IN EXPLAINING THE WRITTEN PROCESS OF DIVISION.

In like manner, solve and explain the following problems.

1. How many bushels in 88 pecks?
2. How many weeks in 77 days?
3. How many dollars in 126 shillings?
4. If 4 horses are required to draw 1 wagon, how many wagons might be drawn by 168 horses?
5. If a man can travel 5 miles an hour, how many hours would it take him to travel 205 miles?
6. A drover received 248 dollars for sheep that he sold for 4 dollars a head. How many were there?
7. If 5 bushels of corn pay for a pair of boots, how many pairs would 255 bushels pay for?
8. Suppose 6 men should contribute 186 dollars, how much would that be apiece?
9. Suppose 355 dollars' bounty were paid at 5 dollars apiece to a company of soldiers. How many soldiers in the company.
10. How many weeks can a man get board for 156 dollars, at 3 dollars a week?
11. How many times is 7 contained in 637?

12. Suppose 3690 to be a dividend, and 9 a divisor, what is the quotient?

13. Divide 1836 by 3.

14. What must I multiply by 8 to make 7288?

15. Into how many parts of 5 each can 555 be divided?

16. If 567 be divided into 7 equal parts, what must be the size of each part?

73. MODEL OF A RECITATION.

Mr. Farmer planted 4785 grains of corn in a field, planting 5 grains in each hill. How many hills did he make?

Since he put 5 grains in each hill, he made as many hills as there are times 5 grains in 4785 grains.

```
5)4785(957 hills.
  45
  --
   28
   25
   --
    35
    35
    --
```

Beginning at the left hand of the dividend, take into consideration the fewest figures that can contain the divisor; as 5 is not contained in 4, take 47 hundreds, in 45 of which 5 is contained 9 hundreds times, (**10**,) 900 hills require 45 hundred grains, which subtracted from 47 hundred leave 2 hundred, with which join the 8 tens, making 28 tens, (**10**,) in 25 of which 5 is contained 5 tens times; 50 hills require 25 tens grains, which subtracted from 28 tens leave 3 tens, with which join the 5 units, making 35 units, in which 5 is contained 7 times; 7 hills require 35 grains, which subtracted from 35 grains, nothing remains. Hence, 957 hills is the answer required.

74. OBSERVATION.

OBSERVE, (**73**,) *that the division is commenced, by dividing the fewest figures on the left of the dividend that will contain the divisor, that the quotient figure will be of the same denomination as that part of the dividend from which it is obtained, that each succeeding figure of the dividend will require an additional figure in the quotient, a cipher if nothing larger, that the products of the divisor, by each quotient figure, are to be subtracted from those parts of the dividend from which the respective quotient figures are obtained, that the remainder in each case is reduced* (**208**) *and united to the units of the next lower order, for division,*

and that the sum of these partial products, or the product of the divisor by the whole *quotient, is equal to the dividend.*

75. PROOF OF DIVISION.

To *prove* the correctness of an operation in Division, multiply the divisor and quotient together; if their product equals the dividend, probably the operation is correct; for, the correct quotient, expressing how many times the divisor there are in the dividend, (**61**,) is *one*, and the divisor the *other* of two factors, whose product *should* be the dividend.

76. EXERCISES REQUIRING SOME UNITS OF EACH ORDER TO BE REDUCED TO A LOWER ORDER FOR DIVISION.

In like manner, solve and explain the following problems.

1. If 9 hills of potatoes yield a bushel, how many bushels of potatoes in a field of 1296 hills?
2. If an army of 2048 men were marching in sections, having 8 men in each section, how many sections would be there?
3. If in an army every ninth man is an officer, how many officers in an army of 4608 men?
4. If a general should divide his army of 12096 men into 7 equal divisions, how many men would be in each division?
5. How many weeks in 364 days?
6. How many Sabbath days in 12852 days?
7. If an acre of land pasture 5 sheep, how many acres could pasture 315 sheep?
8. How many times is 6 contained in 738?
9. How many times is 4 contained in 20012?
10. Divide 3606 by 3.
11. Divide 25634 by 2.
12. If 28028 be a dividend, and 7 a divisor, what is the quotient?
13. If 18675 be a product, and 5 one factor, what is the other factor?
14. What must 11889 be divided by, to give 9 for a quotient?
15. What must 8 be multiplied by, to produce 2496?

77. MODEL OF A RECITATION.

1. If in the month of July a rail-road company received

6284 dollars from passengers, at 2 dollars apiece, how many passengers rode in the cars in that month?

Since each passenger paid 2 dollars, there were as many passengers as there are times 2 dollars in 6284 dollars.

To obtain the answer by a still shorter process, write the dividend and divisor as heretofore, but perform the operation in your mind, writing only the quotient, and write that under the dividend, with each figure under that of its own order.

```
2) 6284
   ————
   3142 passengers.
```

Thus, 2 in 6 thousands 3 thousand times, therefore, write 3 in the thousands' place; 2 in 2 hundreds 1 hundred times, therefore, write 1 in the hundreds' place; 2 in 8 tens 4 tens times, therefore, write 4 in the tens' place; 2 in 4 units 2 times, therefore, write 2 in the units' place: making 3142 times 2 dollars. Hence, 3142 passengers is the answer required.

2. If a stage run 6 miles an hour, how many hours would it take the stage to run 1848 miles?

Since in one hour it runs 6 miles, it will take as many hours as there are times 6 miles in 1848 miles.

```
6) 1848
   ————
    308 hours.
```

6 in 18 hundreds 3 hundreds times; write 3 in the hundreds' place. If 6 were contained in the 4, which is tens, the quotient figure would be tens, but as 6 is not contained in 4, there are no tens in the quotient, therefore, write a cipher in the tens' place, and reduce the 4 tens to units, making 40 units, which, joined with the 8 units, make 48 units, in which 6 is contained 8 times, therefore, write 8 in the units' place: making 308 times 6. Hence, 308 hours is the answer required.

78. Exercises in Abridging the Process of Division.

In like manner, solve and explain the following problems.

1. If 306 dollars be divided among 3 men, what is each man's share?

2. If 4 shares of a bank cost 416 dollars, what would one share cost?

3. If six brothers receive a legacy of 1512 dollars, what would be the share of each?

4. Paid 150 dollars for 6 tons of hay. How much was that for a ton?

5. If there are 1280 inhabitants in a town, and the families average 8 persons apiece, how many families in that town?

6. How many yards of cloth can be bought for 1155 dollars, at 7 dollars a yard?

7. Find a number, which, multiplied by 9, will produce 63234.

8. What number, multiplied by 8, will produce 2464?

9. What number, divided by 9, will give 72 for a quotient?

10. If 7 be a divisor, and 42014 a dividend, what is the quotient?

11. How many times is 5 contained in 1204500890?

12. How many times does 540010 contain 5?

13. How many times 8 are there in 25648?

14. Divide 4004 by 4.

15. Divide 16800 by 8.

16. Divide 36900 by 3.

17. Divide 1800108 by 9.

18. Divide 105105 by 7.

19. If 1836 be a dividend, and 9 the divisor, what is the quotient?

20. If 1728 be divided by 9, what would be the quotient?

21. If 72 be a dividend, and 9 the quotient, what is the divisor?

22. If 63 be a dividend, and 7 the quotient, what is the divisor?

79. Model of a Recitation.

1. How many days in 1728 hours?

Since in one day there are 24 hours, there must be as many days as there are times 24 hours in 1728 hours.

```
24) 1728 (72 days.
    168
    ---
     48
     48
     --
```

24 is contained in 172 tens 7 tens times; 70 times 24 make 168 tens, which, subtracted from 172 tens, leave 4 tens, to which bring down the 8 units, making 48 units, in which 24 is contained 2 times; 2 times 24 make 48, which subtracted from 48, nothing remains. Hence, as there are 72 times 24 hours, 72 days is the answer required.

2. How many times is 64237 contained in 436940074?

The many figures in this divisor, present a difficulty in ascertaining any quotient figure. The best way is to seek

how many times the highest figure only, of the divisor, is contained in the highest one, or two, figures of the dividend; this quotient figure will either be right, or one or two too large; for the greater certainty, however, before multiplying the whole divisor by it, multiply mentally only one or two of the highest figures of the divisor, and compare the product with the highest figures of the dividend from which this part of the product is to be subtracted; if the appearance is satisfactory, proceed with this quotient figure, otherwise take a smaller figure, and proceed.

If at any time a product prove too large to be subtracted, the last quotient figure is too large; or, if a remainder be larger than the divisor, the last quotient figure is too small. In either case, erase it, and try another figure.

```
64237 ) 436940074 (6802 times.
        385422
        ------
         515180
         513896
         ------
           128474
           128474
           ------
```

6 is contained 7 times in 43, but 7 times 64 is greater than 436; therefore, 7 is too large for the first quotient figure; write 6 in the quotient, and subtract 6 thousand times the divisor, that is, 6 times the divisor from the thousands, and to the remainder bring down the next figure of the dividend; 6 is contained 8 times in 51, and 8 times 64 being less than 515, subtract 8 hundred times the divisor; that is, 8 times the divisor from these hundreds, and to the remainder bring down the next figure; this number being smaller than the divisor, there can be no tens in the quotient; therefore, write a cipher in the tens' place, (**77**,) and bring down the next figure; 6 is contained in 12 twice; subtract 2 times the divisor, and nothing remains. Hence, 6802 times, is the answer required.

80. General Exercises in Division.

In like manner, solve and explain the following problems.

1. How many days in 360 hours?

2. If a man travel 45 miles a day, in how many days will he travel 1125 miles?

3. A butcher gave 875 dollars for 35 cows. What was the cost of each cow?

4. If a field of 34 acres produce 1020 bushels of corn, how much would that be per acre?

5. Suppose an acre of land to produce 38 bushels of corn, how many acres must be cultivated to produce 4902 bushels?

6. How many horses, at 75 dollars apiece, can be bought for 1125 dollars?

7. A school-district paid a teacher 144 dollars for teaching, at 36 dollars a month. How long was the school kept?

8. If a man's income be 1095 dollars for 365 days, how much is that per day?

9. How many hogsheads, of 63 gallons each, can be filled from 8379 gallons?

10. How many years in 8395 days, if 365 days be called a year?

11. If 1512 dollars be divided among some brothers, so that each may receive 252 dollars, how many are the brothers?

12. How many bank shares can be purchased with 2912 dollars, at 112 dollars each?

13. How many acres of land will yield 6996 bushels of potatoes, if 212 bushels grow on one acre?

14. How many barrels must a man have to fill from 125440 pounds of flour, if each barrel hold 196 pounds?

15. A man put 17484 pounds of tea into 186 chests. How much in each chest?

16. How many times can 48 be subtracted from 5040?

17. How many times is 75 contained in 23025?

18. How many times 25 is equal to 23025?

19. How many times does 105735 contain 105?

20. How many times does 105735 contain 1007?

21. Divide 144144 into 144 equal parts; what is each part?

22. Divide 172800 nuts among some boys, giving them 1440 nuts apiece. How many boys can you supply with them?

23. What number, multiplied by 754, will produce 18850?

24. The product of two factors is 612060. If one factor is 303, what is the other factor?

25. Divide a city of 78612 inhabitants into 12 equal wards. How many inhabitants in each ward?

26. How many equal parts can be made of 1048576, if 1024 be one of the parts?

27. How many times 4096 is equal to 262144?

28. If 2048 be one of a certain number of equal parts of 131072, how many are the parts?

81. General Exercises in the Fundamental Principles of Arithmetic.

1. There are two numbers, of which the greater is 27 times the less, and the less is contained 9 times in 27. What are the numbers?

2. A was born when B was 26 years old. How old will A be when B is 45?

3. If the sum of 3 numbers be 500, the difference between the least and the greatest be 174, and the difference between the middle number and the sum of the 3 numbers be 350, what are the numbers?

4. A man bought 5 pieces of cloth at 44 dollars each, 974 pairs of shoes at 2 dollars a pair, 600 pieces of calico at 6 dollars each, and sold the whole for 6000 dollars. How much did he gain, or lose?

5. A man exchanged 6 cows at 15 dollars each, a yoke of oxen at 67 dollars, for a horse at 50 dollars, and a chaise. What did the chaise cost?

6. A boy bought some apples, and, after giving away 10, and buying 34 more, he divided half of what he then had among 4 companions, giving them 8 apiece. How many apples did he buy at first?

7. What is that number, to which, if 4 be added, from which 7 be subtracted, the remainder multiplied by 8, and the product divided by 3, the quotient will be 64?

8. A man bought a farm at 25 dollars an acre, and sold half of it, at the same rate, for 1850 dollars. How many acres did he buy?

9. Five men and three boys were paid a sum of money, so large that each man had 43 dollars, and each boy 25 dollars. What was the whole sum?

10. If a trader gain 160 dollars on 544 barrels of flour, that cost him 6 dollars a barrel, besides 25 dollars that he paid for storage; what would he receive for the flour?

11. Suppose 5 bushels of wheat make a barrel of flour, how many barrels can be made from the wheat raised on 75 acres, at 29 bushels per acre?

12. How many times 5 in 75 times 29?

13. A farmer exchanges 44 acres of land, worth 36 dollars

an acre, for 66 acres of land in another place. What does his land cost him per acre ?

14. A man who owned 520 acres, bought 375 acres more, and, reserving 95 acres for himself, divided the remainder into 8 equal farms, and sold them for 2500 dollars apiece. How much did he get per acre for his land ?

15. If a man's income be 1349 dollars a year, and his expenses 20 dollars a week, how much would he save in a year ?

16. A merchant's business brought him, in a year, 2500 dollars; but his expenses were 1772 dollars. How much did he save per week ?

17. If I buy 245 hogsheads of molasses, at 18 dollars each, how much do I gain, or lose, in selling it for 4000 dollars ?

18. If a man's expenses be 2 dollars a day, and his income 17 dollars a week, how many weeks will it take him to save 156 dollars ?

19. If a lot of land be divided into 8 farms, each of 150 acres, and the farms be sold for 3000 dollars apiece, what would one acre cost ?

20. A gentleman bought 2 pieces of land, one contained 96 acres, the other 103 acres. If he should sell 47 acres, at 25 dollars an acre, how much would the rest of the land be worth at the same rate ?

21. A merchant bought a cask of molasses containing 119 gallons, and sold to one man 10 gallons, to another 9 gallons, to another 25 gallons. How much is the remainder worth, at 40 cents a gallon ?

22. What is the difference between 17 times 105 and 3417 divided by 17 ?

23. What is the difference between 20 times 210 and 7 times 2500 divided by 175 ?

24. If I purchase 1200 pounds of butter for 15600 cents, how must I sell it per pound to gain 2400 cents ?

25. If I buy 375 pounds of pork at 7 cents a pound, and sell it for 3000 cents, how much do I gain on a pound ?

26. How many quintals of fish, at 2 dollars each, will pay for 500 hogsheads of salt, at 5 dollars a hogshead ?

27. How much flour, at 7 dollars a barrel, will pay for 224 cords of wood, at 8 dollars a cord ?

28. How many days must 3 brothers work to receive 2475

cents, if one earn 42 cents a day, the second 32 cents, and the youngest 25 cents?

29. If a man earn 6 dollars a week, and his two boys earn 3 dollars apiece a week, how many weeks will it take them all to earn 624 dollars?

30. If a hogshead hold 252 quarts, and two boys work together to fill it with water, one having a pail which holds 12 quarts, the other having a pail which holds 9 quarts, how many times must they empty their pails to fill the hogshead?

31. If a full hogshead should begin to leak in 3 places, at once, from one hole 4 quarts a day, from another 2 quarts a day, and from the other 1 quart a day, how many days before the hogshead would be emptied?

32. A man bought some sheep and calves, and of each an equal number, for 165 dollars, giving for the sheep 7 dollars apiece, and for the calves 4 dollars apiece. How many were there of each sort?

33. How many coats, pantaloons and vests, of each an equal number, can be made from 405 yards, if it take 5 yards for a coat, 3 yards for a pair of pantaloons, and 1 yard for a vest?

34. If 9000 men march in a column of 750 deep, how many march abreast?

35. A man left his estate, valued at 8956 dollars, to his wife and daughters, giving his wife 4688 dollars, and his daughters 1067 dollars apiece. How many daughters had he?

36. The factors of a certain number are the difference between 1632 and 1700, and between 94 and 5 dozen. What is that number?

37. Paid 57600 cents for eggs, at 12 cents a dozen. How many eggs did I buy?

38. A boy bought a sled for 96 cents, exchanged it for 8 quarts of nuts, sold half of his nuts at 12 cents a quart, and gave the rest of his nuts for a penknife, which he sold for 34 cents. How much did he gain, or lose?

39. Three men owned farms situated together; the first had 64 acres, the second had 20 acres more than the first, and the third had as many acres as both the first and second; the three farms were worth 7400 dollars. What is that per acre?

40. If a man owe 728 dollars to Mr. Saveall, and works for him to pay the debt; how many years, of 52 weeks each, will it take him, if he pay only one dollar a week?

41. If a man earn 40 dollars a month, and spend 13 dollars of it each month, how long will it take him to pay for a house worth 1620 dollars?

42. A farmer sold some pork at 17 dollars a barrel to the amount of 510 dollars, and some at 19 dollars a barrel to the amount of 380 dollars, how many barrels did he sell?

43. A drover exchanged 42 horses worth 72 dollars apiece, for cows worth 36 dollars apiece, and for his cows he received 36 yoke of oxen, which he sold so as to gain 144 dollars, how much did he get for each yoke of oxen?

44. How much is $72 \times 24 \div 36 + 84 \times 7 \div 12 - 11$?

45. Let 27 be a divisor, and 567 a dividend, what will be the quotient?

46. Suppose 25 is a quotient, and 25 a divisor of the same dividend, what is that dividend?

47. Of what dividend is 15 both divisor and quotient?

VI. FRACTIONS.

82. Origin of Fractions, and Manner of Writing them.

1. At 5 cents a quart for nuts, how many quarts can you buy for 38 cents?

Since 1 quart costs 5 cents, you can buy as many quarts as there are times 5 cents in 38 cents.

5 is contained 7 times in 35, which being subtracted from 38, there remain 3 units, which are not sufficient to contain the *whole* of 5, but if 5 be divided into 5 equal parts, each part is exactly a unit; therefore, your remainder being 3 units, will contain exactly 3 of the parts of 5, which being subtracted, nothing remains. In the quotient write 7 to express the *number of whole* 5s; after it near the top write a small 3 to express the *number of parts*, and under the 3, separated by a line, write a small 5, to express the *size of these parts*, which it will do by showing *how many such parts make a unit*, or a *whole*, of which these are parts.

$$\begin{array}{l} 5\,)\,38\,(\,7\tfrac{3}{5} \text{ quarts.} \\ \quad\;\; 35 \\ \quad\;\; \overline{\;\;\;3} \\ \quad\;\;\;\;\, 3 \\ \quad\;\; \overline{\;\;\;\;} \end{array}$$

Hence, as 38 contains 7 times 5 and 3 such *parts* of a 5 that 5 of these parts would make a whole 5, you can buy 7 *whole*

quarts and 3 such *parts* of a quart that 5 of them would make a whole quart.

Another Explanation.

35 cents will buy 7 quarts, and you have 3 cents remaining, which are not sufficient to buy a *whole* quart; but if a whole quart be divided into 5 equal parts, each part will be worth exactly one cent; and as you have 3 cents remaining, you can buy 3 *of these parts.*

$$\begin{array}{r} 5\,)\,38 \\ \hline 7\tfrac{3}{5}\text{ quarts.} \end{array}$$

Hence, with your 38 cents, you can buy 7 *whole* quarts and 3 such *parts* of a quart that 5 of them would make a whole quart.

After the 7 (whole quarts) near the top, write a small 3 to express the *number of parts*, and under the 3 separated by a line, write a small 5 to express *the size of these parts*, which it will do by showing *how many such parts make a unit*, or whole quart.

2. If 25 apples be given to 7 boys, what would be the share of each boy?

Since giving *one* apple to each boy takes 7 apples, the share of each boy would be as many apples as there are times 7 apples in 25 apples.

7 is contained 3 times in 21, which being subtracted from 25, there remain 4 units, which are not sufficient to contain the *whole* of 7; but if 7 be divided into 7 equal parts, each part will be exactly a unit; therefore the remainder, being 4 units, will contain exactly 4 of these parts of 7, which being subtracted, nothing remains. In the quotient write 3 to express the number of *whole* 7s; after it near the top write a small 4 to express the *number of parts*, and under the 4, separated by a line, write a small 7 to express *the size of these parts*, which it will do by showing *how many such parts make a unit*, or a whole 7.

$$\begin{array}{l} 7\,)\,25\,(\,3\tfrac{4}{7}\text{ apples.} \\ \quad\;\; 21 \\ \quad\;\; \overline{\;\;4\;} \\ \quad\;\;\;\; 4 \\ \quad\;\; \overline{\;\;\;\;\;} \end{array}$$

Hence, as 25 contains 3 times 7 and 4 such *parts* of a 7 that 7 of them would make a whole 7, the share of each boy would be 3 *whole* apples and 4 such *parts* of an apple that 7 of them would make a whole apple.

Another explanation.

21 apples will afford 3 to each boy; but the 4 remaining apples will not afford the boys a *whole* apple apiece. If, however, each of these 4 apples be cut into 7 equal parts, they would make 28 parts, or exactly 4 *parts for each boy.*

$$7\,)\,25$$
$$3\tfrac{4}{7} \text{ apples.}$$

Hence, the share of each boy would be 3 *whole* apples, and 4 such *parts* of an apple that 7 of them would make a whole apple.

This answer may be expressed as before directed.

83. Definition of Terms.

A *fraction* is the expression of one, or more of *the equal parts of a unit.*

A fraction is *composed* of two numbers, called the *terms* of the fraction.

The terms of a fraction are *written* one below the other, separated by a line.

The upper term—called the *numerator*—shows *how many parts* the fraction expresses.

The lower term—called the *denominator*—shows the *size of the parts* expressed by the fraction, by shewing *how many such parts make the unit* of which the fraction expresses one or more parts.

84. Manner of Reading Fractions and Mixed Numbers.

Parts take different *names*, according to their *size*, or the number of them that it takes to make a unit. Thus, the fractions $\frac{2}{3}$, $\frac{2}{5}$, $\frac{2}{10}$, $\frac{2}{100}$, &c., are read, two *thirds*, 2 *fifths*, two *tenths*, 2 *hundredths*, &c.

A fraction may be *considered and read* in four different ways; for instance, $\frac{3}{4}$ may be considered $\frac{3}{4}$ of 1, or $\frac{1}{4}$ of 3, or 3 divided by 4, or 3 such parts that 4 like them would make a unit.

A number which is composed of both an integral and fractional number, is called a *mixed number*. The answers to the above problems (82) $7\frac{3}{5}$, $3\frac{4}{7}$, are mixed numbers, which are read thus, seven and three fifths, three and four sevenths.

Integer is a term applied to a number which expresses only *whole* units.

83. Exercises in Originating and Writing Fractions.

Solve and explain the following problems on the left, like the first, and those on the right, like the second of the above examples (82.)

1. If 1 lead pencil cost 3 cents, how many can you buy for 8 cents?

2. If you divide 11 lead pencils among 3 boys, how many will each boy have?

3. If 4 cents buy an orange, how many can be bought for 25 cents?

4. How many cents does 1 lemon cost, when you give 22 cents for 5 lemons?

5. If the stage fare be 6 cents a mile, how far can you ride for 41 cents?

6. How much does a man earn a week, who receives 65 dollars for 7 weeks?

7. How many slates at 8 cents apiece, can be bought for 93 cents?

8. If 9 men do a job together, and receive 220 dollars, what is the share of each?

9. How many writing books at 10 cents apiece can be bought for 125 cents?

10. What does a single knife cost, at 295 cents a dozen?

11. How many shad at 15 cents apiece, can be bought for 218 cents?

12. What is the price of a barrel of flour, when 18 barrels cost 150 dollars?

13. At 22 cents for an inkstand, how many may be bought for 93 cents?

14. If 25 apple trees yield 183 bushels of apples, how much does each tree yield?

15. At 29 dollars a head, how many cows may be bought for 350 dollars?

16. If in 32 equal loads of potatoes 729 bushels were carried to market, how many bushels in each load?

17. How many acres of land at 37 dollars an acre, will 5555 dollars buy?

18. If 1760 yards make 320 rods, how many yards make 1 rod?

19. If 320 rods make a mile, how many miles in 46100 rods?

20. If 1749 feet make 106 rods, how many feet in one rod?

21. If a ship sail 125 miles per day, how long would it take her to sail round the world, it being 24911 miles?

22. If a ship sail 132 miles a day, in how many days will she sail from Boston to Liverpool, it being 3000 miles?

23. If 63 gallons of water in one hour run into a cistern containing 432 gallons, in what time will it be filled?

24. What would be the cost of 1 hogshead of molasses, if 75 hogsheads cost 2200 dollars?

25. How many boxes would be required to contain 32844 oranges, if each box contain exactly 100 oranges?

26. How many bushels of wheat does a farmer raise on an acre, who raises 2400 bushels on 99 acres?

27. How many days, at 175 cents a day, must a man work to earn 4500 cents?

28. If a man receive 730 dollars a year, how much is that a week?

29. At 365 days a year, how many years old is a boy who has lived 3999 days?

30. How many miles per hour does an engine move, which goes 2600 miles in a week?

31. How many times 199 in 2569?

32. Divide 4657 into 25 equal parts.

33. Divide 2864 by 14.

34. Divide 100000000 by 12478.

86. Observation.

Observe, (**82**,) *that when a number is to be divided which is smaller than the divisor, the quotient will be a fraction, of which the dividend will be the numerator and the divisor will be the denominator.*

Hence, division may be expressed in a fractional form, whether the dividend be larger or smaller than the divisor, and the value of the expression will be the true quotient.

87. Model of a Recitation.

1. If a pie be cut into 8 equal parts, what fractions would express one, three, five, and eight of the parts?

When 8 equal parts make a unit, any number of these parts are so many *eighths;* (**84**) therefore, one part is $\frac{1}{8}$ (one eighth,) three parts are $\frac{3}{8}$ (three eighths,) 5 parts are $\frac{5}{8}$ (five eighths,) and eight parts are $\frac{8}{8}$ (eight eighths,) or the whole pie.

2. What fractions of a foot will express 5, 7, and 11 inches?

When a unit is divided into 12 equal parts, any number of the parts are so many *twelfths*, (**84**,) therefore, 5 inches

are $\frac{4}{12}$ of a foot, 7 inches are $\frac{7}{12}$ of a foot, and 11 inches are $\frac{11}{12}$ of a foot.

3. What parts of 15 are 8, 14, and 19?

Since it takes 15 units to make the *whole* of 15, any number of units are so many fifteenths of 15; therefore, 8 is $\frac{8}{15}$ of 15, 14 is $\frac{14}{15}$ of 15, and 19 is $\frac{19}{15}$ of 15.

88. Exercises in Expressing Division.

In like manner, solve and explain the following problems.

1. If a pie be cut into 6 equal pieces, what fractions will express one, two, and five of the pieces?

2. Two boys divided an orange equally between themselves, what fraction will express each one's part?

3. If an acre of land be divided into 4 equal house-lots, what fractions would express one, three, and four of the lots?

4. If a piece of cloth be sufficient for 7 coats, what parts of the piece of cloth would be sufficient for 1, 3, 5, and 6 coats?

5. If you divide a barrel of flour equally among 9 men, what part of a barrel would each receive?

6. If 18 dollars be paid for a ton of hay, what parts of a ton may be bought for 5, 7, 11, and 17 dollars?

7. At 27 dollars a hogshead for molasses, what parts of a hogshead may be bought for 10, 14, 19, and 25 dollars?

8. At one hundred dollars a share in a bank, what parts of a share may be bought for 16, 29, 67, 89, and 93 dollars.

9. At 75 cents a bushel for corn, what parts of a bushel may be bought for 12, 24, 36, and 58 cents?

10. What fractions of a dollar will express 7, 23, 37, 47, 67, and 97 cents?

11. What fractions of June will express 11, 17, and 29 days?

12. What parts of July are 16, 21, and 27 days?

13. What parts of an hour are 13, 43, and 59 minutes?

14. What parts of a day are 1, 7, 19, and 23 hours?

15. 11, 21, 87, 123, and 219 rods are what parts of a mile?

16. What part of 5 dollars are 3 dollars?

17. What parts of 25 cents are 3, 7, 14, and 21 cents?

18. What parts of 63 gallons are 16, 31, and 44 gallons?

19. What parts of 365 days are 31, 60, 124, 243, and 316 days?
20. 15 weeks are what part of 52 weeks?
21. What fractions of a bushel will express 11, 15, 25 and 32 quarts?
22. What part of 8 is 5?
23. What parts of 11 are 2, 9, 12, 14, and 21?
24. What parts of 33 are 5, 7, 16, 25, and 32?
25. How many times is 15 contained in 34?
26. How many times, or, (more properly,) what part of a time, is 15 contained in 8?
27. What part of a time is 24 contained in 7?
28. What part of 16 is contained in 11?
29. What part of 12 does 5 contain?
30. Divide 21 by 25; what is the quotient?
31. If 17 be a dividend, and 25 the divisor, what must be the quotient?
32. If 4 apples be divided among 5 boys, what part of an apple is each boy's share?
33. If 3 men divide a barrel of apples equally among themselves, what fractions will express the shares of 1, 2, and 3 men?
34. If 15 bushels of potatoes cost 7 dollars, what part of a dollar would 1 bushel cost?
35. If 2 bushels of wheat sow 3 acres, what part of a bushel would sow 1 acre?
36. If a cord of wood last 7 weeks, what part of a cord would last 1 week?
37. Divide 16 by 17; what is the quotient?
38. If 2 be a dividend, and 21 the divisor, what must be the quotient?
39. Divide 17 by 123.
40. Divide 84 by 1725.
41. Divide 1728 by 1837.
42. Express the division of 37 by 25.
43. Express the division of 25 by 36.
44. Express the division of 81 by 75.
45. Express the division of 16 by 9.
46. Divide 7 by 11.

89. MODEL OF A RECITATION.

1. If a man receive 125 dollars for $\frac{1}{4}$ of his annual salary, what is his salary?

Since $\frac{4}{4}$ of anything make the *whole* of that thing, ([illegible],) if $\frac{1}{4}$ of his salary is 125 dollars, $\frac{4}{4}$, or the whole of his salary, will be 4 times 125 dollars, equal to 500 dollars, which is the answer required.

2. 18 is $\frac{1}{8}$ of what number?

Since $\frac{8}{8}$ of any number make the whole of that number, if one eighth of some number is 18, the whole of that number will be 8 times 18, equal to 144, which is the answer required.

90. Exercises in Finding the Whole of a Quantity from a single Part of it.

In like manner, solve and explain the following problems.

1. If $\frac{1}{2}$ of a bushel of corn cost 42 cents, what is that a bushel?

2. 42 is $\frac{1}{2}$ of what number?

3. If $\frac{1}{3}$ of an acre produce 23 bushels of corn, how many bushels would 1 acre of land produce?

4. 23 is $\frac{1}{3}$ of what number?

5. If $\frac{1}{4}$ of the annual rent of a house be 75 dollars, how much is that for a year?

6. 75 is $\frac{1}{4}$ of what number?

7. 25 is $\frac{1}{5}$ of what number?

8. 33 is $\frac{1}{6}$ of what number?

9. 16 is $\frac{1}{7}$ of what number?

10. If $\frac{1}{8}$ of a mile is 40 rods, how many rods in a mile?

11. If $\frac{1}{9}$ of a hogshead be 7 gallons, how many gallons in that hogshead?

12. If $\frac{1}{40}$ of an acre is 4 square rods, how many square rods in an acre?

13. If 60 minutes be $\frac{1}{24}$ of a day, how many minutes in a day?

14. If 1 day be $\frac{1}{365}$ of a year, how many days in a year?

15. At 35 dollars for working $\frac{1}{12}$ of a year, how much is that for a year?

16. If 25 cents make $\frac{1}{4}$ of a dollar, how many cents in a dollar?

17. 62 is $\frac{1}{8}$ of what number?

18. 18 is $\frac{1}{25}$ of what number?

19. John, being 12 years old, was only $\frac{1}{7}$ as old as his grandfather. How old was John's grandfather?

91. Model of a Recitation.

1. John having 100 cents, paid away $\frac{1}{4}$ of them for a penknife. How many cents did his penknife cost?

Since it takes *four* $\frac{1}{4}$s of any thing, or number, to make the *whole* of it, (89,) if 100 be divided by 4, the quotient will be $\frac{1}{4}$ of 100 cents, equal to 25 cents, which is the answer required.

2. What is $\frac{1}{8}$ of 144?

Since it takes $\frac{8}{8}$ of 144 to make the whole of it, divide 144 by 8, and the quotient will be $\frac{1}{8}$ of 144, equal to 18, which is the answer required.

92. Observation.

Observe, *that, the dividend being the product of the divisor and quotient,* (74,) *the divisor shows how many equal parts, such as the quotient,* (63,) *will make the dividend.*

Therefore, to ascertain any single part of a number, *divide it by the number which shows how many such parts make the integer, or given number.*

93. Exercises in Finding a single Part of a Quantity from the Whole of it.

In like manner, solve and explain the following problems.

1. If $\frac{1}{2}$ of 100 cents be paid for a penknife, how many cents would the penknife cost?

2. How many cents in $\frac{1}{3}$ of a dollar?

3. How many cents in $\frac{1}{5}$ of a dollar?—in $\frac{1}{6}$ of a dollar?—in $\frac{1}{8}$ of a dollar?—in $\frac{1}{10}$ of a dollar?—in $\frac{1}{12}$ of a dollar?—in $\frac{1}{16}$ of a dollar?

4. If a ton of hay cost 21 dollars, what would $\frac{1}{7}$ of a ton cost?

5. What is $\frac{1}{9}$ of 63?—of 72?—of 81?—of 90?—of 99?—of 108?

6. What is $\frac{1}{11}$ of each of the following numbers: 11, 22, 33, 44, 55, 99, and 132?

7. If a man, owning 279 acres of land, sell $\frac{1}{9}$ of it; how many acres would he sell?

8. If 160 square rods make an acre, how many rods in $\frac{1}{4}$ of an acre?

9. If 320 rods make a mile in distance, how many rods in $\frac{1}{4}$ of a mile.

10. A furlong being $\frac{1}{8}$ of a mile, how many rods in a furlong?

11. In a day there are 1440 minutes. How many minutes in $\frac{1}{24}$ of a day, or in one hour?

12. In a pound there are 960 farthings. How many farthings in a shilling, which is $\frac{1}{20}$ of a pound?

13. If a slaughtered ox weigh 896 pounds, what would be the weight of each quarter, the quarters being equal?

14. A man hired a farm "at the halves," and raised 624 bushels of potatoes, 150 bushels of rye, 64 bushels of wheat, 75 bushels of oats, 12 bushels of white beans, 50 bushels of turnips, 25 bush. of corn, 45 bush. of winter apples, and 40 bushels of sauce apples. How many bushels in his share of this produce?

15. If you could buy 480 apples for a dollar, how many could you buy up for $\frac{1}{2}$ of a dollar?—for $\frac{1}{4}$ of a dollar?—for $\frac{1}{8}$ of a dollar?—for $\frac{1}{16}$ of a dollar?

16. If a man's salary be 800 dollars a year, how much is that for $\frac{1}{2}$ of a year?—for $\frac{1}{4}$ of a year?—for $\frac{1}{12}$ of a year?

17. If 32 quarts of nuts be divided equally among 4 boys, what part, and how much of them, is each boy's share?

18. Divide 64 by 16; what part of 64 is the quotient?

19. If you divide any number by 4, what part of that number will be the quotient?

20. What is $\frac{1}{4}$ of 1000 dollars?

94. Exercises in the Different Modes of Considering and Reading Fractions.

$\frac{1}{4}$ of 1 is $\frac{1}{4}$, and $\frac{1}{4}$ of 3 is three times as much, or $\frac{3}{4}$ of 1; $\frac{1}{4}$ of 5 is $\frac{5}{4}$ of 1; $\frac{1}{4}$ of 13 is $\frac{13}{4}$ of 1; $\frac{1}{5}$ of 3 is $\frac{3}{5}$ of 1; $\frac{1}{7}$ of 5 is $\frac{5}{7}$ of 1.

1. How many ninths of 1 is $\frac{1}{9}$ of 7?
2. $\frac{1}{3}$ of 10 is how many thirds of 1?
3. $\frac{1}{8}$ of 11 is what part of 1?
4. What part of 1 is $\frac{1}{16}$ of 25?
5. Read the following fractions in the four different modes described (84).

$$\frac{5}{8},\ \frac{7}{11},\ \frac{9}{14},\ \frac{2}{9},\ \frac{18}{20},\ \frac{20}{13},\ \frac{12}{7},\ \frac{7}{18},\ \frac{9}{10},\ \frac{10}{9},\ \frac{17}{25},\ \frac{25}{17},\ \frac{50}{63},\ \frac{45}{8},\ \frac{24}{12},\ \frac{8}{2},$$
$$\frac{175}{180},\ \frac{1}{5},\ \frac{8}{3},\ \frac{5}{5},\ \frac{3}{28},\ \frac{5}{14},\ \frac{11}{7}.$$

6. Which of these fractions expresses the greatest number of parts?

7. Which expresses the largest parts?
8. Which expresses the smallest parts?
9. Which expresses the smallest number of parts?
10. Which express the same number of parts?
11. Which express parts of the same size?
12. Which express just parts enough to make a unit?
13. Which express parts enough to make more than one unit?
14. Considering both the number and size of the parts, which is the largest fraction?
15. Which is the smallest fraction?
16. Why, of two fractions having equal denominators, is that *greatest* which has the *greatest* numerator?
17. Why, of two fractions having equal numerators, is that *greatest* which has the *smallest* denominator?
18. What effect is produced upon the value of a fraction by diminishing its numerator?
19. What effect is produced upon the value of a fraction by increasing its denominator?

95. Expression, Definition, and Reduction of an Improper Fraction.

As there is no limit to the number of parts that may be expressed by a fraction, (**83**,) it is often convenient to express in one fraction, more parts than there are of that size, in one unit.

But a fraction whose *value is equal to, or greater than its unit*, is called an *improper fraction;* and a fraction whose *value is less than its unit*, is called a *proper fraction.*

The value of a fraction being the quotient resulting from the division of its numerator by its denominator, (**86**,) an improper fraction may be reduced (**208**) to its equal integral, or mixed number, by *performing the division*, which is only *expressed* by the fraction.

96. Model of a Recitation.

1. A toll-gatherer took in one week $\frac{165}{16}$ of a dollar, (fourpence-half-pennies;) how many dollars would they make?

Since there were 165 such parts of a dollar, that every 16 of them would make a dollar, (**82**,) they would make as many dollars as there are times 16 in 165. Thus:

$\frac{165}{16} = 10\frac{5}{16}$ dollars, which is the answer required.

97. EXERCISES IN THE REDUCTION OF IMPROPER FRACTIONS.

In like manner, solve and explain the following problems.

1. At a certain contribution, $\frac{75}{8}$ of a dollar (ninepences) were taken; how many dollars were taken?
2. A merchant sold calico for $\frac{1}{6}$ of a dollar a yard, till he received $\frac{1272}{6}$ of a dollar; how many dollars did he receive?
3. At a large party, $\frac{58}{6}$ of a pie were eaten, how many whole pies were eaten?
4. In $\frac{150}{32}$ of a bushel how many bushels?
5. In $\frac{387}{20}$ of a pound how many pounds?
6. In $\frac{437}{12}$ of a shilling how many shillings?
7. In $\frac{322}{28}$ of a guinea how many guineas?
8. In $\frac{1728}{24}$ of a day how many days?
9. In $\frac{9737}{60}$ of an hour how many hours?
10. In $\frac{43842}{365}$ of a year how many years?
11. Reduce $\frac{144}{12}$ to units.
12. Reduce $\frac{1728}{144}$ to an integral number.
13. Reduce $\frac{512}{15}$ to a mixed number.
14. Reduce $\frac{487}{45}$ to a mixed number.
15. Reduce $\frac{8753}{17}$ to an integral, or mixed number.
16. Change $\frac{1836}{25}$ to an integral, or mixed number.
17. Change $\frac{234}{225}$ to an integral, or mixed number.
18. Reduce $\frac{2016}{16}$ to an integral, or mixed number.
19. How many units in $\frac{16128}{32}$?
20. What mixed number is equal to $\frac{999}{125}$?
21. What is the value of $\frac{555}{444}$ in a mixed number?

98. MODEL OF A RECITATION.

1. Reduce $5\frac{3}{16}$ to an improper fraction, that is, to *sixteenths.*

Since there are 16 sixteenths in *one* unit, there will be 16 times as many sixteenths as units in any number.

16 times 5 are 80 *sixteenths*, and the other 3 sixteenths are $\frac{83}{16}$, which is the answer required.

$$5\tfrac{3}{16}=\tfrac{83}{16}.$$

99. EXERCISES IN REDUCING INTEGRAL AND MIXED NUMBERS TO FRACTIONS.

In like manner, solve and explain the following problems.

1. Reduce 7 to sixteenths.
2. Reduce $25\frac{3}{4}$ to an improper fraction.

3. Change $12\frac{4}{5}$ to an improper fraction.
4. What fraction is equal to $3\frac{11}{15}$?
5. What is $10\frac{4}{7}$ equal to in a fractional form?
6. $13\frac{4}{9}$ are how many ninths?
7. How many *eighths* of *one* dollar are 9 *whole* dollars?
8. How many $\frac{1}{4}$s of a yard are 32 yards?
9. $15\frac{11}{24}$ days are how many $\frac{1}{24}$s of a day?
10. $8\frac{7}{20}$ pounds are how many $\frac{1}{20}$s of a pound?
11. $17\frac{43}{60}$ hours are equal to how many $\frac{1}{20}$s of an hour?
12. $6\frac{12}{63}$ hogsheads are equal to how many $\frac{1}{63}$s of a hogshead?
13. How many $\frac{1}{365}$s of a year are equal to 10 years?
14. Reduce $437\frac{2}{11}$ to an improper fraction.
15. Reduce $10\frac{161}{365}$ to an improper fraction.
16. Reduce $25\frac{47}{100}$ to an improper fraction.
17. What fraction is equal to $50\frac{11}{50}$?
18. Change 20 to sevenths.
19. Reduce 36 to twelfths.
20. Reduce 15 to fifths; also to sixths.
21. Change 4 to halves, to thirds, to fourths, to fifths, and to sixths.
22. Reduce 16 to halves, to thirds, to fourths, and to fifths.
23. Reduce 1 to halves, to fifteenths, and to seventy-fifths.
24. Reduce 1 to halves, thirds, fourths, fifths, sixths, and to sevenths.

100. MODEL OF A RECITATION.

1. A man bought 25 yards of calico, at $\frac{3}{16}$ of a dollar (3 fourpence-half-pennies) a yard; how many dollars did his calico cost?

Since 1 yard cost $\frac{3}{16}$ of a dollar, 25 yards would cost 25 *times as many sixteenths* of a dollar, which are $\frac{75}{16}$ of a dollar; equal to $4\frac{11}{16}$ dollars, (**95**,) the answer required.

$$\frac{3\times 25}{16}=\frac{75}{16}=4\frac{11}{16}\text{ dollars.}$$

101. OBSERVATION.

OBSERVE, (**100**,) *that in* MULTIPLYING THE NUMERATOR ONLY *by* 25, *retaining the same denominator, you* MULTIPLY THE FRACTION; *for thus, you produce* 25 TIMES AS MANY PARTS (**83**) *of the same size.*

102. Exercises in Multiplying a Fraction by an Integral Number.

In like manner, solve and explain the following problems.

1. How many dollars will 25 penknives come to, at $\frac{3}{8}$ of a dollar apiece?

2. How many dollars would pay a man to work 5 days, at $\frac{3}{4}$ of a dollar per day?

3. How many dollars should Mr. Farmer receive for 12 bushels of corn, at $\frac{5}{6}$ of a dollar a bushel?

4. At $\frac{1}{12}$ of a dollar a pound for beef, how much would 11 pounds cost?

5. If a family consume $\frac{1}{7}$ of a barrel of flour in a week, how much flour would last them a year?

6. If it take $\frac{5}{8}$ of a bushel of rye to sow an acre, 15 acres would require how many bushels?

7. If a horse eat $\frac{1}{3}$ of a bushel of oats in a day, how much would keep him through December?

8. If 1 bushel of apples cost $\frac{1}{4}$ of a dollar, what would be the value of a load containing 33 bushels?

9. At $\frac{1}{2}$ of a dollar a day for board, what would be the cost of board for 365 days?

10. How far can I ride in 1 hour at the rate of $\frac{4}{11}$ of a mile per minute?

11. How much is 5 times $\frac{11}{16}$?
12. Multiply $\frac{7}{24}$ by 13.
13. Multiply $\frac{7}{100}$ by 43.
14. Multiply $\frac{11}{365}$ by 36.
15. Multiply $\frac{41}{50}$ by 3.
16. How much is 15 times $\frac{75}{76}$?
17. Multiply $\frac{1}{365}$ by 366.
18. How much is 3 times $\frac{53}{1728}$?

103. Model of a Recitation.

1. At $32\frac{5}{8}$ dollars apiece, what would 7 cows cost?

$$\begin{array}{r} 32\frac{5}{8} \\ 7 \\ \hline 228\frac{3}{8} \text{ dolls.} \end{array}$$

Since 1 cow cost $32\frac{5}{8}$ dollars, 7 cows would cost 7 times $32\frac{5}{8}$.

Seven times $\frac{5}{8}$ are $\frac{35}{8}$, equal to $\frac{3}{8}$ which write, and 4 units, which add with the units, &c. (**27**)

2. How much is 83 times $16\frac{2}{3}$ feet?

It will be most convenient, in this example, to multiply the

integral and fractional parts separately, and add the products together. Thus:

$$\begin{array}{r} 16\frac{2}{3} \\ 83 \\ \hline 48 \\ 128 \\ 55\frac{1}{3} \\ \hline 1383\frac{1}{3} \end{array} \text{ feet.} \qquad \frac{2}{3}\times 83 = \frac{166}{3} = 55\frac{1}{3} \text{ feet.}$$

104. EXERCISES IN MULTIPLYING A MIXED NUMBER BY AN INTEGRAL NUMBER.

In like manner, solve and explain the following problems.

1. If $1\frac{7}{8}$ yards are sufficient for one coat, how many yards will be sufficient for 10 coats?
2. How many feet in 25 rods, there being $16\frac{1}{2}$ feet in 1 rod?
3. How many yards in 40 rods, there being $5\frac{1}{2}$ yards in 1 rod?
4. How many cents in 6 shillings, there being $16\frac{2}{3}$ cents in 1 shilling?
5. How old is John, if he is 3 times as old as Charles, and Charles is $3\frac{31}{365}$ years old?
6. What would be the cost of 15 barrels of flour, at $6\frac{5}{8}$ dollars per barrel?
7. If $31\frac{1}{2}$ gallons make a barrel, how many gallons in 50 barrels?
8. What is the price of a dozen bibles at $2\frac{5}{6}$ dollars apiece?
9. What is the cost of 10 dozen pairs of shoes at $1\frac{1}{8}$ dollars a pair?
10. What would 7 tons of Lehigh coal cost at $9\frac{5}{8}$ dollars a ton?
11. What would 17 grind-stones come to at $3\frac{5}{12}$ dollars apiece?
12. Multiply $6\frac{11}{12}$ by 35.
13. How much is 100 times $2\frac{7}{541}$?
14. What is the product of $1\frac{25}{49}$ multiplied by 5?
15. How much is $16\frac{2}{3} \times 10$?
16. Multiply $1728\frac{144}{3333}$ by 7.

105. MODEL OF A RECITATION.

1. At $\frac{3}{16}$ of a dollar (3 fourpence-half-pennies) apiece, what would be the postage of 4 letters?

Since the postage of 1 letter is $\frac{3}{16}$ of a dollar, the postage of 4 letters would be 4 times as much.

This product can be ascertained, either by *multiplying the numerator* by 4, retaining the *same denominator;* (**101**,) or, far better, by *dividing the denominator* by 4, retaining the *same numerator.*

For, by the former process, you make the *number* of parts 4 times as large, the parts retaining the *same size;* and, by the latter process, you make the *size* of the parts 4 times as large, retaining the *same number of parts.*

It is evident that, by the latter process, *the parts are made 4 times as large,* from the fact that, *it will take only $\frac{1}{4}$ as many of them to make the unit as before.*

$\frac{12}{16}$ of a dollar are 12 fourpence-half-pennies, and $\frac{3}{4}$ of a dollar are also 12 fourpence-half-pennies; for $\frac{1}{4}$ of a dollar is equal to 4 fourpence-half-pennies, and $\frac{3}{4}$ of a dollar will be 3 times as many, or 12 fourpence-half-pennies.

$\frac{3 \times 4}{16} = \frac{12}{16}$ of a dollar.

$\frac{3}{16 \div 4} = \frac{3}{4}$ of a dollar.

The two processes giving the same result, the latter is to be adopted in all cases when the multiplier is a factor (**25**) of the denominator; because it will give the result in *lower terms,* $\frac{3}{4}$ being in lower terms, and, consequently, a more simple fraction than its equal $\frac{12}{16}$.

2. At $\frac{3}{8}$ of a dollar a bushel, what would be the price of 8 bushels of potatoes?

Since the price of 1 bushel is $\frac{3}{8}$ of a dollar, the price of 8 bushels would be 8 times as much, which is 3 dollars, the answer required.

$\frac{3}{8 \div 8} = \frac{3}{1} = 3$ dollars.

For, by dividing the denominator by 8, the parts become 8 times as large, and such that each one of them makes a unit.

106. Observation.

Observe, (**105**,) *that by whatever number the* DENOMINATOR IS DIVIDED, *retaining the same numerator,* THE FRACTION IS THUS MULTIPLIED BY THAT NUMBER; *for the denominator showing the number of parts that make a unit,* THEIR SIZE IS INCREASED IN THE SAME RATIO THAT THE DENOMINATOR IS DIMINISHED.

Observe, also, that if a fraction be multiplied by its denominator, the product will be the numerator.

107. Exercises in Multiplying a Fraction by dividing its Denominator.

In like manner. solve and explain the following problems.

1. If 1 yard of calico cost $\frac{1}{4}$ of a dollar, what would be the cost of 2 yards? What would be the cost of 4 yards?

2. At $\frac{1}{6}$ of a dollar a pound, what will 2 pounds of butter cost? What will 3 pounds cost? What will 6 pounds cost?

3. At $\frac{1}{8}$ of a dollar apiece, what would be the postage of 2 letters? of 4 letters? of 8 letters?

4. If 1 ninepence is $\frac{1}{8}$ of a dollar, what part of a dollar is 2 ninepences? is 4 ninepences? is 8 ninepences?

5. If 1 fourpence-half-penny is $\frac{1}{16}$ of a dollar, what part of a dollar is 2 fourpence-half-pennies? is 4 fourpence-half-pennies? is 8 fourpence-half-pennies? is 16 fourpence-half-pennies?

6. If a sixpence is $\frac{1}{12}$ of a dollar, what part of a dollar is 2 sixpences? is 3 sixpences? is 4 sixpences? is 6 sixpences? is 12 sixpences?

7. At $\frac{7}{9}$ of a yard apiece for vests, how much satin would be necessary for 3 vests? for 9 vests?

8. At $\frac{23}{60}$ of a mile a minute, how far would a train of cars run in 2 minutes? in 3 minutes? in 4 minutes? in 5 minutes? in 6 minutes? in 10 minutes? in 12 minutes? in 15 minutes? in 20 minutes? in 1 hour?

9. If it take $1\frac{19}{24}$ yards of broad cloth to make a coat, how much would it take for 3 coats? for 6 coats? for 8 coats? for 12 coats? for 24 coats?

10. Multiply $\frac{4}{25}$ by 5.
11. Multiply $\frac{23}{63}$ by 7.
12. Multiply $\frac{73}{125}$ by 25.
13. Multiply $\frac{23}{100}$ by 100.
14. Multiply $\frac{19}{16}$ by 16.
15. Multiply $7\frac{3}{4}$ by 4.
16. Multiply $4\frac{122}{365}$ by 365.
17. How much is 20 times $9\frac{17}{20}$?
18. How much is 327 times $10\frac{125}{327}$?
19. $\frac{12}{5}$ is $\frac{1}{5}$ of what number?
20. $5\frac{3}{8}$ is $\frac{1}{8}$ of what number?
21. $125\frac{3}{50}$ is $\frac{1}{10}$ of what number?
22. $\frac{52}{7}$ is $\frac{1}{7}$ of what number?
23. Multiply $\frac{3}{40}$ by 5, and that product by 3

24. Multiply $\frac{7}{60}$ by 5, and that product by 5.
25. Multiply $\frac{5}{72}$ by 3, and that product by 5.

108. MODEL OF A RECITATION.

1. Multiply $\frac{4}{45}$ by 36.

Since 36 is not a factor of the denominator, but 9, one of the factors of 36, is also a factor of the denominator, multiply first by 9, (**99**,) by making the parts 9 times as large, (**106**,) and then multiply that product by 4, the other factor of 36, by making 4 times as many parts, (**101**,) which will give 4 times 9 times, or 36 times $\frac{4}{45}$, equal to $\frac{16}{5}$, equal to $3\frac{1}{5}$, which is the product required.

$$\frac{4 \times 4}{45 \div 9} = \frac{16}{5} = 3\frac{1}{5}.$$

109. EXERCISES IN MULTIPLYING A FRACTION BY THE FACTORS OF THE MULTIPLIER.

In like manner, solve and explain the following problems.

1. Multiply $\frac{5}{27}$ by 18.
2. Multiply $\frac{11}{14}$ by 35.
3. Multiply $\frac{17}{32}$ by 48.
4. How much is 24 times $\frac{2}{15}$?
5. How much is 50 times $\frac{18}{75}$?
6. How much is 81 times $\frac{17}{27}$?

110. MODEL OF A RECITATION.

1. If 3 yards of calico cost $\frac{9}{16}$ of a dollar, (9 fourpence-half-pennies,) what would be the price of 1 yard?

Since 1 yard is $\frac{1}{3}$ of 3 yards, the *price* of 1 yard must be $\frac{1}{3}$ of the *price* of 3 yards. Therefore, as the price of 3 yards is $\frac{9}{16}$ of a dollar, the price of 1 yard will be $\frac{1}{3}$ (**92**) *as many sixteenths* of a dollar, equal to $\frac{3}{16}$, the answer required.

$$\frac{9 \div 3}{16} = \frac{3}{16} \text{ of a dollar.}$$

111. OBSERVATION.

OBSERVE that, in dividing *the numerator only* by 3, retaining the *same denominator*, you divide the *fraction;* for thus you obtain $\frac{1}{3}$ *as many parts* (**83**) *of the same size.*

112. MODEL OF A RECITATION.

1. At 6 dollars a barrel, how many barrels of flour may be bought for $45\frac{3}{11}$ dollars?

Since 1 barrel costs 6 dollars, you may buy $\frac{1}{6}$ (92) as many barrels as you have dollars; $\frac{1}{6}$ of 42 is 7. Reduce the remaining 3 to elevenths making 33, these and the other 3 elevenths are $\frac{36}{11}$, $\frac{1}{6}$ of which are $\frac{6}{11}$ of a barrel, which written with the 7 barrels make $7\frac{6}{11}$ barrels, the answer required.

$$6\,)\,45\tfrac{3}{11}$$
$$7\tfrac{6}{11} \text{ barrels.}$$

113. Exercises in Dividing a Fraction by an Integral Number.

In like manner, solve and explain the following problems.

1. If 2 bushels of potatoes cost $\frac{4}{5}$ of a dollar, what would that be a bushel?
2. If a cow consume $\frac{3}{4}$ of a bushel of meal in 3 days, how much would that be per day?
3. At $\frac{12}{50}$ of a dollar for 4 pounds of beef, what would be the cost of 1 pound?
4. If 4 horses consume $\frac{16}{11}$ of a ton of hay in a month, how much would that be for 1 horse?
5. At $\frac{21}{25}$ of a dollar for 7 pounds of coffee, what would be the cost of 1 pound?
6. If 2 yards of cloth cost $8\frac{2}{5}$ dollars, what would 1 yard cost at that rate?
7. What would be the cost of 1 bushel of wheat, if 4 bushels cost $32\frac{4}{5}$ shillings?
8. If I give $23\frac{2}{5}$ bushels of wheat for 3 sheep, how much would that be apiece?
9. If I give $59\frac{1}{8}$ bushels of corn for 7 calves, how many bushels would that be apiece?
10. If $20\frac{5}{8}$ dollars be paid for 15 days' work, how much would that be per day?
11. How far per hour is $88\frac{1}{11}$ miles in 17 hours?
12. How far per day is $476\frac{4}{5}$ miles' travel in 8 days?
13. If 15 men divide among themselves $77\frac{8}{11}$ barrels of apples, what would be the share of each man?
14. If 23 yards of cloth cost $152\frac{3}{8}$ dollars, what would that be a yard?
15. How much is the cost of 1 yard of cotton cloth, when $3\frac{85}{100}$ dollars are given for 35 yards?
16. How many times is 25 contained in $59\frac{3}{8}$?
17. What is $\frac{1}{16}$ (92) of $148\frac{4}{11}$?
18. Divide $5\frac{1}{7}$ by 12?

19. How many times is 9 contained in $47\frac{3}{17}$?
20. What is $\frac{1}{2}$ of $1\frac{1}{5}$?
21. What is $\frac{1}{3}$ of $2\frac{1}{4}$?
22. What is $\frac{1}{4}$ of $18\frac{2}{7}$?
23. Divide $4\frac{3}{8}$ by 5.
24. Divide $1731\frac{3}{7}$ by 12.
25. Divide $65542\frac{10}{41}$ by 256.
26. Divide $16388\frac{4}{31}$ by 128.

114. MODEL OF A RECITATION.

1. If the postage of 4 letters, between the same towns, be $\frac{3}{4}$ of a dollar, how much would that be apiece?

Since 1 letter is $\frac{1}{4}$ of 4 letters, the postage of 1 letter must be $\frac{1}{4}$ of the postage of 4 letters.

$\frac{3}{4 \times 4} = \frac{3}{16}$ of a dollar.

Therefore, as the postage of 4 letters is $\frac{3}{4}$ of a dollar, the postage of 1 letter would be $\frac{1}{4}$ of $\frac{3}{4}$ of a dollar. But the *number* of parts not having 4 for a factor, you must perform the division upon the *size* of the parts, which you can do by *multiplying the denominator by the divisor*, retaining the same numerator.

It is evident that, by *multiplying the denominator* by 4, the parts are made $\frac{1}{4}$ as large, from the fact that, *it will take 4 times as many of them to make the unit as before.* It takes only 4 *fourths* of a dollar (4 quarters of a dollar) to make a dollar; whereas it takes 4 *times as many*, or 16 *sixteenths* of a dollar, (16 fourpence-half-pennies,) to make a dollar.

115. OBSERVATION.

OBSERVE, (**114**,) *that by whatsoever number the* DENOMINATOR *of a fraction be* MULTIPLIED, *retaining the same numerator*, THE FRACTION IS THUS DIVIDED BY THAT NUMBER; *for the denominator showing the number of parts that make the unit*, (**83**,) *their* SIZE IS DIMINISHED IN THE SAME RATIO THAT THE DENOMINATOR IS INCREASED.

Observe, also, that the division of a fraction may be performed, either upon the NUMBER OF THE PARTS, *their size remaining the same*, (**110**,) *or upon* THE SIZE OF THE PARTS, *their number remaining the same*, (**114**;) *but, that the former process is to be adopted in all cases when the divisor is a factor of the numerator, because it will give the result in lower terms.*

116. Exercises in Dividing a Fraction by Multiplying its Denominator.

In like manner, solve and explain the following problems.

1. If 2 boys having $\frac{1}{2}$ of a melon divide it equally between themselves, what would be the share of each?

2. What is $\frac{1}{2}$ of $\frac{1}{2}$?

3. Suppose $\frac{1}{4}$ of a pie to be cut into 2 equal pieces, what part of the whole pie would each piece be? What is $\frac{1}{2}$ of $\frac{1}{4}$?

4. A boy having $\frac{3}{4}$ of a dollar, gave $\frac{1}{3}$ of it for a penknife. What part of a dollar did his knife cost? What is $\frac{1}{3}$ of $\frac{3}{4}$?

5. If 2 shillings are $\frac{1}{4}$ of a dollar, what part of a dollar is 1 shilling?

6. If a boy having $\frac{1}{5}$ of a pie should give $\frac{1}{2}$ of it to his sister, what part of a pie would he give away, and what part would he keep?

7. If $\frac{1}{2}$ of a dollar be divided equally among 3 boys, what part of a dollar is the share of each?

8. If you should make a circle on your slate, and draw a line across it through the centre, how many parts would you make of it? What would be the name of each part?

9. If from the centre of said circle you draw a line through the middle of one half, making two parts of that half, how many such parts would make the whole circle? What is the name for such parts? What is $\frac{1}{2}$ of $\frac{1}{2}$?

10. If from the centre of said circle you draw a line through the middle of one fourth, making two parts of that fourth, how many such parts would make the whole circle? What is the name for such parts? What is $\frac{1}{2}$ of $\frac{1}{4}$?

11. What is $\frac{1}{2}$ of $\frac{1}{8}$? What is $\frac{1}{2}$ of $\frac{1}{16}$?

12. If 3 pounds of butter cost $\frac{3}{8}$ of a dollar, what is that a pound?

13. At $\frac{3}{4}$ of a dollar for 4 bushels of apples, what would be the cost of a bushel?

14. At $\frac{3}{4}$ of a dollar for 7 gallons of vinegar, what would be the cost of a gallon?

15. If 6 bushels of wheat cost $4\frac{5}{8}$ dollars, what would that be a bushel?

16. If 4 dollars buy $5\frac{3}{8}$ bushels of rye, how much would one dollar buy?

If 4 dollars buy $3\frac{1}{3}$ yards of silk, how much might be bought for 1 dollar?

18. If 18 pounds of raisins cost $2\frac{3}{7}$ dollars, what is that a pound?

19. If 16 hats cost $48\frac{1}{2}$ dollars, what would 1 hat cost?

20. What would 1 yard of broadcloth cost, if 25 yards cost $150\frac{3}{4}$ dollars?

21. How far per hour would a train of cars go, if it run $125\frac{1}{3}$ miles in 7 hours?

22. Divide $24\frac{3}{5}$ by 7.

23. What part (**87**) of 5 is 2?

24. What part of 5 is $2\frac{1}{3}$?

25. What part of 5 is $1\frac{1}{4}$?

26. What part of 12 is 7?

27. What part of 12 is $3\frac{3}{11}$?

117. Illustration of the Principle of Dividing by the Factors of the Divisor.

By multiplying one number by another, we introduce into the multiplicand all the factors composing the multiplier, (**29**,) and the product will be composed of all the factors of both multiplier and multiplicand. Also, in dividing one number by another, we take from the dividend all the factors composing the divisor, (**75**,) and the quotient will be composed of all those factors composing the dividend, which are not necessary to compose the divisor. Thus, by multiplying $35 = 7 \times 5$ by $33 = 3 \times 11$, we obtain $1155 = 7 \times 5 \times 3 \times 11$. Now, by dividing $1155 = 7 \times 5 \times 3 \times 11$ by $105 = 7 \times 5 \times 3$, the quotient is the remaining factor, 11; or, if we divide by $35 = 7 \times 5$ the quotient is $33 = 3 \times 11$, the remaining two factors; or, if we divide by 7, the quotient is $165 = 5 \times 3 \times 11$, the product of the remaining three factors.

Consequently, by dividing the product of several factors by some of them, the quotient will be the product of the others.

Also, when convenient, we may separate a divisor into factors, and take them from the dividend, one at a time, that is, divide first by one factor, then divide the quotient, thus obtained, by another factor, and so on, with all the factors of the divisor; the last quotient will be the quotient required. Thus, instead of dividing 1155 directly by 21, we may divide first by 7, obtaining 165, which divided by 3 gives 55, the true quotient.

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118. Model of a Recitation.

1. Divide 875 by 35.

$$\begin{array}{r} 7\,)\,875 \\ \hline 5\,)\,125 \\ \hline 25 \end{array}$$

The quotient is $\frac{1}{35}$ of the dividend. Divide first by 7, one of the factors of the divisor, to obtain $\frac{1}{7}$ of the dividend, and then divide the quotient thus obtained, by 5, the other factor of the divisor, to obtain $\frac{1}{5}$ of $\frac{1}{7}$, or $\frac{1}{35}$ of the dividend, as required.

2. Divide $3\frac{1}{5}$ by 36.

$3\frac{1}{5} = \frac{16}{5}$.

$\frac{16}{5} \div 4 \times 9 = \frac{4}{45}$.

Since 36 is not a factor of the numerator, but 4, one of the factors of 36, is also a factor of the numerator, divide first by 4, by taking $\frac{1}{4}$ as many parts, (**111**,) and then divide that quotient by 9, the other factor of 36, by making the parts $\frac{1}{9}$ as large, (**114**,) which will give $\frac{1}{9}$ of $\frac{1}{4}$, or $\frac{1}{36}$ of $\frac{16}{5}$, equal to $\frac{4}{45}$, which is the true quotient required.

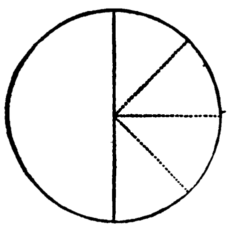
119. Exercises in Dividing by the Factors of the Divisor.

In like manner, solve and explain the following problems.

1. Divide 1421 by 49.
2. How many times is 72 contained in 1728?
3. How many casks of 63 gallons each, may be filled from 7875 gallons?
4. If a horse travel $\frac{8}{9}$ of a mile in 12 minutes, how far would he travel per minute?
5. If 21 dollars buy $3\frac{1}{2}$ barrels of flour, what part of a barrel would 1 dollar buy?
6. How many times is 14 contained in $72\frac{1}{2}$?
7. Divide $12\frac{1}{2}$ by 15.
8. Divide $108\frac{6}{7}$ by 18.
9. How many times is 30 contained in $72\frac{1}{2}$?
10. What part of a time is 27 contained in $3\frac{3}{11}$?
11. What part of a time is 28 contained in $8\frac{2}{5}$?
12. How many times is 36 contained in $42\frac{6}{7}$?
13. Divide 175 by 21.
14. Divide 1836 by 24.
15. What is the quotient of 960 divided by 45?
16. Divide $2\frac{2}{17}$ by 24.
17. Divide $38\frac{18}{19}$ by 36.

18. Divide 3492 by 81.
19. What part of 15 is 10?
20. What part of 18 is $5\frac{1}{4}$?
21. What part of 21 is $4\frac{1}{5}$?
22. If 8512 be the product of three factors, two of which are 8 and 19, what is the third factor?
23. If 17160 be the product of 8, 11, 13, and two other factors, what are the other two factors?
24. One of two factors composing 1625 is 25. What is the other?
25. Divide $17 \times 19 \times 10$ by 19.
26. Divide $12 \times 14 \times 9 \times 6$ by 12×6.
27. How many times is $3 \times 5 \times 7$ contained in $9 \times 10 \times 14$?
28. What is the quotient of $16 \times 39 \times \frac{7}{8}$ divided by $2 \times 7 \times 13$?

120. ILLUSTRATION OF THE PRINCIPLE OF REDUCING A FRACTION TO OTHER TERMS OF EQUAL VALUE.



Make a circle on your slate, and draw a line across it through its centre, making two half-circles. From the centre draw a line through the middle of one of these halves, and from the same point draw a line through the middle of each of the two fourths made of this half by the last line; thus making the $\frac{1}{2} = \frac{4}{8}$. Now erase the three lines last drawn; thus making the $\frac{4}{8} = \frac{1}{2}$ again.

121. OBSERVATION.

OBSERVE, (**120**,) *that both terms in* $\frac{1}{2}$ *are made* 4 *times as large in its equal fraction* $\frac{4}{8}$; *that is, both terms in* $\frac{1}{2}$ *have been multiplied by* 4; *thus making the parts* 4 *times as many, but* $\frac{1}{4}$ *as large in* $\frac{4}{8}$ *as in* $\frac{1}{2}$.

So, multiplying BOTH TERMS *of a fraction by any number, will reduce it to an* EQUAL FRACTION *in higher terms. For, while it* MULTIPLIES *the fraction by* INCREASING THE NUMBER *of parts,* (**101**,) *it also* DIVIDES *it by* DIMINISHING THE SIZE

of the parts (**114**) *in the same ratio as their number is increased.*

OBSERVE, *also, that* BOTH TERMS *in* $\frac{4}{8}$ *are made* $\frac{1}{4}$ *as large in its equal fraction* $\frac{1}{2}$; *that is, both terms in* $\frac{4}{8}$ *have been divided by* 4: *thus making the parts* $\frac{1}{4}$ *as many, but* 4 *times as large in* $\frac{1}{2}$ *as in* $\frac{4}{8}$.

So, dividing BOTH TERMS *of a fraction by any number, will reduce it to an* EQUAL FRACTION *in lower terms. For, while it* DIVIDES *the fraction, by* DIMINISHING THE NUMBER OF PARTS, (**111**,) *it also* MULTIPLIES *it by* INCREASING THE SIZE OF THE PARTS (**106**) *in the same ratio that their number is diminished.*

122. ILLUSTRATION OF THE MODE OF REDUCING A FRACTION TO LOWER TERMS.

1. Reduce $\frac{21}{84}$ to lower terms.

$3)\ \frac{21}{84} = \frac{7}{28}$

$7)\ \frac{21}{84} = 3)\ \frac{3}{12} = \frac{1}{4}$

$21)\ \frac{21}{84} = \frac{1}{4}$

Since 3, 7, and 21 are factors *common* to both terms of the fraction, you may divide both terms (**121**) by either of these *common factors.*

But *observe*, that, the larger the factor used, the lower will be the terms to which the fraction will be reduced; and that, by using *the greatest common factor*, the fraction will be reduced to its *lowest terms.*

When any other than the greatest common factor is used, the new fraction obtained may be reduced lower, by using some other common factor.

123. FACTORING OF NUMBERS.

In small numbers, the factors and common factors may be ascertained by observation; but in larger numbers other means become necessary.

A *multiple* of a number is a number of which the former number is a factor; as multiples of 3 are 6, 9, &c., of which 3 is a factor.

A *common multiple*, of two or more numbers, is a number of which those numbers are factors.

A number composed of factors is a *multiple* of any one of those factors; and, also, of any combination of its *prime* factors. (**29**.)

When one number is a factor of another, all the factors of the former are also factors of the latter. Thus, 21 being a factor of 63, 7 and 3, the factors of 21, are also factors of 63.

And they *should* be; for 63, being 3 times 21, *should* contain 7 and 3 three times as often as 21 contains them.

Hence, a factor of a number is a factor of any *multiple* of that number.

Any common factor of two numbers is a factor of their *sum*, and of their *difference*. For, each of the numbers containing the common factor a certain number of times, their sum must contain it as many times as both of the numbers; and their difference must contain it as many times as the larger of the numbers contains it times more than the smaller.

Thus, 4, being a common factor of 12 and 20, must be a factor of their sum, and of their difference: for 12 is 3 fours, and 20 is 5 fours; their sum will be 5 fours + 3 fours = 8 fours, or 32; and their difference will be, 5 fours — 3 fours = 2 fours, or 8.

2 is a factor of every even number.

Any number ending with a cipher (**10**) is a multiple of 10, consequently, 10, and the factors of 10, are factors of it.

Any number ending with two ciphers (**10**) is a multiple of 100, consequently, 100, and the factors of 100, are factors of it.

5 is a factor of any number ending with 5; for all of the number but 5 is a multiple of 10 of which 5 is a factor.

Any factors of the last two figures of a number, which are also factors of 100, are factors of the whole number; for all of the number but these two figures is a multiply of 100.

8, being a factor of 200, will be a factor of any number which has even hundreds, if it be a factor of the last two figures of the number.

9 is a factor of any number, when it is a factor of the sum of the digits which express that number. For the *excess* in the value expressed by the digits, above what they would express in the units' place, is a multiple of 9; since every removal of a figure one degree higher causes that figure to express *ten times* its former value, (**4**,) it *gains* by each removal 9 times the value it would have before the removal.

Thus, 10 is 9 more than 1; and 100 is $9 \times 10 = 90$ more than 10, or 99 more than 1, &c. Also, 70 is $9 \times 7 = 63$ more than 7; and 700 is $9 \times 70 = 630$ more than 70, or $9 \times 70 + 9 \times 7 = 693$ more than 7.

Also 3, being a factor of any multiple of 9, is a factor of any number when it is a factor of the sum of the digits which express that number.

Every factor of a number has its corresponding factor, which, together, compose the number, (**75.**)

Hence, to find the prime factors of a number, separate it into two factors, by dividing it by any known factor, and proceed in the same manner with each of these factors, and so on, till the prime factors are all obtained.

124. Model of a Recitation.

Find the *prime* factors of 1296.

Here *observe*, that 12 is a factor, the factors of which are 3, 2, 2. The quotient of 1296 divided by 12 is 108, whose factors are 12, the factors of which are 3, 2, 2; and 9, the factors of which are 3, 3. In all, 3, 2, 2 $\times$ 3, 2, 2 $\times$ 3, 3; or, 3, 3, 3, 3, 2, 2, 2, 2; or 3^4, 2^4, the small 4 being an *index* to show the number of factors like that over which it is placed, (**364.**)

125. Exercises in Factoring Numbers.

In like manner, solve and explain the following problems.

Find the *prime* factors of the following numbers. 72, 88, 120, 612, 336, 648, 930, 924, 936, 450, 360, 966, 870, 684, 396, 432, 2480, 8000, 10449, 10503, 24876.

126. Model of a Recitation.

Reduce $\frac{48}{60}$ to an equal fraction in its lowest terms.

12) $\frac{48}{60} = \frac{4}{5}$. Take $\frac{1}{12}$ as many parts, (**111,**) and make them 12 times as large, (**106,**) which gives $\frac{4}{5}$, the answer required.

127. Exercises in Reducing Fractions to Lower Terms.

In like manner, solve and explain the following problems.

1. Reduce $\frac{8}{12}$ to its lowest terms.
2. Reduce $\frac{9}{15}$, $\frac{12}{18}$, $\frac{14}{21}$, $\frac{15}{21}$, $\frac{15}{20}$, $\frac{15}{25}$ and $\frac{18}{24}$ to their lowest terms.
3. Reduce $\frac{14}{42}$, $\frac{15}{30}$, $\frac{25}{75}$, $\frac{24}{96}$, $\frac{20}{100}$ and $\frac{25}{125}$ to their lowest terms.
4. Reduce $\frac{48}{300}$, $\frac{300}{420}$, $\frac{96}{480}$ and $\frac{120}{360}$ to their lowest terms.
5. Reduce $\frac{800}{4800}$, $\frac{180}{5400}$, $\frac{122}{642}$ and $\frac{144}{1728}$ to their lowest terms.
6. Reduce $\frac{918}{1998}$ to its lowest terms.

128. MODEL OF A RECITATION.

It is often most convenient, in reducing a fraction to its lowest terms, to make use of the greatest common factor of its terms, (**122.**) Hence it will be useful to find a more direct way by which the greatest common factor may always be ascertained.

Reduce $\frac{64}{148}$ to its lowest terms.

64) 148 (2
128
—
20) 64 (3
60
—
4) 20 (5
20
—

4) $\frac{64}{148} = \frac{16}{37}$.

The greatest common factor, being a factor of 148 and of 64, consequently, (**123**,) of $64 \times 2 = 128$, will be a factor of $148 - 128 = 20$, (**123.**) Again, this factor, being a factor of 64 and of 20, consequently, (**123**,) of $20 \times 3 = 60$, will be a factor of $64 - 60 = 4$, (**123.**) But 4, being a factor of 20 and of itself, (**123**,) will be a factor of $20 \times 3 + 4 = 64$, one of the given numbers. Again, 4, being a factor of 64 and of 20, will be a factor (**123**) of $64 \times 2 + 20 = 148$, the other given number.

Hence, as 4 *contains* the greatest common factor of the given numbers, and *is a factor of them*, it must be their greatest common factor. Therefore, take $\frac{1}{4}$ as many parts, and make them 4 times as large, which gives $\frac{16}{37}$, the answer required.

129. OBSERVATION.

OBSERVE, (**128**,) *that, the greater of two given numbers being divided by the less, the less by the first remainder, the first remainder by the second, the second by the third, &c., till there be no remainder; the greatest common factor of the given numbers will be a factor of the several remainders; for the remainders are* DIFFERENCES (**123**) *between numbers of which this greatest common factor* IS A FACTOR. *Consequently, the greatest common factor of the given numbers* CANNOT EXCEED THE LAST REMAINDER. *But* THE LAST REMAINDER IS ITSELF THAT FACTOR; *for, retracing the several remainders and given numbers from the last remainder to the larger given number, observe that the last remainder is a factor of the next preced-*

ing; that each of them, added to the next preceding, or a multiple of it, makes the next in order; and that, therefore, the last remainder must be a factor of them all, (**123**;) *hence, as the last remainder*, BOTH CONTAINS THE GREATEST COMMON FACTOR *of the given numbers, and* IS A FACTOR OF THEM, *it must be their greatest common factor.*

130. EXERCISES IN REDUCING FRACTIONS TO THEIR LOWEST TERMS BY THE GREATEST COMMON FACTOR.

In like manner, solve and explain the following problems.

1. Ascertain the greatest common factor of 30 and 72.
2. Reduce $\frac{30}{72}$ to its lowest terms.
3. Ascertain the greatest common factor of 126 and 342.
4. Reduce $\frac{126}{342}$ to its lowest terms
5. Ascertain the greatest common factor of 128 and 176.
6. Reduce $\frac{128}{176}$ to its lowest terms.
7. Reduce $\frac{182}{196}$ to its lowest terms.
8. Reduce $\frac{48}{192}$ to its lowest terms.
9. Reduce $\frac{108}{256}$ to its lowest terms.
10. What is the greatest common factor of 384 and 1152?
11. What are the lowest terms of $\frac{384}{1152}$?
12. What is the greatest common factor of 114 and 285?
13. Reduce $\frac{114}{285}$ to its lowest terms.
14. Reduce $\frac{1428}{2858}$ to its lowest terms.
15. What are the lowest terms of $\frac{96}{544}$?
16. What are the lowest terms of $\frac{36}{1296}$?
17. Reduce $\frac{486}{9720}$ to its lowest terms.

131. ILLUSTRATION OF THE LEAST COMMON MULTIPLE OF NUMBERS.

2 is a factor of 4, 6, 8, 10, 12, 14, 16, 18, &c.,
and 3 is a factor of 6, 9, 12, 15, 18, &c.;
consequently, 4, 6, 8, 10, &c., are multiples of 2; and 6, 9, 12, &c., are multiples of 3. But 6, 12, 18, &c., are multiples of *both* 2 and 3; hence, they are *common multiples* of 2 and 3; and 6 is the *least common multiple* of 2 and 3.

132. ILLUSTRATION OF THE LEAST COMMON DENOMINATOR OF FRACTIONS.

1. Reduce $\frac{1}{2}$, also $\frac{1}{3}$, to equal fractions in higher terms.

By multiplying both terms of each fraction by 2, 3, 4, 5, &c., successively,

$$\left.\begin{matrix}\frac{1}{2}\\[2ex]\frac{1}{3}\end{matrix}\right\}\text{becomes}\left\{\begin{matrix}\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}=\frac{7}{14}=\frac{8}{16}=\frac{9}{18},\ \&c.\\[2ex]\frac{2}{6}=\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{6}{18},\ \&c.\end{matrix}\right.$$

OBSERVE, *that the denominators of the fractions to which* $\frac{1}{2}$ *may be reduced, will be multiples of* 2, *the denominator of* $\frac{1}{2}$; *and that the denominators of the fractions to which* $\frac{1}{3}$ *may be reduced, will be multiples of* 3, *the denominator of* $\frac{1}{3}$.

But, PARTICULARLY OBSERVE, *that the* COMMON MULTIPLES *of* 2 *and* 3, *the denominators of* $\frac{1}{2}$ *and* $\frac{1}{3}$, MAY BE COMMON DENOMINATORS *of fractions to which* $\frac{1}{2}$ *and* $\frac{1}{3}$ *may be reduced; and that* THE LEAST COMMON MULTIPLE *of the denominators of* $\frac{1}{2}$ *and* $\frac{1}{3}$ WILL BE THE LEAST COMMON DENOMINATOR *to which* $\frac{1}{2}$ *and* $\frac{1}{3}$ *can be reduced.*

133. MODEL OF A RECITATION.

2. Reduce $\frac{5}{6}$ and $\frac{3}{8}$ to equal fractions having their least common denominator.

$$\frac{5}{6}=\frac{10}{12}=\frac{15}{18}=\frac{20}{24}$$

$$\frac{3}{8}=\frac{6}{16}=\frac{9}{24}$$

By multiplying both terms of each fraction by 2, 3, 4, &c., successively, you obtain for denominators all the multiples of the given denominators as far as you proceed; consequently, the first common denominator thus obtained, will be the least common denominator of the given fractions.

134. EXERCISES IN REDUCING FRACTIONS TO THEIR LEAST COMMON DENOMINATOR.

In like manner, solve and explain the following problems.

1. Reduce $\frac{2}{3}$ and $\frac{3}{4}$ to equal fractions having their least common denominator.
2. Reduce $\frac{1}{3}$ and $\frac{2}{5}$ to their least common denominator.
3. Reduce $\frac{3}{4}$ and $\frac{1}{5}$ to their least common denominator.
4. Reduce $\frac{1}{3}$ and $\frac{3}{7}$ to their least common denominator.
5. Reduce $\frac{2}{3}$ and $\frac{5}{8}$ to their least common denominator.
6. Reduce $\frac{3}{4}$ and $\frac{2}{7}$ to their least common denominator.
7. Reduce $\frac{1}{6}$ and $\frac{5}{7}$ to their least common denominator.
8. Reduce $\frac{3}{5}$ and $\frac{5}{6}$ to their least common denominator.
9. Reduce $\frac{1}{6}$ and $\frac{3}{10}$ to their least common denominator.

10. Reduce $\frac{3}{8}$ and $\frac{7}{10}$ to their least common denominator.
11. Reduce $\frac{3}{10}$ and $\frac{7}{12}$ to their least common denominator.
12. Reduce $\frac{5}{12}$ and $\frac{5}{18}$ to their least common denominator.
13. Reduce $\frac{7}{10}$ and $\frac{2}{15}$ to their least common denominator.
14. Reduce $\frac{5}{18}$ and $\frac{5}{24}$ to their least common denominator.

135. MODE OF FINDING THE LEAST COMMON MULTIPLE.

If you know the right numbers by which to multiply both terms of each fraction, to reduce the fractions to their least common denominator, *only one* multiplication for each fraction would be necessary.

Hence, as you will often have occasion to reduce fractions to their least common denominator, it is desirable to find a more direct way to ascertain the right multipliers.

Every number which is not a prime number, is composed of *prime factors*, (**29.**) Thus: $24 = 3 \times 2 \times 2 \times 2$.

Though 4, 6, 8 and 12 are factors of 24, yet they themselves are *composed* of prime factors, and, therefore, are *composite factors.*

A multiple, or composite number, is composed of exactly all its prime factors. Hence, a number which contains the prime factors of another number, is a *multiple* of that other number; also, a number which contains the prime factors of two, or more other numbers, is a *common multiple* of those other numbers.

Consequently, *the least common multiple of two or more given numbers, will be composed of such of their prime factors, and only such, as are necessary to compose each of the given numbers.*

Thus: $6 = 3 \times 2$, and $8 = 2 \times 2 \times 2$; now take 3×2, the factors of 6, and 2×2, the factors which 8 has that 6 *has not*, and you have all the factors of 6 and 8; viz: $3 \times 2 \times 2 \times 2 = 24$, the least common multiple of 6 and 8.

Hence, *to ascertain the least common multiple of two or more given numbers, it is only necessary to separate the given numbers into their prime factors, and to select and multiply together such, and only such of the factors as are necessary to compose each of the given numbers.*

136. MODEL OF A RECITATION.

1. Ascertain the least common multiple of 14 and 21.

$14 = 2 \times 7,$
$21 = 3 \times 7,$
$2 \times 7 \times 3 = 42.$

It takes 2 and 7 to compose 14, and this *same* 7, together with the 3, compose 21; therefore, the other 7 being omitted, $2 \times 7 \times 3 = 42$, will be the least common multiple required.

137. MODE OF REDUCING FRACTIONS TO THEIR LEAST COMMON DENOMINATOR.

2. Reduce $\frac{3}{14}$ and $\frac{2}{21}$ to equal fractions having their least common denominator.

Since the least common denominator will be the least common multiple of the given denominators, (**132**,) it will only be necessary to separate the given denominators into their prime factors, and multiply both terms of each fraction by such factors composing the denominator of the other fraction, as are necessary to make each denominator equal to the least common multiple of the given denominators; that is, multiply both terms of each fraction by the factors, composing the denominator of the other fraction, which it has not already in its own denominator.

$$\frac{3}{14} = \frac{3}{2 \times 7} = \frac{3 \times 3}{2 \times 7 \times 3} = \frac{9}{42}.$$

$$\frac{2}{21} = \frac{2}{3 \times 7} = \frac{2 \times 2}{3 \times 7 \times 2} = \frac{4}{42}.$$

Thus; by multiplying both terms of the first fraction by 3, and of the second by 2, the denominators will be composed of the same factors, and only such as are indispensable; consequently, the fractions are reduced to equal fractions having their least common denominator as required.

138. MODEL OF A RECITATION.

3. Reduce $\frac{9}{24}$ and $\frac{15}{36}$ to equal fractions having their least common denominator.

$$\frac{9}{24} = \frac{3}{8} = \frac{3}{4 \times 2} = \frac{9}{24}.$$

$$\frac{15}{36} = \frac{5}{12} = \frac{5}{4 \times 3} = \frac{10}{24}.$$

First, reduce the fractions to their *lowest terms*, then separate the denominators into their prime factors, or, since they have a *common* composite factor, 4, this *need not* be reduced to prime factors; and, finally, multiply both terms of the first fraction by 3, and of the second by 2, and the fraction will be reduced as required.

4. Reduce $\frac{5}{6}$, $\frac{3}{10}$, and $\frac{4}{15}$, to their least common denominator.

$$\frac{5}{6} = \frac{5}{2 \times 3} = \frac{25}{30}.$$
$$\frac{3}{10} = \frac{3}{2 \times 5} = \frac{9}{30}.$$
$$\frac{4}{15} = \frac{4}{3 \times 5} = \frac{8}{30}.$$

Multiply both terms of the first fraction by 5, of the second by 3, and of the third by 2, and the several denominators will be composed of the same factors; consequently, the given fractions will be reduced to their least common denominator as required.

139. Observation.

Observe, *that, to reduce two or more fractions to their least common denominator, we first reduce the fractions to their lowest terms; second, separate these denominators into their prime factors; and third, multiply both terms of each of these fractions by the factors belonging to the other denominators which do* NOT *belong to its own denominator.*

140. Exercises in reducing Fractions to their Least Common Denominator.

In like manner, solve and explain the following problems.

1. Ascertain the least common multiple of 8 and 12.
2. Reduce $\frac{5}{8}$ and $\frac{5}{12}$ to their least common denominator.
3. Ascertain the least common multiple of 8 and 14.
4. Reduce $\frac{3}{8}$ and $\frac{5}{14}$ to their least common denominator.
5. Ascertain the least common multiple of 9 and 15.
6. Reduce $\frac{2}{9}$ and $\frac{7}{15}$ to their least common denominator.
7. Ascertain the least common multiple of 15 and 18.
8. Reduce $\frac{4}{15}$ and $\frac{5}{18}$ to their least common denominator.
9. Ascertain the least common multiple of 5 and 7.
10. Reduce $\frac{2}{5}$ and $\frac{3}{7}$ to their least common denominator.
11. Ascertain the least common multiple of 2, 3, 5, and 7.
12. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{7}$, to their least common denominator.
13. Ascertain the least common multiple of 10, 14, and 15.
14. Reduce $\frac{3}{10}$, $\frac{5}{14}$, and $\frac{2}{15}$ to their least common denominator.
15. Ascertain the least common multiple of 250 and 400.

16. Reduce $\frac{7}{250}$ and $\frac{3}{400}$ to their least common denominator.

17. Ascertain the least common multiple of 15, 24 and 35.

18. Reduce $\frac{4}{15}$, $\frac{5}{24}$, and $\frac{6}{35}$, to their least common denominator.

19. Reduce $\frac{13}{27}$ and $\frac{5}{18}$ to their least common denominator.

20. Reduce $\frac{4}{15}$ and $\frac{10}{25}$ to their least common denominator.

21. Reduce $\frac{18}{24}$, $\frac{21}{28}$, and $\frac{24}{32}$, to their least common denominator.

22. Reduce $\frac{4}{27}$ and $\frac{8}{54}$ to their least common denominator.

23. Reduce $\frac{5}{18}$, $\frac{8}{27}$, and $\frac{14}{90}$, to their least common denominator.

24. Reduce $\frac{115}{1250}$ and $\frac{240}{14400}$ to their least common denominator.

141. Model of a Recitation.

1. John paid $\frac{5}{8}$ of a dollar (5 ninepences) for a reading book, $\frac{3}{8}$ of a dollar for a writing book, and $\frac{7}{8}$ of a dollar for an arithmetic; how many dollars did they all cost?

$$\frac{5+3+7}{8} = \frac{15}{8} = 1\frac{7}{8} \text{ dolls.}$$

Since the parts expressed by these several fractions *are all eighths*, and since the numerator of each fraction shows the number of parts expressed by that fraction, (**88**,) the sum of the numerators will show the number of parts expressed by all of the fractions; therefore, *place the sum of the numerators over their common denominator*, and the result will be the sum of the fractions, as required.

2. If I pay $2\frac{3}{4}$ dollars for a pair of shoes, and $4\frac{5}{6}$ dollars for a pair of boots, what is the whole cost?

$$2\tfrac{3}{4} = 2\tfrac{9}{12}$$
$$4\tfrac{5}{6} = 4\tfrac{10}{12}$$
$$\overline{\quad 7\tfrac{7}{12} \text{ dollars.}}$$

Here are 3 parts and 5 parts making 8 parts, but they are *all* neither *fourths*, nor *sixths*; if, however, you reduce the fractions to their least common denominator, (**139**,) the parts become $\frac{9}{12} + \frac{10}{12} = \frac{9+10}{12} = \frac{19}{12} = 1\frac{7}{12}$. Write the $\frac{7}{12}$, and add the unit with the other units, making $7\frac{7}{12}$ dollars, which is the answer required.

142. Model of a Recitation.

1. A boy, having $\frac{5}{6}$ of a dollar, spent $\frac{2}{6}$ of a dollar for a bunch of quills. How much money had he left?

$$\frac{5-2}{6} = \frac{3}{6} = \frac{1}{2} \text{ of a dollar.}$$

He had left the difference between $\frac{5}{6}$ and $\frac{2}{6}$. Since the parts expressed by the fractions are *all sixths*, and the numerators show the number of the parts, the difference between the numerators will show the number of parts he had left, which continue to be of the same size; therefore, *place the difference of the numerators over the common denominator*, and the result will be the difference between the fractions, as required.

2. If a man earn $14\frac{1}{2}$ dollars, and spend $4\frac{5}{6}$ dollars in a week, what would he save in a week?

$$\begin{array}{rl} 14\frac{1}{2} = & 14\frac{3}{6} \\ 4\frac{5}{6} = & 4\frac{5}{6} \\ \hline & 9\frac{4}{6} \text{ dollars.} \end{array}$$

He would save the difference between what he earned and what he spent.

You cannot take 5 parts from 3 parts of the same size; therefore, reduce 1 of the 14 units to sixths, (**98**,) making 6 sixths, and the 3 sixths, make $\frac{9}{6}$, from which, if $\frac{5}{6}$ be taken, there will remain $\frac{9-5}{6} = \frac{4}{6}$, which write; and then take 4 units, *not* from 14 units, for 1 of them has been disposed of, but take 4 units from 13 units, and there will remain 9 units; making $9\frac{4}{6}$ dollars, which is the answer required.

143. Exercises in adding and subtracting Fractions.

In like manner, solve and explain the following problems.

1. If you buy a lead-pencil for $\frac{1}{16}$ of a dollar, a writing-book for $\frac{3}{16}$ of a dollar, an inkstand for $\frac{3}{16}$ of a dollar, how much must you pay for the whole?

2. At a contribution, John contributed $\frac{3}{16}$ of a dollar, his brother $\frac{1}{8}$ of a dollar, and their sister $\frac{1}{16}$ of a dollar. What did they all contribute?

3. By going in the road, John walks $\frac{7}{8}$ of a mile to school, but by going across the pastures and fields, it is only $\frac{5}{8}$ of a mile to school. How much can he save in distance by going the nearer way?

4. If a writing book cost $\frac{1}{8}$ of a dollar, and a quire of letter

paper cost $\frac{5}{16}$ of a dollar, how much more will the paper cost than the book?

5. If Samuel have $\frac{2}{5}$ of a dollar, and Martin have $\frac{3}{8}$ of a dollar, how much have both of them?

6. If Isaac have $\frac{3}{4}$ of a dollar, and his sister $\frac{2}{5}$, which has the more money, and how much more than the other?

7. How many yards of cloth in 4 pieces which measure as follows, $18\frac{3}{5}$ yards, $27\frac{1}{5}$ yards, $23\frac{4}{5}$ yards, and $25\frac{2}{5}$ yards?

8. If Mr. Farmer hire 2 men and a boy to work for him a week, and pay them as follows, $5\frac{3}{8}$ dollars to one man, $7\frac{5}{8}$ dollars to the other man, and $3\frac{7}{8}$ to the boy; how much would he pay the whole?

9. If it take $1\frac{1}{2}$ yards of cloth to make a coat, and $\frac{3}{8}$ of a yard to make a pair of pantaloons, how much more cloth in the coat than in the pantaloons?

10. A merchant bought a piece of cloth, containing 23 yards, and sold $7\frac{3}{8}$ yards of it. How much of it had he left?

11. In a barrel there are $31\frac{1}{2}$ gallons, and in a hogshead 63 gallons. How many more gallons in a hogshead than in a barrel?

12. If $7\frac{3}{8}$ gallons leak out of a barrel, how much would remain?

13. John works $\frac{1}{6}$ of the time, plays $\frac{1}{4}$ of the time, sleeps $\frac{1}{3}$ of the time, and is at school the rest of the time. What part of the time is he at school?

14. Of the road that John walks to school, $\frac{1}{3}$ is up hill, $\frac{1}{4}$ is down hill, and the rest is level. What part of the way is level road; and how much more of the way is up hill in going to school than in returning home?

15. A pair of oxen and a horse compose a team; one ox draws $\frac{2}{5}$ of the load, the other ox draws $\frac{1}{3}$ of the load, and the horse draws the rest of it. How much more do the oxen draw than the horse?

16. Add together $\frac{2}{7}$ and $\frac{4}{13}$.

17. What is the sum, and difference of $\frac{3}{8}$ and $\frac{1}{4}$?

18. Add together $7\frac{3}{4}$ and $10\frac{4}{5}$.

19. What is the difference between $13\frac{4}{15}$ and $17\frac{9}{20}$?

20. What is the sum, and difference of $16\frac{3}{7}$ and $12\frac{3}{14}$?

21. Subtract $24\frac{3}{10}$ from $25\frac{2}{15}$.

22. How much more is $12\frac{9}{14}$ than both $4\frac{2}{7}$ and $5\frac{3}{28}$?

23. How much less are both $\frac{5}{6}$ and $2\frac{3}{4}$ than 4?

24. How much more is the sum of $10\frac{9}{25}$ and $5\frac{2}{15}$ than their difference?

25. How much is $\frac{10+16-12}{7}$?
26. How much more is $\frac{20+25+12}{8}$ than $\frac{15+10+7}{8}$?
27. How much less is $\frac{1+2+5}{14}$ than $\frac{5+6+10}{21}$?
28. How much are $\frac{16+5}{28}$ and $\frac{11-6}{35}$?
29. Add together $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.
30. Add together $\frac{8}{12}$, $\frac{12}{18}$, $\frac{28}{42}$, and $\frac{32}{48}$.
31. Subtract $\frac{96}{144}$ from $\frac{36}{48}$.

144. Illustration of the Principle of multiplying by a Fraction.

At 4 dollars a yard for broad cloth, what would be the cost of 4 yards?—of 2 yards?—of 1 yard?—of $\frac{1}{2}$ of a yard?—of $\frac{3}{4}$ of a yard?

$4 \times 4 = 16$ dollars.
$4 \times 2 = 8$ dollars.
$4 \times 1 = 4$ dollars.
$4 \times \frac{1}{2} = 2$ dollars.
$4 \times \frac{3}{4} = 3$ dollars.

If 1 yard cost 4 dollars, 4 yards would cost 4 times 4 dollars, equal to 16 dollars. 2 yards would cost 2 times 4 dollars, equal to 8 dollars. 1 yard would cost 1 time 4 dollars, equal to 4 dollars. $\frac{1}{2}$ of a yard would cost $\frac{1}{2}$ time 4 dollars, or, more properly, $\frac{1}{2}$ of 4 dollars, equal to 2 dollars. $\frac{1}{4}$ of a yard would cost $\frac{1}{4}$ time 4 dollars, or, more properly, $\frac{1}{4}$ of 4 dollars, equal to 1 dollar; but $\frac{3}{4}$ of a yard would cost 3 times as much as $\frac{1}{4}$ of a yard, which is 3 dollars.

Observe, *that, since the product must be as many times the multiplicand as there are units in the multiplier,* (**24**,) *when the multiplier is* 1, *the product will not differ from the multiplicand, when the multiplier is greater than* 1, *the product will be greater than the multiplicand;* BUT WHEN THE MULTIPLIER IS LESS THAN 1, THE PRODUCT WILL BE LESS THAN THE MULTIPLICAND, AND SUCH A PART OF THE MULTIPLICAND AS THE MULTIPLIER IS OF A UNIT.

145. Model of a Recitation.

1. At 5 dollars a cord for wood, what would be the cost of 2 cords?—of $\frac{3}{4}$ of a cord?

$5 \times 2 = 10$ dollars.

$5 \times \frac{3}{4} = \frac{5 \times 3}{4} = \frac{15}{4} = 3\frac{3}{4}$ dollars.

If 1 cord costs 5 dollars, 2 cords would cost 2 times 5 dollars, equal to 10 dollars.
$\frac{3}{4}$ of a cord (**144**) would cost $\frac{3}{4}$ of 5

dollars. $\frac{1}{4}$ of 5 is $\frac{5}{4}$, (**94**,) therefore, $\frac{3}{4}$ of 5 will be 3 *times as many fourths*, equal to $\frac{15}{4}$, $= 3\frac{3}{4}$ dollars.

2. At 7 dollars a barrel for flour, what would be the cost of $3\frac{2}{3}$ barrels?

$$7 \times \frac{11}{3} = \frac{7 \times 11}{3} = \frac{77}{3} = 25\tfrac{2}{3} \text{ dollars.}$$

If 1 barrel costs 7 dollars, $3\frac{2}{3} = \frac{11}{3}$ barrels would cost $\frac{11}{3}$ of 7 dollars. Since $\frac{1}{3}$ of 7 is $\frac{7}{3}$, $\frac{11}{3}$ of 7 will be 11 times $\frac{7}{3} = \frac{7 \times 11}{3} = \frac{77}{3} = 25\frac{2}{3}$ dollars, which is the answer required.

Another Explanation.—If 1 barrel cost 7 dollars, $3\frac{2}{3}$ barrels would cost $3\frac{2}{3}$, or $\frac{11}{3}$, times as much. First, multiply by 11 as if it were 11 units, which gives 77 dollars. But, as the right multiplier is $\frac{11}{3}$, only $\frac{1}{3}$ of 11 units, (**94**,) the right product ought to be only $\frac{1}{3}$ of 77 dollars; therefore, divide 77 by 3, which gives $\frac{77}{3} = 25\frac{2}{3}$ dollars, (**86**,) as before.

146. Observation.

Observe, (**145**,) *that, in multiplying by a fraction, the process consists of two steps, on account of the two numbers in the multiplier; and that either step may be taken first, provided the reasoning be suited to the process.*

147. Exercises in multiplying by a Fraction.

In like manner, solve and explain the following problems.

1. If a man earn 10 dollars a week, how much would he earn in $\frac{5}{8}$ of a week?
2. If you can walk 3 miles an hour, how far can you walk in $\frac{3}{4}$ of an hour?
3. If board be 3 dollars a week, what must be paid for board $1\frac{1}{7}$ weeks?
4. At 20 dollars a month, what is a man's wages $\frac{6}{30}$ of a month?
5. If 3 dollars buy a yard of cassimere, what must be paid for $2\frac{1}{3}$ yards?
6. What is $\frac{3}{4}$ of 15?
7. Multiply 15 by $\frac{3}{4}$.
8. If a barrel of mackerel cost 8 dollars, what would $2\frac{3}{8}$ barrels cost?
9. At 2 dollars a day, what would be the wages for $5\frac{5}{12}$ days?

10. If 160 rods make an acre, how many rods in $3\frac{3}{4}$ acres?
11. Multiply 10 by $5\frac{2}{3}$.
12. Multiply $5\frac{2}{3}$ by 10.
13. What is $\frac{3}{10}$ of 16?
14. Multiply 6 by $\frac{4}{11}$.
15. Multiply 5 by $\frac{5}{28}$.

148. MODEL OF A RECITATION.

1. At $\frac{9}{8}$ of a dollar per day, what should a laborer receive for 4 days' work?—for 3 days' work?—for $\frac{1}{3}$ of a day's work?—for $\frac{1}{4}$ of a day's work?

$\frac{9}{8 \div 4} = \frac{9}{2} = 4\frac{1}{2}$ dollars.

$\frac{9 \times 3}{8} = \frac{27}{8} = 3\frac{3}{8}$ dollars.

$\frac{9 \div 3}{8} = \frac{3}{8}$ of a dollar.

$\frac{9}{8 \times 4} = \frac{9}{32}$ of a dollar.

If for 1 day's work he receive $\frac{9}{8}$ of a dollar, for 4 days' work he should receive 4 times $\frac{9}{8}$ of a dollar; which ascertain by making the parts 4 times *as large*, (**106.**) For 3 days' work, he should receive 3 times $\frac{9}{8}$ of a dollar; which ascertain by making 3 times *as many parts*, (**101.**) For $\frac{1}{3}$ of a day's work, he should receive $\frac{1}{3}$ of $\frac{9}{8}$ of a dollar; which ascertain by taking $\frac{1}{3}$ as many parts, (**111.**) For $\frac{1}{4}$ of a day's work, he should receive $\frac{1}{4}$ of $\frac{9}{8}$ of a dollar; which ascertain by making the parts $\frac{1}{4}$ as large, (**114.**)

2. If a horse travel $6\frac{3}{4}$ miles per hour, how far would he travel in 4 hours?—in $\frac{2}{3}$ of an hour?—in $5\frac{1}{3}$ hours?

$\frac{27}{4 \div 4} = 27$ miles.

$\frac{27 \div 3}{4 \div 2} = \frac{9}{2} = 4\frac{1}{2}$ miles.

$\frac{27 \div 3 \times 4}{4 \div 4} = 36$ miles.

If he travel $6\frac{3}{4} = \frac{27}{4}$ miles in 1 hour, in 4 hours he would travel 4 times $\frac{27}{4}$ of a mile; which ascertain by making the parts 4 times as large, (**106.**) In $\frac{2}{3}$ of an hour he would travel $\frac{2}{3}$ of $\frac{27}{4}$ of a mile. Divide the number of parts (**111**) by 3, to obtain $\frac{1}{3}$, and make these parts 2 times as large, (**106**,) to obtain $\frac{2}{3}$, which, reduced, will be the answer required. In $5\frac{1}{3} = \frac{16}{3}$ hours, he would travel $\frac{16}{3}$ of $\frac{27}{4}$ of a mile. Divide the number of parts (**111**) by 3, to obtain $\frac{1}{3}$, and multiply the quotient by 16, to obtain $\frac{16}{3}$; but, since 4, one of the factors of 16, is a factor of the denominator, (**108**,) multiply by 4, by making the parts 4 times as large, and then multiply by 4 again, the other factor

of 16, by making 4 times as many parts, which, reduced, will be the answer required. In reducing this expression of the answer, say: 3 in 27, 9 times, and 4 times 9 are 36. 4 in 4, once; and, since the denominator is 1, the numerator, 36, is units, (**83.**)

Another Explanation.—If he travel $6\frac{3}{4}$, or $\frac{27}{4}$, miles in 1 hour, in $5\frac{1}{3}$ hours he would travel $5\frac{1}{3}$, or $\frac{16}{3}$, times as far. First, multiply by 16, as if it were units, which gives $\frac{27\times4}{4\div4}$; but, as the right multiplier is $\frac{16}{3}$, only $\frac{1}{3}$ of 16 units, the right product ought to be only $\frac{1}{3}$ of what we now have; therefore, divide by 3, which gives $\frac{27\times4\div3}{4\div4}=36$, as before.

149. Observation.

Observe, (**148**,) *that, in multiplying a fraction by a fraction, the process consists of two steps, either of which may be taken first; that, in many cases, there are two ways of performing each part of the process, on account of the two numbers in the multiplicand, but that, of the two ways, that is to be adopted which will give the result in the lower terms; that each part of the process is to be* EXPRESSED *and explained separately; and finally, that the process is to be* PERFORMED *by reducing the* EXPRESSION *of the result to its simplest terms.*

150. Exercises in multiplying Fractions by Fractions.

In like manner, solve and explain the following problems.

1. If a benevolent man, having only $\frac{1}{2}$ of a bushel of wheat, should give $\frac{2}{3}$ of it to his poor neighbors, what part of a bushel would he give away?
2. At $\frac{3}{7}$ of a dollar a yard, what part of a dollar would $\frac{1}{5}$ of a yard cost?
3. What number is equal to $\frac{3}{11}$ of $\frac{22}{3}$?
4. If a yard of cloth cost $5\frac{5}{8}$ dollars, what would $\frac{4}{5}$ of a yard cost?
5. At $\frac{3}{8}$ of a dollar a yard, what will $\frac{8}{9}$ of a yard cost?
6. At $\frac{3}{4}$ of a dollar a pound, what will $\frac{7}{8}$ of a pound of tea cost?
7. At $\frac{1}{6}$ of a dollar a pound, what will $\frac{6}{7}$ of a pound of coffee cost?
8. At $2\frac{1}{4}$ dollars a bushel, what will $6\frac{1}{9}$ bushels of wheat cost?
9. At $\frac{6}{25}$ of a dollar per hour, how much may be earned in $\frac{7}{8}$ of an hour?

10. At $6\frac{4}{5}$ dollars a barrel, what will $\frac{12}{13}$ of a barrel of flour cost?

11. If $7\frac{3}{4}$ yards of satinet be bought at $\frac{5}{8}$ of a dollar per yard, what would be the whole cost?

12. If 1 cord of wood cost $6\frac{2}{3}$ dollars, what will $7\frac{1}{2}$ cords cost?

13. At $\frac{1}{12}$ of a dollar a pound, what will $17\frac{3}{5}$ pounds of sugar cost?

14. At $3\frac{3}{5}$ shillings a yard, what will $8\frac{1}{3}$ yards of ribbon cost?

15. If 1 dollar buy $\frac{7}{8}$ of a gallon of wine, how much would $67\frac{1}{2}$ dollars buy?

16. What is the value of $36\frac{3}{7}$ acres of land, at $40\frac{7}{10}$ dollars per acre?

17. What is the value of $142\frac{1}{2}$ tons of coal, at $7\frac{3}{5}$ dollars per ton?

18. What is the value of $16\frac{2}{3}$ tons of hay at $11\frac{1}{8}$ dollars per ton?

19. What will $7\frac{13}{16}$ bushels of apples cost at $\frac{17}{50}$ of a dollar per bushel?

20. A merchant owning $\frac{9}{16}$ of a ship, sold $\frac{4}{5}$ of his share; what part of the whole ship did he sell?

21. What is $\frac{6}{7}$ of $\frac{7}{8}$?

22. Multiply $\frac{9}{10}$ by $\frac{2}{7}$.

23. Multiply $\frac{3}{4}$ of $\frac{4}{7}$ by $\frac{7}{11}$.

24. What is $\frac{7}{11}$ of $\frac{4}{7}$ of $\frac{3}{4}$?

25. What is $\frac{7}{15}$ of $\frac{15}{28}$ multiplied by $\frac{28}{33}$?

26. Ascertain the product of the following factors, $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$.

27. How much is $\frac{4}{5}$ of $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{3}{4}$?

28. Multiply $8\frac{3}{4}$ by $\frac{3}{7}$.

29. Multiply $\frac{9}{10}$ by $17\frac{3}{11}$.

30. Multiply $11\frac{1}{13}$ by $8\frac{3}{11}$.

31. What is the second power (**49**) of $\frac{3}{4}$?

32. What is the second power of $\frac{2}{3}$?

33. What is the third power of $\frac{2}{3}$?

34. What is the fourth power of $\frac{3}{5}$?

151. ILLUSTRATION OF THE PRINCIPLE OF DIVIDING BY A FRACTION.

If a philanthropist have eight dollars to distribute to the poor, to how many persons could he give 4 dollars apiece?

2 dollars apiece? 1 dollar apiece? $\frac{1}{2}$ of a dollar apiece? $\frac{1}{4}$ of a dollar apiece?

He could give 4 dollars apiece to as many persons as there are times 4 dollars in 8 dollars.

$8 \div 4 = 2$ persons.
$8 \div 2 = 4$ persons.
$8 \div 1 = 8$ persons.
$8 \times 2 = 16$ persons.
$8 \times 4 = 32$ persons.

He could give 2 dollars apiece to as many persons as there are times 2 dollars in 8 dollars.

He could give 1 dollar apiece to as many persons as there are times 1 dollar in 8 dollars.

He could give $\frac{1}{2}$ of a dollar apiece to as many persons as there are times $\frac{1}{2}$ of a dollar in 8 dollars; and, since there are 2 halves in every unit, (**84**,) there will be 2 times as many halves as units; therefore, multiply 8 by 2 to ascertain how many times $\frac{1}{2}$ is contained in it.

He could give $\frac{1}{4}$ of a dollar apiece to as many persons as there are times $\frac{1}{4}$ of a dollar in 8 dollars; and, since there are $\frac{4}{4}$ in a unit, there will be 4 times as many $\frac{1}{4}$s as units; therefore, multiply 8 by 4 to ascertain how many times $\frac{1}{4}$ is contained in it.

OBSERVE, *that, since the divisor shows how many equal parts, such as the quotient, will make the dividend;* (**68**) *when the divisor is* 1 *the quotient will not differ from the dividend; when the divisor is* GREATER *than* 1 *the quotient will be* LESS THAN THE DIVIDEND; *but when the divisor is* LESS *than* 1 *the quotient will be* GREATER THAN THE DIVIDEND.

152. MODEL OF A RECITATION.

1. What would be the price of 1 acre of land, if 25 dollars be paid for 6 acres? for $\frac{3}{4}$ of an acre? for $4\frac{2}{3}$ acres?

$25 \div 6 = 4\frac{1}{6}$ dollars.
$\frac{25 \times 4}{3} = \frac{100}{3} = 33\frac{1}{3}$ dollars.

If 6 acres be bought, paying *one* dollar per acre would require 6 dollars; therefore, the price per acre would be as many dollars as 6 dollars is contained times in 25 dollars.

If $\frac{3}{4}$ of an acre be bought, paying 1 dollar per acre would require $\frac{3}{4}$ of a dollar; therefore, the price per acre would be as many dollars as $\frac{3}{4}$ of a dollar is contained times in 25 dollars. Since there are 4 times as many $\frac{1}{4}$s as units, (**98**,) in any number, multiply by 4 to ascertain how many times $\frac{1}{4}$ is contained; then, (since $\frac{3}{4}$ is 3 times as much as $\frac{1}{4}$, consequently, will be contained only $\frac{1}{3}$ as often as $\frac{1}{4}$,) divide that

quotient by 3 to ascertain how many times $\frac{2}{3}$ is contained, which reduced will be the answer required.

If $4\frac{2}{3} = \frac{14}{3}$ of an acre be bought, paying 1 dollar per acre would require $\frac{14}{3}$ dollars; therefore, the price per acre would be as many dollars as $\frac{14}{3}$ of a dollar is contained times in 25 dollars.

$\frac{25\times3}{14} = \frac{75}{14} = 5\frac{5}{14}$ dollars.

Multiply 25 by 3 to ascertain how many times $\frac{1}{3}$ is contained; (**99**) and, since $\frac{14}{3}$ will be contained $\frac{1}{14}$ as often, divide that quotient by 14 to ascertain how many times $\frac{14}{3}$ is contained, which reduced will be the answer required.

2. How many barrels of flour could a trader buy for 48 dollars, at $6\frac{2}{3}$ dollars per barrel?

$\frac{48\times3\div4}{5} = \frac{36}{5} = 7\frac{1}{5}$

He could buy as many barrels as $6\frac{2}{3} = \frac{20}{3}$ of a dollar is contained times in 48 dollars.

Multiply by 3 to ascertain how many times $\frac{1}{3}$ is contained in 48, and divide that quotient by 20 to ascertain how many times $\frac{20}{3}$ is contained; but, since 4, one of the factors of 20 is also a factor of 48; in dividing by 20, first divide by 4, and then divide that quotient by 5, the other factor of 20, (**117**,) which will give $\frac{1}{5}$ of $\frac{1}{4} = \frac{1}{20}$ of the dividend as required. In reducing this *expression* of the result, say 4 in 48, 12 times, and 3 times $\frac{12}{5}$ are $\frac{36}{5}$, equal to $7\frac{1}{5}$ barrels, which is the answer required.

Another Explanation.—First, divide by 20 as if it were 20 units, which gives $\frac{48\div4}{5}$; but, as the right divisor is $\frac{20}{3}$, only $\frac{1}{3}$ of 20 units, it will be contained 3 times as often as 20 units (**151** $\frac{1}{3}$) therefore, multiply that quotient by 3 to ascertain how many times $\frac{20}{3}$ is contained in 48, which gives $\frac{48\div4\times3}{5} = \frac{36}{5} = 7\frac{1}{5}$ barrels, as before.

153. Observation.

Observe, (**152**,) *that, in dividing by a fraction, the process consists of two steps, on account of the two numbers in the divisor, and that, either step may be taken first, provided the reasoning be suited to the process.*

154. Exercises in dividing by a Fraction.

In like manner, solve and explain the following problems.

1. To how many poor persons could 9 dollars be distributed, giving them $\frac{3}{4}$ of a dollar apiece?

2. If 28 dollars be paid for $1\frac{3}{4}$ tons of hay, what is the price of a ton?

3. If a drunkard drink $\frac{2}{16}$ of a quart of rum per day, how long would 9 quarts last him?

4. If a moderate drinker drink $\frac{1}{2}$ pint of brandy per day, how long would 8 pints last him?

5. How long would 2 barrels of flour last a family that consume $\frac{3}{8}$ of a barrel in each week?

6. If 28 bushels be sown on $9\frac{1}{3}$ acres, how much is that per acre?

7. If it take $\frac{5}{8}$ of a bushel of rye to sow an acre, how many acres would 15 bushels sow?

8. How many bottles of beer holding $\frac{7}{45}$ of a gallon each, could be filled from a hogshead holding 63 gallons?

9. At $1\frac{1}{5}$ dollars a bushel, how much wheat could be bought for 20 dollars?

10. How many acres would it take to produce 96 bushels, at the rate of $15\frac{4}{5}$ bushels per acre?

11. If a man pay 21 dollars for pasturing his horse $16\frac{4}{5}$ weeks, how much is that per week?

12. If a man earn 6 dollars in $\frac{3}{8}$ of a month, how much is that for one month?

13. In what time can a man build 28 rods of wall, if he build $\frac{2}{5}\frac{1}{2}$ of a rod per hour?

14. If $1\frac{7}{8}$ yards of cloth be put into a coat, how many coats may be made from 30 yards?

15. At $\frac{5}{8}$ of a dollar a bushel, how many bushels of corn may be bought for 125 dollars?

16. How many pairs of gloves may be bought for 12 dollars at $\frac{5}{6}$ of a dollar a pair?

17. If $7\frac{13}{16}$ barrels of apples be bought for 20 dollars, what is the cost of one barrel?

18. If $11\frac{2}{11}$ gallons of molasses cost 3 dollars, what would be the cost of one gallon?

19. Divide 128 by $\frac{2}{15}$.

20. How many times is $\frac{1}{2}\frac{4}{3}$ contained in 19?

21. How many times is $\frac{7}{8}$ contained in 14?

22. How many times is $\frac{3}{14}$ contained in 9?

23. Divide 2 by $7\frac{3}{8}$.

24. What part of 7 is 3?

25. What part of 7 is $\frac{3}{4}$?

26. What part of $\frac{3}{4}$ is 7?

27. What part of $2\frac{1}{3}$ is 2?
28. What part of $6\frac{1}{4}$ is 5?

155. MODEL OF A RECITATION.

1. With $3\frac{3}{8}$ dollars how many yards of broadcloth, at 9 dollars per yard, could a merchant buy? — How many yards of cassimere, at 2 dollars per yard? — How many yards of satinet, at $\frac{3}{4}$ of a dollar per yard? — How many yards of camlet at $\frac{5}{8}$ of a dollar per yard? — How many yards of velvet at $2\frac{2}{3}$ dollars per yard?

He could buy as many yards of broadcloth, at 9 dollars per yard, as 9 dollars is contained times in $3\frac{3}{8} = \frac{27}{8}$ dollars, which ascertain by dividing the *number* of parts by 9, (**111.**)

$\frac{27 \div 9}{8} = \frac{3}{8}$ yards of broadcloth.

$\frac{27}{8 \times 2} = \frac{27}{16} = 1\frac{11}{16}$ yards of cassimere.

$\frac{27 \div 3}{8 \div 4} = \frac{9}{2} = 4\frac{1}{2}$ yards of satinet.

$\frac{27}{8 \div 8 \times 5} = \frac{27}{5} = 5\frac{2}{5}$ yards of camlet.

$\frac{27 \times 3}{8 \times 8} = \frac{81}{64} = 1\frac{17}{64}$ yards of velvet.

He could buy as many yards of cassimere, at 2 dollars per yard, as 2 dollars is contained times in $3\frac{3}{8} = \frac{27}{8}$ dollars; which ascertain by dividing the *size* of the parts by 2, (**114**,) that is, making the parts $\frac{1}{2}$ as large.

He could buy as many yards of satinet, at $\frac{3}{4}$ of a dollar per yard, as $\frac{3}{4}$ of a dollar is contained times in $\frac{27}{8}$ of a dollar; multiply by 4, by making the parts 4 times as large, (**106**,) to ascertain how many times $\frac{1}{4}$ is contained in $\frac{27}{8}$; (**151**;) and, since $\frac{3}{4}$ is 3 times as much as $\frac{1}{4}$, and, consequently, will be contained only $\frac{1}{3}$ as often as $\frac{1}{4}$, divide this quotient by 3, by dividing the number of parts, to ascertain how many times $\frac{3}{4}$ is contained, which reduced will be the answer required.

He could buy as many yards of camlet, at $\frac{5}{8}$ of a dollar per yard, as $\frac{5}{8}$ of a dollar is contained times in $3\frac{3}{8} = \frac{27}{8}$ of a dollar; multiply by 8, by multiplying the size of the parts, to ascertain how many times $\frac{1}{8}$ is contained in $\frac{27}{8}$, and, since $\frac{5}{8}$ will be contained $\frac{1}{5}$ as often, divide this quotient by 5, by dividing the size of the parts, to ascertain how many times $\frac{5}{8}$ is contained, which reduced will be the answer required.

He could buy as many yards of velvet, at $2\frac{2}{3} = \frac{8}{3}$ of a dollar per yard, as $\frac{8}{3}$ of a dollar is contained times in $3\frac{3}{8} = \frac{27}{8}$ of a dollar; multiply the number of parts by 3 to ascertain how many times $\frac{1}{3}$ is contained in $\frac{27}{8}$, and divide the size of

the parts in that quotient by 8 to ascertain how many times $\frac{3}{8}$ is contained, which reduced will be the answer required.

Another Explanation.—First, divide $3\frac{3}{8}$, or $\frac{27}{8}$ by 8 as if it were 8 units, which gives $\frac{27}{8\times8}$; but, as the right divisor is $\frac{3}{8}$, only $\frac{1}{3}$ of 8 units, (**94**) it will be contained 3 times as often as 8 units; therefore, multiply that quotient by 3 to ascertain how many times $\frac{3}{8}$ is contained; which gives $\frac{27\times3}{8\times8}=\frac{81}{64}=1\frac{17}{64}$ yards as before.

156. Observation.

Observe, *that, in dividing a fraction by a fraction, the process consists of two steps, either of which may be taken first; that, in many cases there are two ways of performing each part of the process, on account of the two numbers in the dividend; but that, of the two ways, that is to be adopted which will give the result in the lower terms; that, each part of the process is to be* EXPRESSED *and explained separately; and finally, that the process is to be* PERFORMED *by reducing the* EXPRESSION *of the result to its simplest terms.*

157. Exercises in dividing a Fraction by a Fraction.

In like manner, solve and explain the following problems.

1. At $\frac{2}{3}$ of a dollar a bushel, how much rye may be bought for $\frac{3}{5}$ of a dollar?
2. At $\frac{1}{4}$ of a dollar a bushel, how many apples may be bought for $\frac{7}{8}$ of a dollar?
3. How many bushels of turnips, at $\frac{3}{16}$ of a dollar per bushel, may be bought for $\frac{7}{8}$ of a dollar?
4. If 1 bushel cost $\frac{1}{4}$ of a dollar, how many apples may be bought for $\frac{3}{4}$ of a dollar?
5. At $\frac{1}{5}$ of a dollar a dozen, how many dozen of lemons may be bought for $1\frac{2}{5}$ dollars?
6. At $\frac{3}{8}$ of a dollar a dozen, how many oranges may be bought for $5\frac{3}{8}$ dollars?
7. At $\frac{1}{6}$ of a dollar a pound, how many figs may be bought for $2\frac{1}{4}$ dollars?
8. At $\frac{1}{3}$ of a dollar a bushel, how many potatoes may be bought for $4\frac{1}{2}$ dollars?
9. At $\frac{3}{8}$ of a dollar a bushel, how many onions may be bought for $\frac{4}{5}$ of a dollar?
10. With $5\frac{3}{5}$ dollars, how many pounds of butter, at $\frac{4}{15}$ of a dollar a pound, may be bought?

11. If $\frac{3}{8}$ of a pound of fur is sufficient for 1 hat, how many hats would $4\frac{1}{16}$ pounds be sufficient for?

12. If 1 yard of linen cost $\frac{29}{72}$ of a dollar, how much would $3\frac{5}{8}$ dollars buy?

13. If $1\frac{3}{4}$ yards of cloth make 1 coat, how many coats may be made from $9\frac{1}{7}$ yards?

14. If $2\frac{1}{2}$ bushels of oats keep 1 horse a week, how many horses will $18\frac{3}{5}$ bushels keep for the same time?

15. If $2\frac{1}{2}$ bushels of oats keep a horse 1 week, how long would $12\frac{5}{8}$ bushels keep him?

16. Bought $3\frac{1}{4}$ yards of cloth for $14\frac{5}{8}$ dollars; what did I give per yard?

17. At $\frac{3}{8}$ of a dollar a pound, how many pounds of coffee may be bought for $12\frac{1}{2}$ dollars?

18. If $4\frac{3}{8}$ pounds of butter serve a family 1 week, how many weeks would $36\frac{7}{8}$ pounds serve them?

19. If a man walk a mile in $\frac{3}{10}$ of an hour, how far would he walk in $5\frac{3}{4}$ hours?

20. If a barrel of cider last a cider-drinker $3\frac{4}{5}$ months, how many barrels would he drink in $10\frac{2}{3}$ months?

21. If the stage run $8\frac{5}{12}$ miles per hour, how long would it be in running $25\frac{3}{16}$ miles?

22. How many bushels of rye at $\frac{5}{8}$ of a dollar per bushel, may be bought for $12\frac{3}{25}$ dollars?

23. If $4\frac{1}{2}$ pounds of tea cost $3\frac{3}{25}$ dollars, what is that per pound?

24. How many times is $4\frac{1}{2}$ contained in $3\frac{3}{25}$?

25. At $1\frac{3}{7}$ dollars per yard, how much carpeting may be purchased for $33\frac{1}{3}$ dollars?

26. Divide $1\frac{3}{7}$ by $33\frac{1}{3}$.

27. Divide $33\frac{1}{3}$ by $1\frac{3}{7}$.

28. If $\frac{13}{18}$ of a dollar buy a pound of tea, how much would $3\frac{1}{4}$ dollars buy?

29. How many times is $16\frac{2}{3}$ contained in $83\frac{1}{3}$?

30. How many times is $6\frac{1}{4}$ contained in $62\frac{1}{2}$?

31. How many times is $8\frac{1}{3}$ contained in $66\frac{2}{3}$?

32. How many times is $18\frac{3}{4}$ contained in $37\frac{1}{2}$?

33. How many times is $4\frac{1}{6}$ contained in $33\frac{1}{3}$?

34. At $\frac{3}{4}$ of a dollar a bushel, how much corn can be bought for $\frac{1}{2}$ of a dollar?

35. At 3 dollars a yard, how much velvet may be bought for $\frac{1}{8}$ of a dollar?

36. Divide $\frac{1}{3}$ by 3?

37. What part of 10 is 7?
38. What part of 3 is $\frac{1}{6}$?
39. What part of 3 is $2\frac{1}{4}$?
40. Divide $5\frac{1}{6}$ by 10.
41. What part of 10 is $5\frac{1}{6}$?
42. Divide $2\frac{4}{5}$ by $7\frac{3}{8}$.
43. What part of $7\frac{3}{8}$ is $2\frac{4}{5}$?
44. Divide $\frac{2}{5}$ by $\frac{7}{8}$.
45. When corn is $\frac{7}{8}$ of a dollar per bushel, what part of a bushel may be bought for $\frac{2}{5}$ of a dollar?
46. $\frac{2}{5}$ is what part of $\frac{7}{8}$?

158. Review of the several Ways of multiplying a Fraction by a Fraction.

Multiply $\frac{5}{12}$ by $\frac{4}{5}$.

	(*a.*) State the problem.
No. 1. $\frac{5\div 5}{12\div 4} = \frac{1}{3}$.	(*b.*) What may be the first step?
" 2. $\frac{5\div 5\times 4}{12} = \frac{4}{12} = \frac{1}{3}$.	(*c.*) Why need that be done?
" 3. $\frac{5}{12\times 5\div 4} = \frac{5}{15} = \frac{1}{3}$.	(*d.*) How may that be done?
" 4. $\frac{5\times 4}{12\times 5} = \frac{20}{60} = \frac{1}{3}$.	(*e.*) Why may it be done in that way?
————————	(*f.*) Result of the first step?
" 5. $\frac{5\div 5}{12\div 4} = \frac{1}{3}$.	(*g.*) What must be the next step?
" 6. $\frac{5\times 4\div 5}{12} = \frac{4}{12} = \frac{1}{3}$.	(*h.*) Why must that be done?
" 7. $\frac{5}{12\div 4\times 5} = \frac{5}{15} = \frac{1}{3}$.	(*i.*) How may that be done?
" 8. $\frac{5\times 4}{12\times 5} = \frac{20}{60} = \frac{1}{3}$.	(*j.*) Why may it be done in that way?
	(*k.*) Result of both steps?

159. Model of a Recitation.

No. 1. (*a.*) The product should be $\frac{4}{5}$ of the multiplicand, (**144.**) (*b.*) First divide by 5, (*c.*) to obtain (**92**) $\frac{1}{5}$, (*d.*) which is done by dividing the numerator, (**111**) by 5; (*e.*) because that will give $\frac{1}{5}$ as many parts, (*f.*) or $\frac{1}{12}$, (*g.*) Next, multiply by 4, (*h.*) to obtain $\frac{4}{5}$, (*i.*) which is done by dividing the denominator (**106**) by 4; (*j.*) because that will make the parts 4 times as large, (*k.*) or $\frac{1}{3}$, which is the answer required.

In like manner explain the first four of the examples; but explain the last four by using the numerator of the multiplier in the first step of the process.

160. Review of the several Ways of Dividing a Fraction by a Fraction.

Divide $\frac{12}{25}$ by $\frac{4}{5}$.

		(*a.*) State the problem.
No. 1.	$\frac{12}{25}\div\frac{4}{5} = \frac{3}{5}$.	(*b.*) What may be the first step?
" 2.	$\frac{12}{25\div5\times4} = \frac{12}{20} = \frac{3}{5}$.	(*c.*) Why need that be done?
" 3.	$\frac{12\times5\div4}{25} = \frac{15}{25} = \frac{3}{5}$.	(*d.*) How may that be done?
" 4.	$\frac{12\times5}{25\times4} = \frac{60}{100} = \frac{3}{5}$.	(*e.*) Why may it be done in that way?
		(*f.*) Result of the first step.
" 5.	$\frac{12\div4}{25\div5} = \frac{3}{5}$.	(*g.*) What must be the next step?
" 6.	$\frac{12}{25\times4\div5} = \frac{12}{20} = \frac{3}{5}$.	(*h.*) Why must that be done?
" 7.	$\underline{12\div4\times5} = \frac{15}{25} = \frac{3}{5}$.	(*i.*) How may that be done?
" 8.	$\frac{12\times5}{25\times4} = \frac{60}{100} = \frac{3}{5}$.	(*j.*) Why may it be done in that way?
		(*k.*) Result of both steps.

161. Model of a Recitation.

No. 1. (*a.*) It is required to find how many times $\frac{4}{5}$ is contained in the dividend, (**62**); (*b.*) First, multiply by 5, (*c.*) to ascertain how many times $\frac{1}{5}$ is contained, (**99**,) (*d.*) which is done by dividing the denominator (**106**) by 5; (*e.*) because that will make the parts 5 times as large, (**105**); (*f.*) or, $\frac{12}{5}$. (*g.*) Next. divide by 4, (*h.*) to ascertain how many times $\frac{4}{5}$ is contained, (*i.*) which is done by dividing the numerator, (**111**) by 4; (*j.*) because that will give $\frac{1}{4}$ as many parts, (*k.*) or $\frac{3}{5}$, which is the answer required.

In like manner, explain the first four of the examples; but explain the last four by using the numerator of the divisor in the first step of the process.

VII. DECIMAL FRACTIONS.

162. Similarity of Decimal Fractions to Integral Numbers.

In integral numbers you see that there is a uniform law, 10 units of any order making 1 unit of the next higher order, (**10**) or 1 unit of any order making 10 units of the next lower order; that, therefore, the units of the different orders are written together in places appropriated to them, according to their values; and that, hence, the values of the several units are known from the places which they occupy.

But in fractional numbers, you see that there is no such uniformity, since the parts may be of any size, depending upon the number of them that it takes to make a unit; that, therefore, the parts of different sizes cannot be written together in places appropriated to them, according to their values, and that, hence, the values of the parts cannot be known from the places which they occupy; but that the parts, whatever may be their size, are written in the same place, at the right of an integral number, when not written alone, and always accompanied by a denominator to show their size, (**84**.)

You will now give your attention to a kind of fractions in which there prevails the same uniformity as in integral numbers; 10 parts of any size making 1 part of the next larger size; or 1 part of any size making 10 parts of the next smaller size, (**10**); and therefore, the parts of different sizes, are written together in places appropriated to them, according to their sizes; and hence the different sizes of the parts are known, without the presence of their denominator, from the *places* which the parts occupy; moreover, all the operations of addition, subtraction, multiplication, and division, are performed upon them, either alone, or together with integral numbers, precisely as upon integral numbers alone; *care being required only to keep the* POINT *of separation between the integral and fractional parts of numbers.*

163. ILLUSTRATION OF THE LOCAL VALUE OF DECIMAL FIGURES.

$1. = \frac{10}{10}, = \frac{100}{100}, = \frac{1000}{1000}, = \frac{10000}{10000}$, &c.

$\frac{1}{10}, = \frac{10}{100}, = \frac{100}{1000}, = \frac{1000}{10000}$, &c.

$\frac{1}{100}, = \frac{10}{1000}, = \frac{100}{10000}$, &c.

$\frac{1}{1000}, = \frac{10}{10000}$, &c.

$\frac{1}{10000}$, &c.

1.1111.

OBSERVE, in this table, that 1 unit is reduced to *tenths*, &c., $\frac{1}{10}$ to *hundredths*, &c., $\frac{1}{100}$ to *thousandths*, &c., $\frac{1}{1000}$ to *ten-thousandths*, &c., by multiplying both terms (**121**) of each fraction by 10, and by 10 again, &c.; that 1 part of each size makes 10 parts of the next smaller size, or that 10 parts of each size make 1 part of the next larger size, (**10**); and that on the left, 1 part of each size is arranged, without its denominator, according to the values of the parts, 1 unit being written, then 1 *tenth* in the *first* place at the right hand of units, 1 *hundredth* in the *second* place, 1 *thousandth* in the *third* place, 1 *ten-thousandth* in the *fourth* place, &c. Any other digit written in any of these places, would express parts of the size for which that place is appropriated. Hence, the values of these parts, or any number of parts arranged in this way, according to their values, may be known without their denominator, since the different parts will always occupy places at the same relative distances from the unit's place; but a POINT (.) must be prefixed to a fraction to distinguish it from an integral number, or the integral part of a mixed number.

Such fractions are called *Decimal Fractions*, because the parts expressed by them are always such, that it takes 10 of them, or some power (**49**) of 10 to make a unit. They differ from Common Fractions, only in the uniformity in the values of the parts expressed by them, and consequently, in the manner of writing them, and operating by them.

164. MODE OF READING DECIMAL NUMBERS.

Since, as you may observe, the places equidistant from the units, on each side, correspond in name, except that the ter-

mination of the fractional names is *ths*, the manner of reading decimal fractions is similar to that of reading integral numbers.

Observe, *also*, in the table, that $1 = 10000$ ten-thousandths, $\frac{1}{10} = 1000$ ten-thousandths, $\frac{1}{100} = 100$ ten-thousandth, $\frac{1}{1000} = 10$ ten-thousandth, and $\frac{1}{10000} = 1$ ten-thousandth; consequently, the whole mixed number, 1.1111, may be read, eleven thousand one hundred and eleven *ten-thousandths*, precisely the same as an integral number, except at last, speaking the denominator of the last figure, which is also the common denominator of this and the other figures in the number, as may be observed in the table. But the better way is to read the integral and fractional parts separately. Thus: *One*, and one thousand one hundred and eleven *ten-thousandths.*

The denominator of the last figure, or the common denominator of all the figures in the numerator, may be known from the fact that it will always consist of one more figure than the decimal places occupied by the numerator, or 1 with as many ciphers as the numerator occupies decimal places.

165. Exercises in Reading Decimal Numbers.

In like manner, read the following numbers.

1.	5.111.	11.	252.5.	21.	.05.	31.	2.40003.
2.	3.12.	12.	25.25.	22.	.005.	32.	2.400305.
3.	2.6.	13.	2.525.	23.	.0005.	33.	.5.
4.	.2.	14.	.2525.	24.	.00005.	34.	.50.
5.	.25.	15.	40.5.	25.	.000005	35.	.500.
6.	.75.	16.	4.05.	26.	.007.	36.	.5000.
7.	.125.	17.	.405.	27.	.00007.	37.	.050.
8.	17.3.	18.	306.1.	28.	.072.	38.	8.0900.
9.	144.16.	19.	30.61.	29.	3.0407.	39.	.0000009.
10.	3456.4.	20.	3.061.	30.	3.4007.	40.	1.00000080.

166. Mode of writing Decimal Numbers.

In writing a decimal fraction, it should be made to occupy as many places as it requires ciphers in its denominator. Therefore, following the point, write as many ciphers as the number of decimal places required exceeds the number of figures expressing the numerator; then write such figures as will express the numerator; and the fraction will be properly expressed.

167. MODEL OF A RECITATION.

Write twelve, and one thousand and sixteen ten-millionths.

First, write twelve and the point, thus; 12.; then, as the decimal must occupy *seven* places, and it requires only *four* figures to express the numerator, write three ciphers, and one thousand and sixteen, thus; 12.0001016, which is the number required.

168. EXERCISES IN WRITING DECIMAL NUMBERS.

In like manner, write the following numbers, expressing the fractions decimally.

1. $27\frac{6}{10}$.	6. $16\frac{1}{1000}$.	11. $13\frac{23}{100}$.	16. $\frac{420}{1000}$.
2. $14\frac{7}{100}$.	7. $\frac{6}{10}$.	12. $1\frac{43}{1000}$.	17. $\frac{300}{1000}$.
3. $108\frac{5}{10}$.	8. $\frac{5}{100}$.	13. $17\frac{573}{10000}$.	18. $\frac{80}{1000}$.
4. $73\frac{9}{100}$.	9. $\frac{7}{1000}$.	14. $\frac{807}{10000}$.	19. $\frac{2000}{100000}$.
5. $4\frac{6}{100}$.	10. $\frac{2}{10000}$.	15. $\frac{40}{100}$.	20. $\frac{205}{10000000}$.

21. Seventeen, and four hundred and nine thousandths.
22. Six, and sixty-five thousandths.
23. Seven, and seven ten-thousandths.
24. Ten thousand eight hundred and nine hundred-thousandths.
25. Twenty-six, and fifteen millionths.
26. Three, and one hundred and one ten-thousandths.
27. Four, and twenty-five hundred-thousandths.
28. Eight, and six hundred and four millionths.
29. One, and sixty thousand and five ten-millionths.
30. Two, and thirty thousand hundred-thousandths.
31. How many thousandths in .2?
32. How many hundredths in 2.5?
33. Reduce $\frac{3}{10}$ to thousandths.
34. Reduce $\frac{125}{1000}$ to its lowest terms.
35. Reduce .25 to its lowest terms in a common fraction.
36. Reduce .3125 to its lowest terms.
37. Reduce $\frac{3}{10}$, $\frac{3}{100}$, and $\frac{3}{1000}$, to thousandths and add them.
38. Reduce $\frac{3}{10}$, $\frac{7}{1000}$, and $\frac{8}{100}$, to a common denominator, and add them.

169. FEDERAL MONEY EXPRESSED BY DECIMAL NUMBERS.

Federal money is the metallic money which is coined by the authority of the United States. It consists of eagles, dollars, dimes, cents, and mills; the values of which, as you may observe in the following table, correspond to decimal numbers; 1 coin of either denomination equaling 10 of the next lower; or 10 coins of either denomination equaling 1 of the next higher, (**10.**) But the mill is only an imaginary coin.

In commerce, eagles are expressed in dollars, and dimes in cents. *The dollar is considered the unit*, and cents and mills, decimal fractions of a dollar. Hence, numbers expressing Federal money are precisely like numbers in decimal fractions, and they are made to express Federal money by prefixing to them this character ($.)

Eagle.	Dollars.	Dimes.	Cents.	Mills.	Dollars. Cents. Mills.	Read.
1 =	10 =	100 =	1000 =	10000 =	$10. . . .	Ten dollars.
	1 =	10 =	100 =	1000 =	$ 1. . . .	One dollar.
		1 =	10 =	100 =	$ .10 .	Ten cents.
			1 =	10 =	$ .01 .	One cent.
				1 =	$ .001	One mill.
				11111 =	$11.111	Eleven dollars, eleven cents and one mill.

170. REDUCTION OF FEDERAL MONEY ILLUSTRATED.

1. In 25 how many hundredths?—how many thousandths?

2500 hundredths.

25000 thousandths.

Since there are $\frac{100}{100}$ in 1, there will be 100 times as many hundredths as units; therefore, multiply the units by 100, by annexing two ciphers, (**34.**)

There will be 1000 times as many thousandths as units; therefore, annex three ciphers to 25, which will give the answer required.

2. In $25, how many cents?—how many mills?

2500 cents.

25000 mills.

Since there are 100 cents in $1, there will be 100 times as many cents as dollars; therefore, annex two ciphers, (**10**) to 25, making 2500 cents, which is the answer required.

There will be 1000 times as many mills as dollars; therefore, annex three ciphers to the dollars, which will reduce the dollars to mills, as required.

OBSERVE *that, by pointing off the ciphers annexed in these examples, that is, putting a point between the 25 and the ciphers, the hundredths and thousandths will be reduced to units again, and the cents and mills to dollars again.*

171. MODEL OF A RECITATION.

1. In 25125 mills how many cents?—how many dollars?

2512.5 cents.

$25.125

Since in 1 cent there are 10 mills, there will be $\frac{1}{10}$, or .1 as many cents as mills; therefore, divide by 10, by pointing off one figure at the right hand, (**10**); for thus the tens become units, and the other figures, also, are all brought one degree lower

There will be $\frac{1}{1000}$, or .001 as many dollars as mills, therefore, point off three figures, (**10**); for thus the thousands become units, and all the other figures are also brought three degrees lower.

172. EXERCISES IN THE REDUCTION OF FEDERAL MONEY.

In like manner, solve and explain the following problems.

1. In $16, how many cents?
2. How many mills in $16?
3. In 12000 mills how many cents?
4. How many dollars in 12000 mills?
5. In 75 cents how many mills?
6. In $8.25 how many cents?
7. In $5.125, how many mills?
8. How many cents in $5.125?
9. In $3.375, how many dollars, cents and mills?
10. In 16125 mills how many dollars?
11. In 12548 cents, how many dollars?
12. Reduce $37.50 to cents.
13. Reduce 75625 mills to dollars?
14. Reduce 984 mills to dollars.
15. Reduce $.75 to cents.
16. Reduce $.125 to mills.
17. How many mills in $1.25?
18. How many cents in $2.375?
19. In 12345 mills how many dollars, cents and mills?
20. How many times 10 in 85?

21. Divide 625 by 100.
22. Divide 1836 by 1000.
23. What is $\frac{1}{100}$ of 1728?
24. How many times 100 in 1276?
25. Reduce 12.25 to hundredths.

173. MODEL OF A RECITATION.

1. Bought 1 barrel of flour for \$6.75, 10 pounds of coffee for \$2.20, 7 pounds of sugar for \$.875, 12 pounds of butter for \$2, 1 pound of raisins for \$.125, and 2 oranges for \$.06. What was the whole amount?

Arrange the numbers together so that the figures of each denomination, may stand in a column by themselves, and proceed as in the addition of integral numbers, (**20.**)

6.75
2.20
.875
2.
.125
.06
\$12.01

The 10 mills of the first column make 1 cent, (**169,**) which added with the first column of cents make 21 cents, equal to 1 cent, which write, and 2 dimes; which add with the other dimes, making 20 dimes, equal to 2 dollars; write a cipher in the dimes' place, or second place of cents, and add the 2 dollars with the other dollars, making 12 dollars, which written at the left of the point, make \$12.01, the answer required.

2. Mr. Farmer having a pasture of 25 acres, fenced off 2.375 acres to plant with potatoes; how many acres remained in the pasture?

Write the subtrahend under the minuend, placing the figures of each denomination under those of the same denomination, and proceed as in the subtraction of integral numbers, (**53.**) Since there are no thousandths from which to take the 5, reduce 1 of the 5 units to tenths, (**10,**) making 10, one of which (leaving 9,) reduce to hundredths, making 10, one of which (leaving 9,) reduce to thousandths, making 10, from which subtract the 5, and 5 thousandths remain, which write; 7 hundredths from 9 hundreths leave 2 hundredths, which write; 3 tenths from 9 tenths leave 6 tenths, which write; 2 units from 4 units leave 2 units, which write; and blank from 2 tens leaves 2 tens, which write; making 22.625 acres, which is the answer required.

25.
2.375
22.625 acres.

174. Exercises in adding and subtracting Decimal Numbers.

In like manner, solve and explain the following problems, taking care to keep a point between the integral and fractional parts of every number.

1. Bought a pair of oxen for $76.50, a horse for $75, and a cow for $25.75; what was the whole amount?

2. A man gave $4.75 for a pair of boots, and $2.25 for a pair of shoes; how much more did the boots cost than the shoes?

3. A man bought a cow and calf for $28.375, and sold the calf for $3.625, what did the cow cost him?

4. Bought a horse for $92, but sold him so as to lose $15.25; for how much was he sold?

5. What is the whole cost of a cart at $17.625, a wagon at $48.50, a plough at $7.333, a rake at $.42, a hoe at $.60, and a pitchfork at $.875?

6. How much cloth in 6 pieces measuring as follows, 25.5 yards, 27.75 yards, 28.125 yards, 30 yards, 29.375 yards, and 26.5 yards?

7. A merchant having a piece of cloth measuring 25 yards, sold from it 1.875 yards for a coat, and 1.125 yards for a pair of pantaloons; how much was there left in the piece?

8. Mr. Farmer took to market 32 bushels of potatoes in one load, and peddled them as follows: 5.5 bushels for $2.75, 4.25 bushels for $2.125, 6.75 bushels for $3.375, 10.5 bushels for $5.25, and the rest of the load for $2.50; how much did he sell at the last sale; and how much did he get for his load?

9. A man owing $253, paid $187.375, how much did he then owe?

10. Add together 10.0625, 5.1875, 6.5, and 4.25.

11. How much is $15.5 + 2.75 + 3.75 - 12$?

12. What is the sum of 192.423 and 20.58?

13. What is the difference between 12.5 and 6.25?

14. What is the sum and difference of 245.0075 and 234.9925?

15. Subtract $2\frac{1}{10}$ from $4\frac{25}{100}$

175. Model of a Recitation.

I. If 1.75 yards be required for 1 coat, how much would be required for 7 coats?

1.75
7
12.25 yards.

Arrange the multiplicand and multiplier, and proceed as in the multiplication of integral numbers, (**27**) and the product would be 1225; but, since 7 times 175 things of *any* kind will be 1225 things of the *same* kind, 7 times 175 *hundredths* will be 1225 *hundredths*, or 12.25.

But, to analyze it, say: 7 times 5 hundredths are 35 hundredths, equal to 5 hundredths, which write in the place of hundredths, and 3 tenths, (**10**,) which add with 7 times 7 tenths, making 52 tenths, equal to 2 tenths, which write in the place of tenths, and 5 units, which add with 7 times 1 unit, making 12 units, which write at the left of the point, and the result will be the answer required.

2. At $175 per acre, what would be the cost of .7 acres, or, more properly, .7 of an acre?

$175
.7
$122.5

Since 1 acre costs $175, .7 of an acre would cost .7 times as much, or .7 as much. First, multiply by 7, as if it were 7 units, which gives $1225. But, the right multiplier being .7, only $\frac{1}{10}$ of 7 units, the right product should be only $\frac{1}{10}$ of 1225; therefore, divide this product by 10, by removing the point one place farther to the left, (**10**,) which gives $122.50, the answer required.

3. What would be the price of 2.25 cords of wood, at $5.375 per cord?

$5.375
2.25
26875
10750
10750
$12.09375

Since 1 cord costs $5.375, 2.25 cords would cost 2.25 times as much. 225 times 5.375 would be 1209.375. But, the multiplier being only $\frac{1}{100}$ of 225, the product will be only $\frac{1}{100}$ of 1209.375; therefore, divide by 100, by pointing off two more figures (**171**) for decimals, making $12.09375, which is the answer required.

4. Multiply .125 by .03.

.125
.03
.00375

3 times .125 would be .375. But the multiplier being $\frac{1}{100}$ of 3, the product will be $\frac{1}{100}$ of .375; therefore, divide by 100, by removing the point (**163**) two places farther to the left. But you must make those places in this example, by prefixing ciphers.

176. Proof of the Pointing in the Multiplication of Decimals.

Observe, *that, in the preceding examples,* (**175,**) *each product has as many decimal figures as all its factors. This will hold true in all cases; and this truth may be applied to prove the pointing of the product; for, if the factors be considered as integral numbers, the product would be integral; and, since for every removal of the point one place to the left, in either factor, that factor becomes* $\frac{1}{10}$ *as large,* (**10,**) *and, consequently, the product also becomes* $\frac{1}{10}$ *as large,* THE PRODUCT MUST BE DIVIDED BY 10, (*which is done by removing the point* (**10**) *one place to the left,*) FOR EVERY DECIMAL FIGURE IN ALL THE FACTORS.

177. Exercises in the Multiplication of Decimal Numbers.

In like manner, solve and explain the following problems.

1. How many yards of cloth would be required for 5 pairs of pantaloons, if 1.25 yards be put into each pair?
2. What cost 8 yards of cloth, at $2.875 per yard?
3. How many dollars in 8 ninepences, if $.125 make 1 ninepence?
4. If $.0625 make 1 fourpence-halfpenny, how much is 16 fourpence-halfpennies?
5. How much would a man receive for 5 barrels of pork at $17.25 per barrel?
6. At $5.50 per yard, what cost 10 yards of broad cloth?
7. At $.05 per pound, what cost 100 pounds of rice?
8. At $.20 per pound, what cost 1000 pounds of butter?
9. What cost 60 pounds of candles, at $.17 per pound?
10. What cost 12 dozen of eggs, at $.125 per dozen?
11. Multiply 5.333 by 8.
12. Multiply .464 by 25.
13. How much is 50 times .05?
14. What is the amount of the following bill?

Lowell, June 2, 1846.

Mr. John Debtor,

Bought of Charles Creditor,

7	yds.	Broad Cloth,	@	$5.50	per yard,
5	"	Cassimere,	@	1.50	" "
12	"	Striped Jean,	@	.375	" "
15	"	Bleached Sheeting,	@	.14	" "
27	"	Brown "	@	.125	" "

15. If a barrel of flour cost $6, what cost .5 barrels?
16 At $25 a ton, what cost .7 of a ton of hay?
17. At $6 per yard, what cost .25 of a yard of cloth?
18. At $8 per cord, what cost .75 of a cord of wood?
19. At $50 per acre, what cost .125 of an acre of land?
20. What is the amount of the following bill?

Salem, June 2, 1846.

Mr. Jacob Shem,

Bought of Israel Ham,

37.5	yards	German Broad Cloth,	@ $9	per yd.
25.75	"	French " "	@ $7	" "
18.125	"	English Cassimere,	@ $3	" "
24.375	"	American "	@ $2	" "

21. Multiply 144 by .5.
22. What is .5 of 1728?
23. Multiply 512 by .25.
24. What is .75 of 856?
25. Multiply 1840 by .125.
26. What is .625 of 1000?
27. Multiply 75 by .004.
28. What is .0003 of 3000?
29. At $.96 a gallon, what costs .4 gallons of oil?
30. At $.50 a yard, what costs .5 yards of cloth?
31. Multiply .3 by .6.
32. What is .4 of .7?
33. Multiply .25 by .5.
34. What cost 1.5 yards, at $.12 per yard?
35. What cost 4.12 yards, at $.50 per yard?
36. What is the amount of the following bill?

Boston, June 2, 1846.

Mr. Reuben Retail,

Bought of Warren Wholesale,

6.5	dozen	Spelling-Books,	@ $ 1.75	per doz.
8.25	"	Young-Readers,	@ $ 2.875	" "
10.75	"	National-Readers,	@ $ 7.50	" "
9.25	"	Testaments,	@ $ 3.625	" "
4.5	"	Polyglot Bibles,	@ $10.50	" "

37. What is the product of .204 multiplied by 1.4?
38. How much is 11.03 times .1109?
39. Multiply .04 by .004.
40. What is .0006 of .0012?
41. Multiply 1.006 by .002.
42. Multiply .062 by .003.
43. How much is .0004 of .025?
44. What is the second power (**49**) of .5?
45. What is the second power of .25?
46. What is the third power of .5?
47. What is the fourth power of .5?

178. Model of a Recitation.

1. If 4 books cost $3, how much would that be apiece?

```
4) 3.0 ($.75
   2 8
   ---
     20
     20
     --
```

One book being $\frac{1}{4}$ of 4 books, the *price* of 1 book should be $\frac{1}{4}$ *of the price* of 4 books. $\frac{1}{4}$ of 3 dollars would be $\frac{3}{4}$ of a dollar, (**94**,) in a common fraction; but the answer may be obtained in a decimal form. Thus, $\frac{1}{4}$ of 3 dollars not being a *whole* dollar, reduce the 3 dollars to dimes, or tenths, (**170**,) making 30 tenths, $\frac{1}{4}$ of which is .7, and 2 dimes, or tenths, remaining, which reduce to cents, or hundredths, making 20 hundredths, $\frac{1}{4}$ of which is .05, which, written with the .7, makes $.75, the answer required.

2. A man, having 3 acres of land, divided it into 8 equal house-lots. How many acres in each lot?

```
3
8) 3.0 (.375 acres.
   2 4
   ---
     60
     56
     --
      40
      40
      --
```

Each lot would contain $\frac{1}{8}$ of 3 acres, which is $\frac{3}{8}$ of an acre; but this common fraction may be reduced to a decimal fraction. Thus, $\frac{1}{8}$ of 3 acres not being a whole acre, reduce the 3 acres to tenths, making (**163**) 30 tenths, $\frac{1}{8}$ of which is .3, and .6 remaining, which reduce to hundredths, making 60 hundredths, $\frac{1}{8}$ of which is .07, which write, and .04 remaining, which reduce to thousandths, making 40 thousandths, $\frac{1}{8}$ of which is .005, which write, making .375 of an acre, the answer required.

3. At $3.375 for 9 gallons of molasses, what would be the cost of 1 gallon?

$$\begin{array}{r} 9\,)\,\underline{3.375} \\ \$.375 \end{array}$$

One gallon would cost $\frac{1}{9}$ of the price of 9 gallons. $\frac{1}{9}$ of 3 dollars not being a *whole* dollar, reduce the 3 to tenths, making, with the 3 tenths, 33 tenths, $\frac{1}{9}$ of which is .3, and .6 remaining, which reduce to hundredths, making, with the 7 hundredths, 67 hundredths, $\frac{1}{9}$ of which is .07, which write, and .04 remaining, which reduce to thousandths, making, with the 5 thousandths, 45 thousandths, $\frac{1}{9}$ of which is .005, which write, making $.375, which is the answer required.

179. Exercises in reducing Common Fractions to Decimal Fractions.

In like manner, solve and explain the following problems.

1. If 12.25 yards be required for 7 coats, how much would be required for 1 coat?
2. If $.375 be paid for 3 boy's tickets for admission to a concert, what would be the price of 1 ticket?
3. If 4 rides in a car cost $6, what is the cost of 1 ride?
4. If 3 bushels of apples be divided among 4 men, what would be each man's share?
5. If 12 dozen of eggs cost $1.50, how much is that per dozen?
6. If 3 acres of land be fenced off into 5 equal parts, how many acres in each part?
7. Reduce $\frac{3}{5}$ to a decimal fraction.
8. Reduce $\frac{3}{4}$ to a decimal fraction.
9. Reduce $\frac{1}{4}$ to a decimal fraction.
10. Reduce $\frac{1}{8}$ to a decimal fraction.
11. Reduce $\frac{5}{16}$ to a decimal fraction.
12. How many times is 24 contained in 6?
13. How many times is 12 contained in 28.8?
14. Divide 17.28 by 48.
15. Divide 1.44 by 72.
16. Divide 4.096 by 64.
17. Reduce $\frac{16}{256}$ to a decimal fraction.
18. How many times is 50 contained in 2.5?
19. How many times is 100 contained in 5?
20. Divide 3.75 by 8.
21. Divide 2.5 by 4.

180. MODEL OF A RECITATION.

1. Divide 54.32 by 40.

4) 5.432
1.358

The factors 10 and 4 composing 40, first divide by 10, by removing the point (**10**) one place farther to the left, to obtain $\frac{1}{10}$ of the dividend, which divide (**118**) by 4, to obtain $\frac{1}{4}$ of $\frac{1}{10}$, or $\frac{1}{40}$ of the dividend, which will be the answer required.

2. How many times is 35000 contained in 31.5?

7) .0315
5) .0045
.0009

The factors 1000, 7, and 5, composing 35000, first divide by 1000, by removing the point (**10**) three places farther towards the left, to obtain $\frac{1}{1000}$ of the dividend, which divide by 7, to obtain $\frac{1}{7}$ of $\frac{1}{1000}$, or $\frac{1}{7000}$ (**118**) of the dividend, which divide by 5, to obtain $\frac{1}{5}$ of $\frac{1}{7000}$, or $\frac{1}{35000}$ of the dividend, which will be the answer required.

181. OBSERVATION.

OBSERVE *that, when the divisor is a certain number of tens, hundreds, or thousands, &c., it is more convenient to divide first by* ONE *ten, hundred, or thousand, &c.; then divide that quotient by the other factor of the divisor.*

182. EXERCISES IN DIVIDING BY UNITS OF THE HIGHER ORDERS.

In like manner, solve and explain the following problems.

1. How many times is 500 contained in 1775?
2. Divide 6.25 by 250.
3. How many times 9000 in 63459?
4. What is the quotient of 129.6 divided by 1200?
5. Divide 3.651 by 30.
6. How many tons, of 2000 pounds each, in 16948.25 pounds?
7. What part of an hour is 21 minutes?
8. How many years would it take a man to save $5750, at $500 per year?
9. How many months, at $60 per month, would it take a man to earn $1296?
10. Divide 45.6855 by 1500.

11. Divide 8943.75 by 75000.
12. What part of a ton, or 2000 pounds, are 1728 pounds?
13. How many times is 42000 contained in 586.488?
14. Reduce 14784 minutes to hours.
15. How many eagles, of $10 each, in $362.50?

183. Illustration of Infinite Decimals.

1. *If $1 be paid for 9 writing-books, how much would that be apiece?*

$$\begin{array}{l} \frac{1}{9})\ 1.0\ (\$.111+ \\ \quad\ \ 9 \\ \quad\ \overline{\ \ 10} \\ \quad\ \ \ \ 9 \\ \quad\ \overline{\ \ \ 10} \\ \quad\ \ \ \ \ 9 \\ \quad\ \overline{\ \ \ \ \ 1} \end{array}$$

If 9 books cost $1, one book would cost $\frac{1}{9}$ of a dollar. But to reduce $\frac{1}{9}$ to a decimal form, (178) annex a point and a cipher to the numerator, and divide it by the denominator, which will give .1 for the first quotient figure. A cipher annexed to the remainder, gives .01 for the quotient; a cipher annexed to this remainder, gives .001 for the quotient; and thus, continuing without limit, the same remainder would recur, and the same figure would be repeated in the quotient. This quotient, and the like, are called *Infinite Decimals.*

When a quotient figure thus repeats, it is called a *repetend;* and the fact of its being a repetend, is denoted by placing a point *over* the first figure, omitting the rest. Thus, $.\dot{1} = .111$ &c. $= \frac{1}{9}$; $.\dot{2} = .222$ &c. $= \frac{2}{9}$; $.\dot{5} = .55$ &c. $= \frac{5}{9}$; and any figure, thus repeating, is so many times $\frac{1}{9}$. Therefore, to reduce any repetend of one repeating figure to a common fraction, you need only make the repeating figure the numerator, and 9 the denominator, and the result will be a fraction of 1 in the next higher place.

2. Reduce $\frac{1}{99}$ to a decimal.

$$\begin{array}{l} \frac{1}{99})1.00(.0101\ \&c. = .\dot{0}\dot{1} \\ \quad\ \ \ 99 \\ \quad\ \overline{\ \ \ 100} \end{array}$$

When two or more figures repeat, as in this example, the repetend is denoted by a point over both the *first* and *last* of the repeating figures.

Since $.\dot{0}\dot{1} = \frac{1}{99}$, any two figures thus repeating, equal so many times $\frac{1}{99}$; and in like

manner, three repeating figures equal so many times $\frac{1}{999}$, &c.

Therefore, *to reduce any repetend to a common fraction, take the repeating figures for a numerator, and as many 9s, for a denominator, and the result will be an equal fraction of* 1 *in the next higher place.*

184. Model of a Recitation.

1. Reduce $\frac{1}{6}$ to a decimal fraction, and back again to a common fraction.

$$\tfrac{1}{6} = .1\dot{6} = \tfrac{1}{10} + \tfrac{6}{9} \text{ of } \tfrac{1}{10} = \tfrac{1}{10} + \tfrac{6}{90} = \tfrac{9}{90} + \tfrac{6}{90} = \tfrac{15}{90} = \tfrac{1}{6}.$$

2. Reduce $\frac{71}{330}$ to a decimal, and back again.

$$\tfrac{71}{330} = .2\dot{1}\dot{5} = \tfrac{2}{10} + \tfrac{15}{99} \text{ of } \tfrac{1}{10} = \tfrac{66}{330} + \tfrac{5}{330} = \tfrac{71}{330}.$$

185. Exercises in the Reduction of Infinite Decimals.

In like manner, solve and explain the following problems.

1. Reduce $\frac{1}{12}$ to a decimal, and back again to a common fraction.
2. Reduce $\frac{5}{12}$ to a decimal, and back again to a common fraction.
3. Reduce $\frac{7}{12}$ to a decimal, and back again to a common fraction.
4. Reduce $\frac{11}{12}$ to a decimal, and back again to a common fraction.
5. Reduce $\frac{5}{6}$ to a decimal, and back again to a common fraction.
6. Reduce $\frac{23}{180}$ to a decimal, and back again to a common fraction.
7. Reduce $.5\dot{3}$ to a common fraction.
8. Reduce $.4\dot{6}$ to a common fraction.
9. Reduce $.3\dot{2}\dot{5}$ to a common fraction.
10. Reduce $.\dot{2}\dot{4}$ to a common fraction.
11. Reduce $\frac{5}{111}$ to a decimal, and back again to a common fraction.
12. Reduce $\frac{887}{900}$ to a decimal, and back again to a common fraction.

186. MODEL OF A RECITATION.

1. Reduce $\frac{1}{72}$ to a decimal fraction.

$$\begin{array}{l} 1 \\ \overline{72})1.00(.014- \\ 72 \\ \overline{} \\ 280 \\ 288 \\ \overline{} \end{array}$$

It will generally be sufficiently accurate to extend the quotient only to four or five places of decimals, and write in the last place the figure that will make the quotient the nearer correct, with (—) after it if the figure be too large, and (+) if it be too small. But if it be required to multiply such a quotient, some allowance should be made for its incorrectness.

2. How much would 125 cords of wood come to, at $\$5.1\dot{6}$ per cord?

$$\begin{array}{r} 5.1\dot{6} \\ 125 \\ \hline 258\dot{3} \\ 1033\dot{3} \\ 5166\dot{6} \\ \hline \$645.8\dot{3} \end{array}$$

5 times 6 are 30, but if another 6 of the repetend were written and multiplied by 5, it would afford 3 to be added to this 30, making 33; therefore, write 3 in the first place. 2 times 6 are 12, but if another 6 were written and multiplied by 2, it would afford 1 more to be added to this 12, making 13; write the 3, and, since this 3 is a repetend, continue it to the lowest place; so, also, continue the 6 down to the lowest place. The sum of the first column is 12, but as there might be a lower column like this, which would afford 1 more for this column, write 3 in the first place, &c.

3. What would be the cost of 1 pair of boots, if 5 pairs cost $\$1\dot{6}$?

$$\begin{array}{l} 5)1\dot{6}.(\$3.333+ \\ 15 \\ \overline{} \\ 16 \\ 15 \end{array}$$

5 is contained 3 times in 16, and 1 remains, which reduce to tenths, making with the .6 belonging to the repetend, 16 tenths, which will give .3 for the quotient; 1 tenth remains, which reduced to hundredths, and added to the .06, will give .03 for the quotient, &c.

187. EXERCISES IN THE USE OF INFINITE DECIMALS.

In like manner, solve and explain the following problems.

1. If there are $\$\dot{4}$ in 1 sterling pound, what is the value of 1000 sterling pounds?

2. If $\$.1\dot{6}$ make a shilling, what is the value of 6 shillings?

3. If $\$.08\dot{3}$ make a sixpence, what is the value of 12 sixpences?

4. If $\$.041\dot{6}$ make a threepence, what is the value of 24 threepences?

5. What is the value of 100 yards of silk, at $\$.8\dot{3}$ per yard?

6. If 5 spelling books cost $\$.8\dot{3}$, what is the price of 1 of them?

7. What is the value of 1 sterling pound, if 12 pounds make $\$5\dot{3}.$?

8. If 6 yards of silk cost $5, what is that per yard?

9. If 24 oranges cost $1, how much is that apiece?

10. What would 1 comb cost, if 60 combs should cost $5.00?

188. MODEL OF A RECITATION.

1. At $.12 a pound for raisins, how many pounds may be bought for $2.88?

```
.12)2.88(24 pounds.
     24
     ——
      48
      48
      ——
```

As many pounds may be bought as $.12 is contained times in $2.88; that is, 12 cents in 288 cents, or, 12 hundredths in 288 hundredths, which will be as many times as 12 things of *any kind* is contained in 288 things of the *same kind;* that is, 24 times; therefore, 24 pounds is the answer required.

2. If a charitable person distribute 3 barrels of flour to the poor, giving them .125 of a barrel apiece, to how many persons could he give a portion?

```
.125)3.000(24 persons.
      250
      ———
       500
       500
       ———
```

To as many persons as .125 is contained times in 3.

OBSERVE, *that, in the first example, both divisor and dividend being of the* SAME DENOMINATION, *the quotient was an integral number, and necessarily so, from the fact, that if both divisor and dividend be of the same denomination, it cannot affect the quotient, whether that denomination be pounds, barrels, miles,*

days, dollars, cents, mills, tenths, hundredths, or thousandths. For instance; 6 *things of* ANY DENOMINATION *are contained in* 12 *things of the* SAME *denomination*, 2 *whole times.*

Therefore, if this dividend be reduced to the same denomination as the divisor, that is, to thousandths, the quotient *so far* must be an integral number. The quotient being 24, so many persons could receive a portion of the flour.

3. How many bushels of apples at \$.5 per bushel may be bought for \$.375?

```
.5).375(.75 bushels.
    35
    --
     25
     25
     --
```

As many bushels as .5 is contained times in .375. 5 tenths is not contained in 3 tenths; therefore, there can be no units in the quotient; write the point, and take into consideration one more figure of the dividend, and the quotient figure thence obtained will be *tenths*, since tenths follow next to units; then come hundredths, &c., in their own order.

4. At \$6.25 per barrel for flour, how many barrels, or what part of a barrel may be bought for \$.03125?

```
6.25).03125(.005 barrels.
      3125
      ----
```

As many barrels as 6.25 is contained times in .03125. The divisor being hundredths only the hundredths of the dividend will afford units for the quotient, but 625 (hundredths) not being contained in the 3 (hundredths,) there will be no units in the quotient. Write the point, and take into consideration one more figure of the dividend, which gives 0 tenths for the quotient; the next figure gives 0 hundredths for the quotient, and the next figure gives 5 thousandths, which, as there is no remainder, is the answer required.

189. OBSERVATION.

In the division of decimals by integral, or decimal numbers, you need have little difficulty in ascertaining the right place for the point, if you OBSERVE *understandingly, that the figures of the dividend, as low as the lowest figure of the divisor, and no farther, will give integral quotient figures; and if you are careful to write the point in the quotient as soon as you have come to its place. Should not the dividend already be as low as the divisor, make it so by annexing ciphers.*

190. PROOF OF THE POINTING IN THE DIVISION OF DECIMALS.

To *prove* whether you have pointed the quotient correctly, consider that the divisor and quotient being the two factors of the dividend, (**75**) must together have the same number of decimal figures as the dividend, (**176**). If your work stands this test, probably you have put the point in its right place.

191. EXERCISES IN THE DIVISION OF DECIMALS.

In like manner, solve and explain the following problems.

1. How many umbrellas, at $1.25 apiece, may be bought for $3.75?
2. How many pairs of half-hose, at $.35 a pair, may be bought for $1.40?
3. How many pounds of coffee, at $.12 a pound, may be bought for $13.44?
4. How many pounds of cheese, at $.07 a pound, may be bought for $5.25?
5. At $1.50 per yard, how many yards of cassimere may be bought for $24.?
6. At $.80 per yard, how many yards of kersey may be bought for $20?
7. At $.40 per yard, how many yards of flannel may be bought for $12?
8. At $.20 per yard, how many yards of calico may be bought for $32?
9. If $108.50 be paid for cassimere, at $4 per yard, how many yards were bought?
10. If $18.50 be paid for broad-cloth, at $5 a yard, how many yards were bought?
11. If $28.35 be paid for 4.5 barrels of flour, how much is that a barrel?
12. If $153.525 be paid for 26.7 cords of wood, what would 1 cord cost?
13. What would 1 bushel of wheat cost, if 14.75 bushels cost $18.4375?
14. What would 1lb. of sugar cost, if 375.6 pounds cost $46.95?
15. What would 1 ton of potash cost, if 28.75 tons cost $3616.175?

16. What would 1 bushel of corn cost, if 63.5 bushels cost $49.53?
17. What costs 1 yard of cloth, if 79.4 yards cost $187.384?
18. How much sugar, at $.125 per pound, can be bought for $15.50?
19. If 112 pounds of iron cost $7.28, what is the cost of 1 pound?
20. How many times is $.06 contained in $33.60?
21. How many times is .46 contained in 18.4?
22. How many times is .18 contained in 7.02?
23. How many times is .4 contained in 2.5?
24. Divide 13.2 by 17.6.
25. Divide 61.512 by 2.4.
26. Divide .063 by 10.5.
27. Divide 1.8144 by 10.5.
28. Divide 5.38575 by 1.075.
29. What part of 8 is 3?
30. What part of 8 is .3?
31. What part of 2.4 is .6?
32. What part of 10.35 is 5.175?
33. How many times is .2 contained in .06?
34. How many times is .04 contained in .008?
35. Divide .00003 by .003.
36. Divide .000011021 by .0107.
37. Divide .00001 by .025.
38. Divide 47 by .1.
39. Divide 3. by .0003.
40. What part of 1.006 is .002012?

VIII. COMPOUND NUMBERS.

192. COMPOUND NUMBERS DEFINED AND ILLUSTRATED.

In simple numbers, such as we have heretofore employed, the several orders of units increase, or decrease, by the uniform ratio 10 or $\frac{1}{10}$, that is, one unit of each order equals 10 units of the next lower order (**10,**) or $\frac{1}{10}$ of a unit of the next higher order. But *a compound number is a number which expresses a quantity in several denominations having no uniform ratio.* Thus, 4 yards 2 feet 6 inches, is a compound number; 12 inches making a foot, and 3 feet a yard.

The ratios (**264**) *of the denominations of compound numbers, are exhibited in the following tables, which should be thoroughly committed to memory.*

193. Long Measure illustrated.

Long measure is used in measuring *distances* between two points. *Its unit is the mile*, which is divided and subdivided according to the following

Table.

m.		fur.		rods.		yds.		ft.		in.
1	=	8	=	320	=	1760	=	5280	=	63360.
		1	=	40	=	220	=	660	=	7920.
				1	=	$5\frac{1}{2}$	=	$16\frac{1}{2}$	=	198.
						1	=	3	=	36.
								1	=	12.

12 inches	= 1 foot.	40 rods	= 1 furlong.
3 ft.	= 1 yard.	8 fur.	= 1 mile.
$5\frac{1}{2}$ yd.	= 1 rod.		

194. Cloth Measure illustrated.

Cloth measure is used in measuring *cloths, laces, ribbons, &c. Its unit is the yard* of long measure, which is divided, and subdivided, according to the following

Table.

yd.		qr.		na.		in.
1	=	4	=	16	=	36.
		1	=	4	=	9.
				1	=	$2\frac{1}{4}$.

$2\frac{1}{4}$ inches	= 1 nail.
4 na.	= 1 quarter.
4 qr.	= 1 yard.

195. Square Measure illustrated.

Square measure is used in measuring *surfaces. The unit of measure is a square*, which is a plain surface having four equal sides and angles (**429**). It is called a square inch, foot, yard, &c., according as its side is one inch, foot, yard, &c., in length.

If lines one inch, or foot, &c., apart, be drawn parallel (**429**) to two opposite sides of a square, and, in like manner, lines be drawn parallel to the other two sides, the number of squares thus formed, or the superficial contents of the square, will be the second power (**49**) of the inches, or feet, &c., in a side of the square; for the number of inches, or feet, &c., in a side of the square, will be equal to the number of rows of squares, and to the number of squares in each row, the

product of which will express the contents of the square. Thus, a square foot would make 12 rows of 12 square inches each, equal to 144 square inches, which is the second power (**363**) of 12, the number of inches in the side of a square foot.

Hence, *the contents of any rectangular surface* (**429**) *is the product of its length and breadth.*

The *square mile* is divided and subdivided according to the following

Table.

m.	acres.	roods.	rods.	yds.	ft.	in.
1 =	640 =	2560 =	102400 =	3097600 =	27878400 =	4014489600.
	1 =	4 =	160 =	4840 =	43560 =	6272640.
		1 =	40 =	1210 =	10890 =	1568160.
			1 =	$30\frac{1}{4}$ =	$272\frac{1}{4}$ =	39204.
				1 =	9 =	1296.
					1 =	144.

144 inches = 1 foot.	40 rods = 1 rood.
9 ft. = 1 yard.	4 roods = 1 acre.
$30\frac{1}{4}$ yd. = 1 rod.	640 acres = 1 mile.

196. CUBIC MEASURE ILLUSTRATED.

Cubic measure is used in measuring *solids* and *capacities*, or anything that has *three dimensions, length, breadth, and thickness.*

The unit of measure is a cube, which is a solid having six equal square faces. (**429**). It is called a cubic inch, or foot, &c., according as a side of a face of it is one inch, or foot, &c., in length.

If 12 boards, each a foot square and one inch thick, be piled together, they would make a cubic foot; but each board may be divided into 12 equal pieces one inch wide and thick, and each piece into 12 cubic inches; therefore, each board would make $12 \times 12 = 144$ cubic inches, and the 12 boards, or cubic foot, would make $12 \times 12 \times 12 = 1728$ cubic inches.

Hence, *the contents of a cube is the product of its three dimensions*, or the product of its base (**429**) and height, or *the third power* (**49**) *of a side of one of the cube's faces.*

Hence also, *the contents of any solid, &c., having rectangular faces,* (**429**) *is the product of its three dimensions.*

A cubic *yard* is divided and subdivided, according to the following table.

Table.

yd.		ft.		in.	
1	=	27	=	46656.	1728 inches = 1 foot.
		1	=	1728.	27 ft. = 1 yard.

128 feet make 1 cord; but it is usual to consider $\frac{1}{8}$ of a cord, or 16 cubic feet, 1 cord-foot; hence, 8 cord-feet make 1 cord.

50 feet of timber, make 1 ton. But a ton of round timber will make only 40 feet of square timber, as $\frac{1}{5}$ is allowed for waste in squaring.

197. DRY MEASURE ILLUSTRATED.

Dry measure is used in measuring *grain, fruit, salt,* and *similar dry goods. Its unit is the bushel,* which is divided, and subdivided, according to the following

Table.

bu.		pk.		gal.		qt.		pt.	
1	=	4	=	8	=	32	=	64.	2 pints = 1 quart.
		1	=	2	=	8	=	16.	4 qts. = 1 gallon.
				1	=	4	=	8.	2 gals. = 1 peck.
						1	=	2.	4 pks. = 1 bushel.

One gallon contains 268$\frac{4}{5}$ cubic inches (**196.**)

198. LIQUID MEASURE ILLUSTRATED.

Liquid measure is used in measuring *all kinds of liquids. Its unit is the gallon,* which is divided, and subdivided according to the following

Table.

gal.		qt.		pt.		gill.	
1	=	4	=	8	=	32.	4 gills = 1 pint.
		1	=	2	=	8.	2 pt. = 1 quart.
				1	=	4.	4 qt. = 1 gallon.

One gallon contains 231 cubic inches.

One gallon of milk, and malt liquors contains 282 cubic inches.

199. TROY WEIGHT ILLUSTRATED.

Troy weight is used in weighing *precious metals,* and *liquids. Its unit is the pound,* which is divided, and subdivided, according to the following

Table.

lb.		oz.		dwt.		gr.		
1	=	12	=	240	=	5760.	24 grains	= 1 pennyweight.
		1	=	20	=	480.	20 dwt.	= 1 ounce.
				1	=	24.	12 oz.	= 1 pound.

200. APOTHECARIES' WEIGHT ILLUSTRATED.

Apothecaries' weight is used in *compounding medicines. Its unit is the pound*, which is divided, and subdivided, according to the following

Table.

℔		℥		ʒ		℈		gr.		
1	=	12	=	96	=	288	=	5760.	20 grains	= 1 scruple.
		1	=	8	=	24	=	480.	3 ℈	= 1 dram.
				1	=	3	=	60.	8 ʒ	= 1 ounce.
						1	=	20.	12 ℥	= 1 pound.

201. AVOIRDUPOIS WEIGHT ILLUSTRATED.

Avoirdupois weight is used in weighing *coarse goods*, such as are not weighed by the Troy, or Apothecaries' weight. *Its unit is the ton*, which is divided, and subdivided, according to the following

Table.

ton		lb.		oz.		dr.		gr.		
1	=	2000	=	32000	=	512000	=	14000000.	$27\frac{11}{32}$ grs.	= 1 dram.
		1	=	16	=	256	=	7000.	16 dr.	= 1 ounce
				1	=	16	=	437½.	16 oz.	= 1 pound.
						1	=	$27\frac{11}{32}$.	2000 lb.	= 1 ton.

The hundred-weight and quarter are generally dispensed with; and 2240 pounds are no longer considered a ton.

The grain in the three weights is the same, but the other denominations, though agreeing in name, differ in weight, excepting in Troy and Apothecaries' weight, where they are identical.

The weight of anything together with the container, is called *gross* weight; and the remainder, after deduction has been made for the container, &c., is called *net* weight, (**307.**)

202. TIME ILLUSTRATED.

Time is divided into years by the revolutions of the earth about the sun, and years into days by the revolutions of the earth upon its axis. A *year* is divided, and subdivided, according to the following

Table.

Year		days,		hours,		minutes,		seconds.
1	=	$365\frac{1}{4}$	=	8766	=	525960	=	31557600
		1	=	24	=	1440	=	86400
				1	=	60	=	3600
						1	=	60

60 seconds = 1 minute,	24 h. = 1 day,
60 m. = 1 hour,	$365\frac{1}{4}$ d. = 1 year.

$365\frac{1}{4}$ days, though not exactly a year, is sufficiently accurate for ordinary purposes, and will be considered a year, unless it be otherwise specified. It is usual in calendars to reckon 365 days to all years, except those divisible by 4, to which 366 days are allowed; but centennial years, though divisible by 4, have only 365 days, except the years which are divisible by 400, which have 366 days. Years having 366 days are called *leap years*, in which February has 29 days, otherwise, only 28. The calendar months, April, June, September, and November, have each 30 days; and January, March, May, July, August, October, and December, have each 31 days. But *in calculations involving dates*, 30 *days are considered a month, and* 12 *months a year.*

203 CIRCULAR MEASURE ILLUSTRATED.

Circular Measure is used in measuring *circles*, (**429**,) *and their circumferences*, particularly angles, latitude and longitude, and the relative situations of the heavenly bodies.

If from the centre of a circle straight lines be drawn, dividing the circumference into 360 equal parts, or arcs, each of these arcs is called a degree, as is also each of the spaces comprehended by two of the straight lines, or radii.

Hence, a degree being $\frac{1}{360}$ of a circle, or of its circumference, its extent will be greater, or less, according to the size of the circle.

The divisions, and subdivisions, of a *circle and its circumference* are exhibited in the following

Table.

C.	signs,	degrees,	minutes,	seconds.
1 =	12 =	360 =	21600 =	1296000
	1 =	30 =	1800 =	108000
		1 =	60 =	3600
			1 =	60

60″ (seconds) = 1 minute,	30° = 1 sign,
60′ = 1 degree,	12s. = 1 circle.

204. ENGLISH MONEY ILLUSTRATED.

English Money is the national currency of Great Britain. It was the currency of the United States till the establishment of *Federal Money*, in 1786, and is partially used here at present. *Its unit is the pound*, which is divided, and subdivided, according to the following

Table.

£.	*s.*	*d.*	*qr.*	
1 =	20 =	240 =	960	4 farthings = 1 penny.
	1 =	12 =	48	12 pence = 1 shilling.
		1 =	4	20 shillings = 1 pound.

205. CURRENCIES OF ENGLISH MONEY ILLUSTRATED.

The term pound represents different values in the different currencies; so also do the other denominations of English money, according to the following

Table.

£1, Sterling,	= 4\frac{4}{9}$,	used in England.	
£1, Can. Cur.	= $4,	" " Canada and Nova Scotia.	
£1, N. E. "	= 3\frac{1}{3}$,	" " N. E., Va., Ky., and Tenn.	
£1, N. Y. "	= 2\frac{1}{2}$,	" " N. Y., Ohio, and N. C.	
£1, Penn. "	= 2\frac{2}{3}$,	" " Penn., N. J., Del. and Md.	
£1, Georgia "	= 4\frac{2}{7}$,	" " Ga. and S.C.	
$1 = 4*s*. 6*d*.	= £$\frac{9}{40}$,	Sterling.	
$1 = 5*s*.	= £$\frac{1}{4}$,	Can. Currency.	
$1 = 6*s*.	= £.3,	N. E.	"
$1 = 8*s*.	= £.4,	N. Y.	"
$1 = 7*s*. 6*d*.	= £$\frac{3}{8}$,	Penn.	"
$1 = 4*s*. 8*d*.	= £$\frac{7}{30}$,	Ga.	"

$4.84 is the *present legal* value of the pound sterling.

206. NEW ENGLAND CURRENCY ILLUSTRATED.

New England Currency is still much used in appraising articles of merchandise. The most common prices are exhibited, reduced to Federal money, and aliquot parts of a dollar, in the following

Table.

S.	d.			S.	d.		
0	3	= \$.04 1/6	= \$ 1/24	3	3	= \$ .54 1/6	= \$ 13/24
0	4½	= .06 1/4	= 1/16	3	6	= .58 1/3	= 7/12
0	6	= .08 1/3	= 1/12	3	9	= .62 1/2	= 5/8
0	9	= .12 1/2	= 1/8	4		= .66 2/3	= 2/3
1		= .16 2/3	= 1/6	4	3	= .70 5/6	= 17/24
1	3	= .20 5/6	= 5/24	4	6	= .75	= 3/4
1	6	= .25	= 1/4	4	9	= .79 1/6	= 19/24
1	9	= .29 1/6	= 7/24	5		= .83 1/3	= 5/6
2		= .33 1/3	= 1/3	5	3	= .87 1/2	= 7/8
2	3	= .37 1/2	= 3/8	5	6	= .91 2/3	= 11/12
2	6	= .41 2/3	= 5/12	5	9	= .95 5/6	= 23/24
2	9	= .45 5/6	= 11/24	6		= 1.00	= 1.
3		= .50	= 1/2				

207. MODEL OF A RECITATION.

1. How many inches long is a road, which measures 3 miles, 6 furlongs, 25 rods, 2 yards and 1 foot?

```
  3m. 6f. 25rds. 2yds. 1ft.
   8
  30 furlongs.
  40
1225 rods.
  11
2)13475 half-yards.
  6739½ yards.
     3
 20219½ feet.
    12
242634 inches.
```

Since there are 8 furlongs in a mile, there will be 8 times as many furlongs as miles; 8 times 3 are 24, which, together with the 6, make 30 furlongs. Since there are 40 rods in a furlong, there will be 40 times as many rods as furlongs; 40 times 30 are 1200, which, together with the 25, make 1225 rods. Since there are 5½, or $\frac{11}{2}$, yards in a rod, there will be $\frac{11}{2}$ as many yards as rods; $\frac{11}{2}$ of 1225 are 6737½, which, together with the 2, make 6739½ yards. Since

there are 3 feet in a yard, there will be 3 times as many feet as yards; 3 times $6739\frac{1}{2}$ are $20218\frac{1}{2}$, which, together with the 1, make $20219\frac{1}{2}$ feet. And, since there are 12 inches in a foot, there will be 12 times as many inches as feet; 12 times $20219\frac{1}{2}$ are 242634 inches, which is the answer required.

2. Reduce 242634 inches to higher denominations.

$$
\begin{array}{rl}
12) & \underline{242634} \text{ inches.} \\
3) & \underline{20219} \text{ ft. 6 inches.} \\
5\frac{1}{2} = \frac{11}{2}) & 6739 \text{ yds. 2ft.} \\
 & \underline{2} \\
11) & \underline{13478} \text{ half-yards.} \\
40) & \underline{1225} \text{ rods, 3 half-yards.} \\
8) & \underline{30} \text{ fur. 25 rods.} \\
 & \text{3m. 6fur. 25rds. 2 yds. 1ft.}
\end{array}
$$

Since it takes 12 inches for 1 foot, there will be $\frac{1}{12}$ as many feet as inches; $\frac{1}{12}$ of 242634 are 20219 feet, and 6 inches remaining. Since it takes 3 feet for 1 yard, there will be $\frac{1}{3}$ as many yards as feet; $\frac{1}{3}$ of 20219 are 6739 yards, and 2 feet remaining. Since it takes $5\frac{1}{2}$, or $\frac{11}{2}$, yards for 1 rod, there will be as many rods as $\frac{11}{2}$ is contained times in 6739; multiply by 2, to ascertain how many times $\frac{1}{2}$ is contained, (**152**,) and divide that product by 11, to ascertain how many times $\frac{11}{2}$ is contained, which gives 1225 rods, and 3 half-yards remaining. Since it takes 40 rods for 1 furlong, there will be $\frac{1}{40}$ as many furlongs as rods; $\frac{1}{40}$ of 1225 are 30 furlongs, and 25 rods remaining. And, since it takes 8 furlongs for 1 mile, there will be $\frac{1}{8}$ as many miles as furlongs; $\frac{1}{8}$ of 30 are 3 miles, and 6 furlongs remaining: making, in all, 3 miles, 6 furlongs, 25 rods, 2 yards, and 1 foot, which is the answer required. The 2 yards are obtained thus: 1 of the 2 feet, and the 6 inches, make $\frac{1}{2}$ yard, which, with the $\frac{3}{2}$ yards, make $\frac{4}{2} = 2$ yards.

208. REDUCTION DEFINED.

Reduction is the changing of a compound number into a simple number of the same value, as in the first example, (**207**,) or the changing of a simple number into a compound number of the same value, as in the second example; or, the

changing of a number of any kind into another of the same value, as there are frequent examples in this book.

209. OBSERVATION.

OBSERVE, (**207**,) *that, in the first example, the simple number of the highest denomination in the given compound number, is reduced to the next lower denomination, to which is added what there may be of this lower denomination; that this sum is reduced to the next lower denomination still, and increased as before, and so on, till all is reduced as low as desired: the reduction in each case being performed by* MULTIPLYING *by the number which expresses how many units of the next lower denomination make a unit of the simple number to be reduced.*

OBSERVE, *also, that, in the second example, the number in each denomination is* DIVIDED *by the number which expresses how many units of its own denomination make a unit of the next higher denomination; that the last quotient, together with the several remainders, form the compound number required.*

210. EXERCISES IN THE REDUCTION OF COMPOUND NUMBERS.

Solve and explain the following problems, on the LEFT, *like the* FIRST, *and those on the* RIGHT, *like the* SECOND, *of the above examples* (**207**).

LONG MEASURE.

1. Reduce 6 m. 3 fur. 20 rds. 3 yds. and 2 ft. to feet.

2. Reduce 34001 feet to higher denominations.

3. How many rods in 25 miles?

4. How many miles in 16000 rods?

211. CLOTH MEASURE.

1. Reduce 27 yds. 1 qr. and 3 n. to nails.

2. Reduce 3627 inches to higher denominations.

3. What will 54 yds. 3 qrs. of cloth cost, at $.25 per yard?

4. How much cloth may be bought for $75.25, at $.25 per nail?

212. SQUARE MEASURE.

1. How many rods in a pasture which measures 2 m. 320 acres, and 80 rods?

2. Reduce 1548800 yards to acres.

3. What will 1 acre and 20 rods of land cost, at $.05 per foot?

4. How much land may be bought for $1575.25, at $.05 per foot?

213. Cubic Measure.

1. Reduce 4 yds. 15 ft. and 144 inches to inches.

3. How many inches in 3 tons of timber?

5. What cost 5 cords and 3 cord-feet of wood, at $.75 per cord-foot?

2. Reduce 81648 inches to higher denominations.

4. How much timber in 267840 inches?

6. How much wood may be bought for $79.20, at $.75 a cord-foot?

214. Dry Measure.

1. Reduce 5 bu. 3 pks. and 1 gal. to quarts.

3. What cost 2 bu. 1 pk. and 3 qts. of chestnuts, at $.04 per quart?

2. How many bushels in 10752 cubic inches?

4. What would be the price per bushel, if $60 be paid for 1920 pints of shag-barks?

215. Liquid Measure.

1. How many pints in 25 gals. 3 qts.?

3. What would a milk-man receive for 300 cans of milk, each holding 2 gals. and 2 qts., at $.05 per quart?

2. Reduce 1732 gills to higher denominations.

4. How many gallons of molasses in a cask gauging 7238 cubic inches?

216. Troy Weight.

1. In 7 lb. 11 oz. 3 dwt. 9 grs. how many grains?

3. What will 1 lb. 10 oz. 15 dwt. 20 grs. of jewelry cost, at $.04 per grain?

2. Reduce 45681 grains to higher denominations.

4. How much jewelry may be bought for $975.20, at $.02 per grain?

217. Apothecaries' Weight.

1. In 1 lb. 7℥, 2ʒ, 1℈, 12 grs. how many grains?

2. Reduce 9876 grains to higher denominations.

3. Reduce 6 lb. 10℥, 7ʒ, 2℈, 16 grs., to grains.

4. In 39836 grains, how many pounds?

218. Avoirdupois Weight.

1. Reduce 2 tons, 1200 lbs. 13 oz., to ounces.

2. How many tons, &c., in 1539000 drams?

3. What cost 20 tons, 500 lbs. of hay, at $.0075 per pound?

4. How much hay may be bought for $34.05, at $.005 per pound?

219. Time.

1. How many seconds old is a boy who has lived 12 y. 90 d. 15 h. 20 m. 30 s.?

2. How many days has a child lived, whose age is 31536000 seconds?

3. If a clock tick 60 times a minute, how many times would it tick in 16 years?

4. How long has a watch run, whose minute hand has turned round 46975 times?

220. Circular Measure.

1. Reduce 1 sign, 15° to seconds.

2. Reduce 749408 seconds to higher denominations.

3. If Massachusetts extends 3° 41′ in longitude, what is its extent in seconds?

4. What is the extent of Massachusetts in latitude, it being 5940 seconds from south to north?

221. English Money.

1. Reduce £25 10s. 6d. 3qrs. to farthings.

2. Reduce 9750qrs. to higher denominations.

3. In £125 15s. Canada currency, how much Federal money?

4. In 9600qrs. N. Y. currency, how much Federal money?

222. Model of a Recitation.

1. Reduce £$\frac{25}{144}$ to farthings.

$\frac{25\times20}{144\div12\div4}=\frac{500}{3}$ qrs. $=166\frac{2}{3}$ qrs.

There will be 20 times as many shillings as pounds, or $\frac{500}{144}$s.; 12 times as many pence as shillings, or $\frac{500}{12}$d.; *and*

4 times as many farthings as pence, or $\frac{500}{3}$ qrs. $= 166\frac{2}{3}$ qrs., which is the answer required.

2. Reduce £$\frac{83}{720}$ to shillings, pence, and farthings.

36) 83 (2s.
72
3) 11 (3d.
9
2
4
3) 8 (2$\frac{2}{3}$ qrs.
6
2
2
0

Perform the multiplications by dividing the denominator, (106,) or divisor, when practicable. Thus :

There will be 20 times as many shillings as pounds, or $\frac{83}{720 \div 20} = \frac{83}{36} = 2\frac{11}{36}$s. $\frac{11}{36}$ shillings will make 12 times as many pence, $\frac{11}{36 \div 12} = \frac{11}{3} = 3\frac{2}{3}$d. $\frac{2}{3}$ penny will make 4 times as many farthings, $\frac{2 \times 4}{3} = \frac{8}{3} = 2\frac{2}{3}$ qrs. In all, 2s. 3d. $2\frac{2}{3}$qrs., the answer required.

223. EXERCISES IN REDUCING A FRACTION TO UNITS OF LOWER DENOMINATIONS.

In like manner, solve and explain the following problems.

1. Reduce £$\frac{1}{1440}$ to the fraction of a farthing.
2. How many shillings and pence in $\frac{3}{8}$ of a pound?
3. Reduce $\frac{1}{3216}$ of a mile in length to the fraction of a rod.
4. What is the value of $\frac{5}{6}$ of a mile?
5. What fraction of a rod is $\frac{1}{1728}$ of an acre?
6. What is the value of $\frac{5}{6}$ of an acre?
7. Reduce $\frac{5}{108}$ lb. Troy to the fraction of an ounce.
8. Reduce $\frac{5}{7}$ of a Troy pound to ounces, pennyweights, and grains.
9. What fraction of a grain is $\frac{1}{512}$ of an ounce, Apothecaries' weight?
10. Reduce $\frac{7}{11}$℥ to drams, scruples, and grains.
11. Reduce $\frac{5}{960}$ of a pound, Avoirdupois weight, to the fraction of a dram.
12. Reduce $\frac{5}{72}$ of a ton to lower denominations.
13. What fraction of a quart is $\frac{3}{160}$ of a bushel?
14. Reduce $\frac{3}{8}$ of a bushel to quarts.
15. Reduce $\frac{1}{231}$ of a gallon to gills.
16. How many quarts, pints, &c., in $\frac{5}{12}$ of a gallon?
17. How many cubic inches in $\frac{1}{23328}$ of a yard?
18. Reduce $\frac{2}{81}$ of a cubic yard to inches.

19 What fraction of a day is $\frac{3}{1461}$ of a year?
20. What is the value of $\frac{9}{10}$ of a day?
21. Reduce $\frac{12}{360}$ of a degree to seconds.
22 Reduce $\frac{7}{8}$ of a circle to lower denominations.

224. Model of a Recitation.

1. What part of a pound is $\frac{500}{3}$ qrs.?

$£\frac{500 \div 20}{3 \times 4 \times 12} = \frac{25}{144}$.

There will be $\frac{1}{4}$ as many pence as farthings, or $\frac{500}{12}$d. (**114**,) $\frac{1}{12}$ as many shillings as pence, or $\frac{500}{144}$s., and $\frac{1}{20}$ as many pounds as shillings, or $£\frac{25}{144}$, the answer required.

2. What part of a pound is 4s. 6d.?

There will be 12 times as many pence as shillings, and 6 added, giving 54d., and there will be $\frac{1}{240}$ as many pounds as pence, which (**86**) gives $£\frac{54}{240} = £\frac{9}{40}$, as required.

225. Exercises in reducing lower Denominations to the Fraction of a higher.

In like manner, solve and explain the following problems.

1. Reduce $\frac{3}{5}$d. to the fraction of a pound.
2. What part of a pound is 10s. 6d.?
3. Reduce $\frac{5}{8}$ of a rod in length to the fraction of a mile.
4. Reduce 6 fur., 26 rods, 11 feet, to the fraction of a mile.
5. What fraction of an acre is $\frac{2}{33}$ of a rod?
6. Reduce 3 roods, 13 rods, 90 feet, to the fraction of an acre.
7. What part of a Troy pound is $\frac{5}{8}$ of an ounce?
8. Reduce 7 ounces, 4 dwt., to the fraction of a Troy pound.
9. What fraction of a pound is $\frac{7}{8}$ of a grain, Apothecaries weight?
10. Reduce $\frac{4}{5}$℈ to the fraction of an ounce.
11. What fraction of an ounce is 3ʒ, 2℈, 10 grs.?
12. Reduce $\frac{3}{4}$ of a pound to the fraction of a ton.
13. Reduce 1000 lb. 12 oz. 12 dr. to the fraction of a ton.
14. What fraction of a bushel is $\frac{2}{3}$ of a pint?
15. Reduce 3 pks. 1 gallon to bushels.
16. Reduce $\frac{128}{282}$ of a gill to gallons.
17. What part of a gallon is 1 qt., 1 pt., 1 gill?

18. How many yards in $\frac{1}{12}$ of a cubic foot?
19. Reduce 81 cubic inches to yards.
20. What part of a year is 24 hours?
21. Reduce 25 minutes, 30 seconds to days.
22. Reduce $\frac{2}{3}$ of a minute to degrees.
23. What part of a circle is 4 signs, 15 degrees, 15 minutes, and 15 seconds?

226. MODEL OF A RECITATION.

1. Reduce $.\dot{6}$ (**183**) of a bushel to lower denominations.

$.\dot{6}$ bushels.
$\underline{\quad 4}$
$2.\dot{6}$ pecks.
$\underline{\quad 2}$
$1.\dot{3}$ gallons.
$\underline{\quad 4}$
$1.\dot{3}$ quarts.

There will be 4 times as many pecks as bushels, or $2.\dot{6}$ pecks; the fraction $.\dot{6}$ pecks, will make 2 times as many, or $1.\dot{3}$ gallons; the fraction $.\dot{3}$ gal. will make 4 times as many, or $1.\dot{3}$ quarts; — in all, 2 pecks, 1 gal. $1.\dot{3}$ qts., which is the answer required.

227. EXERCISES IN REDUCING DECIMAL FRACTIONS TO UNITS OF A LOWER DENOMINATION.

In like manner, solve and explain the following problems.

1. What is the value of .625 of a bushel?
2. Reduce .3125 of a mile to lower denominations.
3. Reduce $.8\dot{3}$ yards to lower denominations.
4. Reduce .375 acres to lower denominations.
5. How many cubic feet in .1875 of a cord?
6. How many quarts, pints, &c., in $.\dot{6}$ of a gallon?
7. What is the value of .875℥.?
8. What is the value of $.\dot{4}$ of a ton?
9. Reduce .75 of a year to units of lower denominations.
10. If the moon advance in its orbit .406779661 + of a sign in a day, what is its daily advance?
11. Reduce .625£ to shillings and pence.

228. MODEL OF A RECITATION.

1. Reduce 2 pks. 1 gal. $1.\dot{3}$ qts. to the decimal of a bushel.

$$\begin{array}{r|l} 4 & 1.\dot{3} \text{ quarts.} \\ 2 & 1.\dot{3} \text{ gallons.} \\ 4 & 2.\dot{6} \text{ pecks.} \\ \hline & .\dot{6} \text{ bushels.} \end{array}$$

The $1.\dot{3}$ quarts will make $\frac{1}{4}$ as many gallons, that is, $.\dot{3}$ gallons, (**183**,) which annexed to the 1 gallon, make $1.\dot{3}$ gallons; $1.\dot{3}$ gallons will make $\frac{1}{2}$ as many, or $.\dot{6}$ pecks, which annexed to the 2 pecks make $2.\dot{6}$ pecks; $2.\dot{6}$ pecks will make $\frac{1}{4}$ as many, or $.\dot{6}$ bushels, which is the answer required.

Or, 2 pks. 1 gal. $1.\dot{3}$ qts., equal to $21.\dot{3}$ qts., which will make $\frac{1}{32}$ as many bushels, or $\frac{21\frac{1}{3}}{32}$ of a bushel, (**224**, 2,) this common fraction reduced to a decimal fraction, (**178**,) makes $.\dot{6}$ of a bushel as before.

229. Exercises in reducing lower Denominations to the Decimal of a higher.

In like manner, solve and explain the following problems.

1. Reduce 1 pk. 1 gal. 1 qt. 1 pt. to the decimal of a bushel.
2. Reduce 3 fur. 15 rods, 2 yds. to the decimal of a mile.
3. Reduce 1 qr. 3 n. 2 in. to the decimal of a yard.
4. Reduce 25 yds. 3 ft. 72 inches to the decimal of a square rod.
5. Reduce 10 ft. 144 inches to the decimal of a ton of timber.
6. What part of a gallon is 1 qt. 1 pt. 3 gills?
7. What part of a pound, Troy, is 1 oz. 1 dwt.?
8. What part of a ton is 1500lbs?
9. Reduce January to the decimal of a year.
10. What part of a revolution will a person's shadow make in 1 hour, if its hourly motion be 15 degrees?
11. Reduce 10s. 6d. 3qr. to the decimal of a pound.

230. Illustration of the Mode of reducing English Money by Inspection.

Shillings, pence, and farthings, may be reduced to the decimal of a pound; or, a decimal of a pound, to shillings, pence, and farthings, more expeditiously by *inspection*, as follows.

1. Reduce 15s. 7d. 3qrs., to the decimal of a pound.

14s.	$=.7$£
1s.	$=.05$£
7d. 3qrs.	$=.032+$ £
15s. 7d. 3qrs.	$=.782+$ £

Since 20s. $=$ 1£; 2s. $= \frac{2}{20}$£ $= \frac{1}{10} = .1$£; therefore, write .1£ for every 2 shillings, or 2 shillings for every .1£. Thus, 14 of the 15s. will make .7£.

Since 1s. $=\frac{1}{20}$£ $=\frac{5}{100}$ £ $=$ .05£, write .05£ for an odd shilling, or 1 shilling for .05£. Thus, 15s. make .75£.

Since 24qrs. $=\frac{24}{960}$£ $=$ .025£, add 1 to the farthings for every 24 farthings in the given pence and farthings, and they will become thousandths of a pound; or subtract 1 from the thousandths of a pound for every 25 thousandths in the given thousandths, and the remainder will be farthings. Thus, 7d. 3qrs. $=$ 31qrs., which will be a trifle more than .032£; — in all, £.782 + which is the answer required.

231. MODEL OF A RECITATION.

1. Reduce £.782 to units of lower denominations.

.7£	$=$ 14s.	
.05£	$=$ 1s.	
.032£ $=$		7d. 3qrs.
.782£ $=$	15s.	7d. 3qrs.

Since .1£ is 2 shillings, (**230**,) .7£ will be 14 shillings, and .05£ being 1 shilling, .75£ will be 15 shillings; also, .025£ being 24 farthings, or 1 subtracted from every 25 thousandths leaving farthings, the .032£ will be a trifle less than 31qrs. $=$ 7d 3qrs.;—in all, 15s. 7d. 3qrs., which is the answer required.

232. OBSERVATION.

OBSERVE, (**230**,) *that it will be sufficiently accurate, in reducing farthings to thousandths of a pound, to add* 1 *to the farthings if they are* MORE THAN ONE HALF OF 24, *or* 2 *if they are more than* $1\frac{1}{2}$ *times* 24; *and, in reducing thousandths of a pound to farthings,* (**231**,) *to subtract* 1 *from the thousandths if they are* MORE THAN ONE HALF OF 25; *or* 2, *if they are more than* $1\frac{1}{2}$ *times* 25; *since in either case the error will be less than one half of a farthing.*

233. EXERCISES IN REDUCING ENGLISH MONEY BY INSPECTION.

In like manner, solve and explain the following problems.

1. What is the value of £.25?
2. Express 6s. 6d. in a decimal.
3. Reduce £.625 to shillings and pence.
4. Reduce 7s. 6d. to pounds.
5. Reduce £.875 to lower denominations.
6. Reduce 4s. 5d. 3qrs. to the decimal of a pound.
7. Reduce £.323 to units of lower denominations.

8. Express 5s. 4d. in a decimal of a pound.
9. What is the value of £.116?
10. Express 4½d. in pounds.
11. What is the value of £.075.
12. Express 19s. 6d. 1 qr. in pounds.
13. What is the value of £.48?
14. Express 12s. 11d. 3 qrs. in pounds.
15. Reduce £.1875 to shillings, &c.
16. Reduce 1s. 2d. 3 qrs. to pounds.
17. Reduce £.6̇ to shillings, &c.
18. Reduce 6s. 8d. to pounds.
19. Reduce £12.18 to pounds, shillings, pence, and farthings.
20. Reduce £125 11s. 3d. to the decimal of a pound.

224. MODEL OF A RECITATION.

1. If from a piece of broad-cloth, measuring 35 yds. 3 nls., a tailor cut 6 yds. 1 qr. for a cloak; 3 yds. 3 qrs. 1 n. for a surtout; 2 yds. 2 qrs. 2 nls. for a frock-coat; 1 yd. 2 qrs. for a pair of pantaloons; and 2 qrs. 2 nls. for a vest; how much cloth would he use for these garments?

yds.	qrs.	nls.
6	1	
3	3	1
2	2	2
1	2	
	2	2
14	3	1

To ascertain how much cloth he would use, you must add together the several quantities cut off from the piece.

Arrange together the numbers to be added so that the simple numbers of each denomination may stand in a column by themselves. Add the numbers of each denomination separately, beginning with the lowest. The 5 nails of the first column are equal to 1 nail, which write in its own column, and 1 quarter, (**194**,) which add with the other quarters, making 11 quarters, equal to 3 quarters, which write in their own column, and 2 yards, which add with the other yards, making 14 yards, which write in their own column,—making in all, 14 yds. 3 qrs. 1 nl., which is the answer required.

225. OBSERVATION.

OBSERVE, (**224**,) *that, in the addition of compound numbers, the amount of each denomination must be reduced to units of the next higher denomination, and added there, and*

that in each column, only the excess over exact units of the next higher denomination are to be written.

236. EXERCISES IN ADDING COMPOUND NUMBERS.

In like manner, solve and explain the following problems.

1. How much cloth in 4 pieces, measuring as follows: 25 yds. 3 qrs.; 27 yds. 1 qr. 2 nls; 30 yds. 3 nls; and 28 yards?

2. How far would a horse trot in 5 hours, if he should trot the first hour 12 miles, 3 fur. 25 rods; the second hour, 11 miles, 7 fur. 20 rods; the third hour, 12 miles, 5 fur. 36 rods, 5 yds.; the fourth hour, 11 miles, 6 fur.; and the fifth hour, 13 miles, 15 rods?

3. If by one road, from Lowell to Boston, the distance be 25 m. 2 fur. and 20 rods, and by another road, the distance be 24 m. 7 fur. 12 rods; how much distance is traveled in riding to Boston by one road and returning by the other?

4. How much land in a farm which consists of 50 acres, 2 roods, 33 rods, wood-land; 25 acres, 14 rods, mowing-land; 30 acres, tillage; 20 acres, pasturing; 12 acres, 1 rood, covered with water; and 10 acres, 25 rods, swamp?

5. How much timber in two sticks, one of which measures 2 tons, 20 feet, 1642 inches; the other, 1 ton, 15 feet, 1295 inches?

6. How much wheat does that man raise, who has three fields, and raises on the first, 45 bush. 3 pks.; on the second, 36 bush. 1 pk. 7 qts.; and on the third, 30 bush. 2 pks. 1 quart?

7. How much molasses in two casks containing as follows: 70 gals. 3 qts., and 126 gals. 1 quart?

8. If a johannes weigh 18 dwts. a doubloon 16 dwts. 21 grs., a moidore 6 dwts. 18 grs. and an English guinea 5 dwt. 6 grs.; what is the weight of them all?

9. If an apothecary mix of one kind, 7℥, 5ʒ, 2℈; of another kind, 2℥, 3ʒ; and, of a third kind, 2℈, 10 grs.; what is the weight of the mixture?

10. If a load of hay weigh, without the wagon, 1 ton, 1200 lbs., and the weight of the wagon is 1984 lbs.; what is the weight of the whole?

11. If the load of hay mentioned in the last problem, were drawn over a bridge by two oxen and a horse, the oxen weighing 1 ton, 187 lbs., the horse weighing 1160 lbs., and the

driver 165 lbs. 12 oz.; how much more did the bridge sustain from this team passing over it?

12. If £15 14s. 6d. be paid for a pair of oxen, £14 for a horse, and £6 9s. 3d. for a cow; what would be the whole cost?

13. How old would a man be when his eldest child is 12 years, 25 days, and 16 hours old, if he was 25 years, 344 days, and 10 hours old, at the birth of this son?

237. Model of a Recitation.

How much cloth would remain, if 14 yds. 3 qrs. 1 nl. be cut from a piece measuring 35 yds. 3 nails?

yds.	qrs.	nls.
35	0	3
14	3	1
20	1	2

To ascertain how much cloth would remain, you must subtract the sum of what he used from the whole piece.

Write the subtrahend under the minuend, placing the simple numbers of each denomination under those of the same denomination.

Beginning with the lowest denomination, take 1 nail from 3 nails, and 2 nails remain, which write in their own column; reduce 1 of the 35 yards to quarters, making 4 quarters, (**194**,) from which take the 3 quarters, and 1 quarter remains, which write in its own column, and take 14 yards from 34 yards, and 20 yards remain, which write in their own column,—making in all, 20 yds. 1 qr. 2 nls., which is the answer required.

238. Observation.

Observe, *that, in subtraction of compound numbers, when a number of any denomination in the minuend is* less *than the corresponding number in the subtrahend, a unit, or in some cases, a part of a unit, of a higher denomination in the minuend must be reduced to make up the deficiency.*

239. Exercises in subtracting Compound Numbers.

In like manner, solve and explain the following problems.

1. If 5 yds. 1 qr. 3 nls. be cut from a piece of cloth measuring 20 yds. 3 qrs., how much would remain?

2. What is the difference between two piles of wood, one of which measures 15 cords, 3 cord-feet, 12 feet, and the other, 10 cords, 7 cord-feet, 6 feet?

3. If a farmer raise on one field 150 bush. of potatoes, and on another, 90 bush. 3 pks.; how much more does he raise on the large field than on the other.

4. If a merchant draw from a cask of molasses, containing 126 gals. 1 qt.; at one time, 13 gals. 3 qts.; at another, 10 gals.; and at a third time, 25 gals. 3 qts.; how much would remain in the cask?

5. How much more does a johannes weigh than a doubloon? (**236**, 8.)

6. If for a horse worth £18 10s. a man should give a cow worth £8 7s. 6d., and a calf worth £1 16s. 4d., and the rest in money; how much money would it require?

7. How much quicker could a person travel from Lowell to Boston, in a car, than in a stage, if it should take the car 1 h. 20 m. 30 sec., and the stage 4 h. 15 m. and 45 sec. in the passage?

8. How much longer is the day than the night, when the sun rises 56 minutes past 4 o'clock, and sets 4 minutes past 7 o'clock?

9. What is the difference of latitude between Boston and Cape Horn, Boston being 42° 28′ north, and Cape Horn being 55° 2′ south latitude? and how much farther from the Equator is Cape Horn than Boston?

10. What is the third angle of a triangle, (**429**, 14,) if the three angles equal 180°, the first 44° 13′ 24″, and the second, 79° 46′ 38″?

240. Model of a Recitation.

1. How much silver would be required for 15 spoons, if 1 oz. 14 dwt. 12 grs. be put into each spoon?

lbs.	*oz.*	*dwt.*	*grs.*
	1	14	12
			15
2	1	17	12

It would require for 15 spoons, 15 times as much as for 1 spoon. 15 times 12 grs. is 180 grs., equal to 12 grs., which write in their own column, and 7 dwts., (**199**,) which add with 15 times 14 dwts. making 217 dwts., equal to 17 dwts., which write in their own column, and 10 oz., which add with 15 times 1 oz., making 25 oz., equal to 1 oz., which write in its own column, and 2 lbs., which write in their own place,—making in all, 2 lbs. 1 oz. 17 dwts. 12 grs., which is the answer required.

241. EXERCISES IN MULTIPLYING COMPOUND NUMBERS.

In like manner, solve and explain the following problems.

The contractions practised in the multiplication and division of simple numbers, may be adopted here, whenever found more convenient.

1. If a silver thimble weigh 12 dwt. 12 grs., what would be the weight of 25 thimbles?

2. If Lowell railroad be 25m. 5 fur. 30 rods in length, how far would a locomotive run on this road in June, if it perform 3 trips per day?

3. How much cloth in 35 pieces, each piece containing 27 yds. 2 qrs. 3 nails?

4. How much land in a man's farm which is fenced into 9 fields, each containing 3 acres, 2 roods, 25 rods?

5. How much gravel could a man remove in 18 loads, at 1 yd. 25 feet each?

6. How much would that cask hold, which could be filled with 35 pailfuls, each pailful being 9 qts. 1 pt. 2 gills?

7. How many bushels of wheat in 135 bags, each containing 2 bush. 3 pks.?

8. What would be the weight of a box of 115 pills, if each pill should weigh 1℈, 4 grs.

9. What would be the weight of 15 barrels of flour, at 196 lbs. each?

10. How much sterling money in $25, if 1 dollar make 4s. 6d.?

11. If a person rise 1 h. 20 m. later than he ought to every morning for 12 years, how much time would be thus wasted?

12. If the sun appear to move 15° per hour, what is its apparent motion in a day of 15.1 hours in length?

13. Multiply 5 oz. 10 dwts. 15 grs. by 10.
14. Multiply 40 m. 3 fur. 25 rods by 15.
15. Multiply 34 yds. 1 qr. 1 n. by 64.
16. Multiply 57 acres 3 roods 20 rods by 100.
17. Multiply 17 tons 40 ft. 512 in. by 500.
18. Multiply 50 gals. 2 qts. 1 pt. 1 gill by 25.
19. Multiply £84 15s. 3d. 2 qr. by 12.
20. Multiply 7 tons 1800 lbs. 12 oz. by 2.5.
21. Multiply 5 y. 212 d. 10 h. 15 m. by 100.

242. MODEL OF A RECITATION.

1. If 2 lb. 1 oz. 17 dwt. 12 grs. of silver be put into 15 spoons, what would be the weight of each spoon?

```
   lbs. oz. dwts. grs.
    2    1   17   12
   12
   —
15)25
   —
    1 oz. 10
          20
          ——
      15)217
          —
          14 dwts. 7
                  24
                  ——
              15)180
                  ——
 1 oz. 14 dwts. 12 grs.
```

Each spoon would weigh $\frac{1}{15}$ as much as 15 spoons. As 15 is not contained in the 2 lbs. reduce them to ounces, making with the 1 oz. 25 ounces, which divided by 15 gives 1 oz. for the quotient, and 10 oz. remainder, which reduced to pennyweights, make with the 17 dwts. 217 dwts., which divided by 15 gives 14 dwts. for the quotient, and 7 dwts. remainder, which reduced to grains, make with the 12 grs. 180 grs., which divided by 15 gives 12 grains for the quotient, —making in all, 1 oz. 14 dwts. 12 grs., which is the answer required.

243. EXERCISES IN DIVIDING COMPOUND NUMBERS.

In like manner, solve and explain the following problems.

1. What would be the weight of 1 dollar, the weight of 8 dollars being 6 oz. 18 dwts. 16 grs.?

2. How far does that boy live from his school-house, who has to travel 170 m. 2 fur. in attending school twice a day, for 60 days?

3. How long is that room which requires 27 yds. 2 qrs. of carpeting cut into 5 pieces, to carpet it?

4. If 135 acres be fenced off into 16 equal lots, what would be the size of each lot?

5. If a man team to market 25 cords, 5 cord-feet of wood, at 20 loads; how much would that be a load?

6. What is the contents of each of such bottles that 160 of them could be filled from a cask holding 115 gallons?

7. How many apples in a barrel, if 101 bush. 1 pk. make 45 barrels?

8. What would be the weight of a dose of medicine, if 4 ℥, 5 ʒ, 2 ℈, 12 grs. be taken at 12 doses?

9. If 33 lbs. of steel be put into 256 axes, how much would that be apiece?

10. If 147 bushels cost 47£. 12s. 5d., what does it cost per bushel?

11. If a teacher devote 5 hrs. 30 min. per day to 50 scholars, how much would that be for each scholar?

12. What would be the daily motion of the moon, if it complete a revolution in $29\frac{1}{2}$ days?

244. MODEL OF A RECITATION.

How many rods round a pasture, measuring on the first side $\frac{11}{32}$ of a mile, on the second 1.17 furlongs, on the third $2\frac{3}{20}$ furlongs, and on the fourth 1 furlong 18.8 rods.

$\frac{11}{32}$ m.	$= \frac{11 \times 8 \times 40}{32}$ rods	$= 110$ rods.
1.17 fur.	$= 1.17 \times 40$ rods	$= 46.8$ "
$2\frac{3}{20}$ fur.	$= 2\frac{3}{20} \times 40$ rods	$= 86$ "
1 fur. 18.8 rods	$= 40$ rods $+ 18.8$ rods	$= 58.8$ "
		301.6 rods.

These quantities being expressed in different ways, *must*, before they can be added, *be reduced to numbers of the same kind.*

Multiply $\frac{11}{32}$ m. by 8 to reduce it to furlongs, and that product, also, 1.17 furlongs, $2\frac{3}{20}$ furlongs, and 1 furlong, by 40 to reduce them to rods; these several quantities, being made alike and added, make 301.6 rods, which is the answer required.

245. EXERCISES IN THE USE OF NUMBERS VARIOUSLY EXPRESSED.

In like manner, solve and explain the following problems.

1. How much is $\frac{5}{16}$ of a week and $\frac{1}{5}$ of a day?

2. What is the difference between two fields, one of which measures $2\frac{4}{5}$ acres, the other 1 acre $2\frac{1}{5}$ roods?

3. How much cloth in three remnants, the first measuring 2.4 yards, the second 1 yd. 3 qrs., and the third $\frac{7}{8}$ of a yard?

4. How many cubic inches in 4 bushels, 1.375 bushels and $\frac{4}{5}$ of a peck?

5. If $2\frac{3}{4}$ pecks be taken from a bag holding 2.75 bushels; how much would be left?

6. How much water in a pail measuring 10.75 quarts, but wanting $\frac{3}{16}$ of a gallon of being full?

7. How much silver in a large and small spoon, sugar-tongs, and butter-knife, weighing severally, 2.9 oz., 1 oz. $12\frac{2}{3}$ dwt., $1\frac{5}{8}$ oz. and 18.5 dwt.?

8. If an apothecary should mix a medicine at a cost of \$.48 per ounce, and should sell it at \$.48 per ounce avoirdupois; how much would he gain in selling 10 lbs.?

9. Add $\frac{4}{15}$ of a ton, $16\frac{3}{16}$ lbs., $\frac{3}{25}$ of a ton, and $.8\dot{3}$ of a ton together.

10. Subtract $\frac{4}{5}$ of a shilling from £1.25.

11. What is the difference between 52 wks. 1 d. 6 hrs. and 365.25 days?

12. If two ships sail from the same point, one north $18\frac{7}{12}$ degrees, the other south 25° $33\frac{1}{3}'$; what then would be the latitude between them?

246. Model of a Recitation.

How many years, months and days from the first resistance with arms in the American revolution, April 19th, 1775, to the declaration of Independence, July 4th, 1776?

years.	mo.	days.
1775	6	3
1774	3	18
1	2	15

To answer this question you must subtract the time between the Christian era and the *earlier* date, which is 1774 years, 3 months and 18 days, from the time between the Christian era and the *later* date, which is 1775 years, 6 months and 3 days, the hours, &c., being disregarded.

Reduce 1 of the 6 months to days, making, (**202**,) with the 3 days, 33 days, from which 18 being subtracted, 15 days remain; 3 months from the other 5 months leave 2 months; and 4 years from 5 years leave 1 year, making, in all, 1 year, 2 months and 15 days, which is the answer required.

1776	7	4
1775	4	19
1	2	15

But a more convenient way of obtaining the same result is, instead of writing the *number* of years, months and days, to write the *order* of the year, month and day, that is, *the dates themselves*. Thus, from the 1776th year, 7th month and 4th day, subtract the 1775th year, 4th month and 19th day;

precisely as so many years, months and days. This increases each number in each denomination by the same quantity, 1, and, consequently, does not affect the difference.

247. EXERCISES IN FINDING THE DIFFERENCE OF TIME BETWEEN DATES.

In like manner, solve and explain the following problems.

1. How long from the time that Washington entered upon the command of the American army at Cambridge, July 2d, 1775, till the disbanding of the army at West Point, November 3d, 1783?
2. How long was General Harrison's victory at Tippecanoe, November 7th, 1811, before General Jackson's victory at New Orleans, January 8th, 1815?
3. How long from the date of a note, May 10th, 1835, till its payment, June 5th, 1840?
4. How old is that man, June 27th, 1840, who was born March 23d, 1807?
5. What is the date of that note, which was paid December 31st, 1839, 2 y. 3 m. 11 d. after its date?
6. When was that note paid, which was dated August 4th, 1836, and paid 3 y. 3 m. 30 d. after date?
7. How much older is Lizzie, born Sept. 11th, 1843, than Mary, born April 15th, 1846?
8. How long was John absent, having left town July 1st, 1839, and returned August 25th, 1840?

248. MODEL OF A RECITATION.

What cost 4 bush. 3 pks. 1 gal. of wheat, at 5s. 6d. per bushel?

```
  4.875 bushels.
   .275 £.
  ------
  24375
 34125
 9750
 ---------
1.340625£ =
1£. 6s. 9¾d.
```

The whole cost will be the product of the price of one bushel by the number of bushels; but, *before they can be multiplied together, they must be reduced to simple numbers;* 4 bush. 3 pks. 1 gal. reduced to bushels is 4.875 bushels, (**228.**) and 5s. 6d. reduced to pounds is £.275 (**230.**) Now, since 1 bushel costs £.275, the whole cost will be .275 as many pounds as bushels, which is £1.340625 equal (**231**) to 1£. 6s. $9\frac{3}{4}$d.

249. EXERCISES IN THE REDUCTION OF COMPOUND NUMBERS FOR MULTIPLICATION.

In like manner, solve and explain the following problems.

1. What is the value of 15 acres, 2 roods, 20 rods, $62.25 being the cost of each acre?

2. What would 12 miles, 3 furlongs, 32 rods of road cost, at 175£. 10s. 6d. per mile?

3. Goliah, measuring $6\frac{1}{2}$ cubits of 1 ft. 7.168 in. in height, was how tall in feet and inches?

4. What is the cost of 5 yds. 1 qr., 2 nls. of broadcloth, at $5.50 per yard?

5. What is the value of $2\frac{1}{2}$ tons, $1\frac{3}{8}$ tons, and $2\frac{4}{5}$ tons of hay, at $12.25 per ton?

6. What will $4\frac{3}{8}$ tons of iron come to, at 20£. 15s. 6d. per ton?

7. What will $8\frac{3}{5}$ hogsheads of molasses, at 63 gallons each, come to, at 2s. 6d. per gallon?

8. At 5s. per bushel, what will 4 bush. 2 pks. 1 qt. of corn come to?

9. Bought a silver cup, weighing 9 oz. 4 dwt. 16 grs., at 6s. 8d. per ounce, what was the whole cost?

250. MODEL OF A RECITATION.

How many square feet in a square, measuring 16 ft. 6 in. on each side?

```
  16.5
  16.5
  ----
   825
  990
 165
 ------
272.25 feet.
```

Since the contents is the product of the length by the breadth, (**195**,) and 16 feet 6 inches being 16.5 feet, the contents will be $16.5 \times 16.5 = 272.25$ square feet, which is the answer required, (**429**, 26.)

251. EXERCISES IN THE MENSURATION OF SURFACES AND SOLIDS.

In like manner, solve and explain the following problems.

1. How many square inches (**195**) on the page of a book 8 inches long and 5 inches wide?

2. How many square yards in a square, measuring $5\frac{1}{2}$ yards on each side?

3. How many feet in a floor which is 16½ feet long and 15 feet wide?

4. How many square yards will carpet a floor which is 5 yds. 1 ft. 6 in. long, and 5 yards wide?

5. How many rods in a garden 5 rods, 2¾ yards long, and 4.5 rods wide?

6. How much land in a field 26 rods, 11 feet long, and 6 rods wide?

7. How many feet in a board 17 ft. 9 in. long, and 1 ft. 6 in. wide?

8. If from a square stick of timber 1 foot wide and 1 foot thick, you saw off a piece 1 foot long, that block would contain exactly 1 cubic foot; how many cubic feet in such a stick of timber 16 feet long?

9. How many boards 1 inch thick could be made of that stick, allowing no waste in sawing?

10. How many cubic inches in one of the boards?

11. How many cubic inches in all of the boards?

12. How many cubic inches in a stick of timber 1 foot wide and thick, and 16 feet long?

13. How many feet in a stick of timber 24 feet long, 1.8 feet wide, and 1.5 feet thick?

14. How many feet in 2 sticks of timber, each 36 feet long, 2 ft. 6 in. wide, and 2 ft. 3 in. thick?

15. How many feet in a load of wood 8 ft. long, 3 ft. 6 in. wide, and 3 ft. 9 in. high?

16. How many feet in a load of gravel 7 ft. 6 in. long, 4 ft. 3 in. wide, and 2 ft. 3 in. high?

17. How many yards of gravel must be removed to make a cellar 2.5 yards deep, 6 yards long, and 5.6 yards wide?

18. How many yards of stone work in a wall 38½ yards long, 4 ft. 6 in. high, and .8 of a yard thick?

19. How many feet in a room 17 ft. 6 in. long, 15 ft. 3 in. wide, and 10 ft. 9 in. high?

20. How many cord-feet in a load of wood 8 feet long, 4 feet wide, and 4 feet high?

21. How many cords of wood in a pile 32 feet long, 4 feet wide, and 7 feet high?

252. Illustration of the mode of Abridging the Process of solving Problems.

1. What is the value of a pile of wood 64 feet long, 4 feet wide, and 6 ft. 6 in. high, at $5.25 per cord?

In questions like this, involving both multiplication and division, it will be most convenient, and will generally much abridge the process, *to express all the operations before performing any of them.* Thus; the length 64 feet multiplied by the breadth 4 ft. will give the

$$64 \times 4 \times 6.5 \div 16 \div 8 \times 5.25 = \$68.25.$$

square contents of the base, which multiplied by the height 6 ft. 6 in., or 6.5 feet, will give the cubic contents (**196**) in feet; this divided by 16 will give the contents in cord-feet, and this quotient divided by 8 will give the contents in cords, which multiplied by the price of 1 cord, will give the whole value, or the answer required.

The whole process being thus expressed and explained, perform the operations indicated by the signs, in such order as will require the fewest figures; thus, divide the 64 by the 16; the quotient, 4, multiply by the 4; the product, 16, divide by the 8; multiply 6.5 by the quotient, 2, and multiply $5.25 by that product, 13, making $68.25, which is the answer required.

Perhaps it will be more convenient still, to express the process in a fractional form, (**86**,) by making the divisors factors of the denominator, and then to reduce the fraction to its lowest terms. (**121**.) Thus, the 16 in the denominator, being a factor of the 64 in the numerator, may be canceled

$$\frac{\overset{4}{\cancel{64}} \times \overset{2}{\cancel{4}} \times 6.5 \times 5.25}{\cancel{16} \times \cancel{8}} = \$68.25.$$

from both terms (**121**); and 4, the other factor of 64, multiplied into the other 4 of the numerator, makes 16, of which the 8 in the denominator is a factor; consequently, 8 may be canceled from both terms; and 2, the other factor of 16, multiplied into 6.5 makes 13, which multiplied into $5.25 makes $68.25, as before.

253. Exercises in solving Problems involving both Multiplication and Division.

In like manner, solve and explain the following problems.

1. How many cords of wood in a pile 40 feet long, 4 feet wide, and 9 ft. 3 in. high?

2. What is the value of a load of wood, measuring 8 feet in length, 4 ft. 6 in. in width, and 5 ft. 3 in. in height, at $8 per cord?

3. What is the value of a stick of timber, measuring 50 feet in length, 2 ft. 6 in. in width and thickness, at $4 per ton?

4. What would be the cost of digging a cellar 19 ft. 6 in. long, 15 feet wide, and 10 ft. 6 in. deep, at $.25 per yard?

5. How many acres in a pasture 36 rds. 8.25 ft. long, and 30 rods wide?

6. How many yards in a floor 28 ft. 9 in. long, and 22 ft. 4 in. wide?

7. How many yards, in length, of carpeting, which is 4 ft. 6 in. wide, will cover a floor 17 feet long, and 15 ft. 6 in. wide?

8. How many days would Samuel have to go to school, twice per day, to travel 1000 miles, if he live 5 furlongs from school?

9. How many times would a wagon wheel, 13 ft. 9 in. in circumference, revolve in running 25 miles, 6 furlongs?

10. How many times could a coal-basket, holding 1 bush. 1 gal. 2 qts., be filled from a coal-cart, containing 65 bush. 1 pk. 2 qts.?

11. What would a hogshead of cider, containing 62 gals. 2 qts. come to, at $.04 per bottle of 1 pt. 2 gills?

12. What is the value of a lot of spoons, weighing 9 lbs. 10 oz. 4 dwt., each spoon weighing 16 dwt. 10 grs., and worth $1.25?

13. How many loads of hay, each weighing 1750 lbs., in a stack, weighing 16 tons, 875 lbs?

14. How many pills may be made of a mixture of 10℥ 4ʒ, each weighing 1℈, 10 grs.?

15. In 67£. 11s. 7d., how many crowns, at 6s. 7d. each?

16. How many yards of English cassimere, at 12s. 8d. per yard, may be bought for 395£. 4s. sterling?

17. What part of £1, or 20s. is 15s.?

18. If a yard of broadcloth cost 17s. 6d.; what part of a yard might be bought for 13s. 6d.?

19. What part of £1 15s. 9d. is 15s. 9d.?

254. MODEL OF A RECITATION

Reduce £64 17s. 6d. of sterling money, and of Canada, New England, New York, Pennsylvania, and Georgia, currencies to Federal money; and the results back again as before.

£64.875
40
9) 2595.000
288.33\frac{1}{3}$
9
40) 2595.00
£64.875 = £64. 17s. 6d.

Reduce 17s. 6d. to the decimal of a pound, (**230**,) and, since in sterling money (**205**) one pound equals 4\frac{4}{9}$, there will be $4\frac{4}{9}$, or $\frac{40}{9}$ times as many dollars as pounds, which gives 288.33\frac{1}{3}$.

Since one dollar equals £$\frac{9}{40}$, there will be $\frac{9}{40}$ as many pounds as dollars; which gives £64.875.

£64.875
4
4) $259.5000
£64.875.

Since, in Canada currency, (**205**,) one pound equals $4, there will be 4 times as many dollars as pounds; which gives $259.50.

Since 1 dollar equals £$\frac{1}{4}$, there will be $\frac{1}{4}$ as many pounds as dollars; which gives £64.875.

3) £64.875
$216.25
.3
£64.875.

Since, in N. England currency, (**205**,) one pound equals 3\frac{1}{3}$, there will be $3\frac{1}{3}$, or $\frac{10}{3}$ times as many dollars as pounds; divide by 3 to obtain $\frac{1}{3}$, and remove the point one place farther to the right to obtain $\frac{10}{3}$; which gives $216.25.

Since $1 = £.3, there will be .3 as many pounds as dollars, or £64.875.

4) £64.875
162.18\frac{3}{4}$
.4
£64.875.

Since, in New York currency, (**205**,) £1 = 2\frac{1}{2}$, there will be $2\frac{1}{2}$, or $\frac{10}{4}$ times as many dollars as pounds; which gives 162.18\frac{3}{4}$.

Since $1 = £.4, there will be .4 as many pounds as dollars; which gives £64.875.

£64.875
8
———
3) 519.000
———
$173.
3.
———
8) 519.
———
£64.875.

Since, in Pennsylvania currency, (**205**,) £1 = 2\frac{2}{3}$, there will be 2$\frac{2}{3}$, or $\frac{8}{3}$ times as many dollars as pounds; which gives $173.

Since $1 = £$\frac{3}{8}$, there will be $\frac{3}{8}$ as many pounds as dollars; which gives £64.875.

£64.875
30
———
7) 1946.250
———
278.03\frac{4}{7}$
7
———
30) 1946.25
———
£64.875.

Since, in Georgia currency, (**205**,) £1 = 4\frac{2}{7}$, there will be 4$\frac{2}{7}$, or $\frac{30}{7}$ times as many dollars as pounds; which gives 278.03\frac{4}{7}$.

Since $1 = £$\frac{7}{30}$, there will be $\frac{7}{30}$ as many pounds as dollars; which gives £64.875.

255. Exercises in the Reduction of Currencies.

In like manner, solve and explain the following problems.

1. One dollar is what part of a pound in sterling money, and in Canada, New England, New York, Pennsylvania, and Georgia currencies?
2. What part of one dollar is one pound of sterling money, and of each of the currencies?
3. How many dollars in £1 of sterling money, and of each of the currencies?
4. Reduce £25 10s. from sterling to federal money.
5. Reduce $25.375 to sterling money.
6. How much Federal money would pay for a farm, in Canada, worth £500.
7. If a Canadian lumber merchant sell, in New Orleans lumber that cost him £875 for $3750, whether, and how much, would he gain, or lose?

8. If a farm, in Cambridge, Massachusetts, which, in 1740, cost £125 7s. 6d., be now worth $3000, how much has it increased in value?

9. How many dollars should a New York merchant receive for 125 yards of flannel, at 3s. 6d. per yard?

10. If a wholesale dealer, in Philadelphia, receive $1500 for a quantity of cloth, at 3s. 9d. per yard; how many yards in the quantity?

11. If a Georgia planter sell his wheat at 3s. 6d. per bushel, and receive $387.50; how many bushels would he sell?

12. Where can the same kind of penknives be bought the cheapest, if 2s. 3d. apiece be the price?

13. How much Federal money would a horse cost in each of the several currencies; if the price be £20 18s.?

14. Reduce $.25 to each of the several currencies.

15. Reduce 1s. 6d. of the several currencies to Federal money.

Note. Whenever any of the denominations of English money occur in the following pages, they will be in New England currency, unless otherwise specified.

256. Model of a Recitation.

1. A merchant sold 3545 yards of cotton cloth, at 9d. per yard; what was the amount of it in Federal money?

Since the price of 1 yard was $\frac{1}{8}$ of a dollar, (**206**,) the amount of the whole quantity must have been $\frac{1}{8}$ as many dollars as there were yards; therefore, divide (**92**) the number of yards by 8 to ascertain how many dollars there were in the amount.

$$\begin{array}{r} 8)\underline{3545} \\ \$443.125. \end{array}$$

2. A merchant paid $35.31¼ for cotton cloth, at 4½d. per yard; how many yards did he purchase?

Since the price of 1 yard was $\frac{1}{16}$ of a dollar, (**206**,) he must have purchased 16 times as many yards as he paid dollars (**151**); therefore, multiply the number of dollars that he paid by 16 to ascertain how many yards he purchased.

$$\begin{array}{r} \$35.31\frac{1}{4} \\ \underline{16} \\ 565.\text{ yards.} \end{array}$$

257. Observation.

Hence, observe, *that,* (**256,**) *in the following questions, and whenever the multiplier, or divisor, is such that it can be reduced to an aliquot part of a dollar, the process may be much abridged, by using the aliquot part.*

258. Exercises in the Use of Aliquot Parts.

In like manner, solve and explain the following problems.

1. What cost 4872 oranges, at 3d. apiece?
2. How many pounds of sugar, at 6d. per pound, may be purchased with \$12.58⅓?
3. How many dollars would it take to pay for 144 yards of calico, at 1s. per yard?
4. How many times is 1s. 3d. contained in \$5?
5. What is the value of 1728 bushels of apples, at 1s. 6d. a bushel?
6. How many bushels of potatoes would it take to come to \$24.66⅔, at \$.33⅓ per bushel?
7. What would be the cost of 24 yards of muslin, at 2s. 3d per yard?
8. Bought 36 yards of bombazine, at 2s. 6d. per yard. What was the bill?
9. How many pairs of half-hose, at \$.45⅚ a pair, may be bought for \$11?
10. What would be the cost of 40 yards of linen, at 3s. per yard?
11. Sold children's shoes, at 3s. 3d. a pair, to the amount of \$54.16⅔; how many pairs were sold?
12. How many palm-leaf hats, at 3s. 6d. apiece, may be bought with \$14.58⅓?
13. What may I receive for 576 lbs. of wool, at \$.62½ a pound?
14. How much flannel, at 4s. a yard, may be bought with \$4.66⅔?
15. If the expense of cultivating an acre of corn be \$20, what would be the profits from a field of 12 acres, each yielding 50 bushels, worth 4s. 6d. per bushel?
16. If a farmer sell his rye at \$.83⅓ per bushel, and receive \$95 for it, how many bushels would he sell?
17. How many days, at 5s. 3d. per day, must a man work to earn \$63.87½?

18. If a merchant sell 4 dozen pairs of gloves, at 5s. 6d. a pair, what would he receive for them?

259. MODEL OF A RECITATION.

At 7s. 6d. a bushel, what would 100 bushels of wheat come to?

4) \$100 = cost at \$1. per bushel.
25 = cost at \$ .25 per bushel.

\$125 = cost at \$1.25, or 7s. 6d. per bushel.

Since 7s. 6d., the price of 1 bushel, is \$1¼, (**206**,) the number of bushels would also be the number of dollars in the cost at 1 dollar per bushel, and ¼ of the number of bushels would be the number of dollars in the cost, at ¼ of a dollar per bushel; therefore, write the number of bushels, and to it add ¼ of itself, and the sum will be equal to the whole cost in dollars.

260. EXERCISES IN MULTIPLYING BY UNITS AND ALIQUOT PARTS.

In like manner, solve and explain the following problems.

1. How much would 12 pairs of ladies' shoes come to, at 6s. 6d. a pair?
2. At 6s. 9d. a pair for silk hose, what would be the price per dozen?
3. What would be the cost of 54 gallons of oil, at 7 shillings per gallon?
4. How much would 5½ pounds of green tea, at 8s. a pound, amount to?
5. Bought 2.875 yards of satinet, at 8s. 3d. per yard; what was the cost?
6. How much would 11 pitchforks, at 9s. apiece, amount to?
7. Multiply 1840 by 1⅛.
8. What would 12 weeks' work come to, at 10s. 6d. per day, Sundays excepted?
9. How much would 2$\frac{1}{16}$ yards of cassimere cost, at 11s. 3d. per yard?
10. What would be the cost of 52 weeks' board, at 15s. per week?

11. Paid for 18 weeks' board, at 13s. 6d. per week; how much was the bill?

261. ILLUSTRATION OF THE PRINCIPLE OF REDUCING FRACTIONS BY INSPECTION.

Since 100 cents make the *unit*, 1 dollar, any number of cents are so many hundredths of a unit; thus, $12\frac{1}{2}$ cents is \$.125. But $12\frac{1}{2}$ cents is a ninepence, or $\frac{1}{8}$ of a dollar (**206**); therefore, the decimal for any number of eighths will be the number of cents in so many ninepences; and, for any number of twenty-fourths, sixteenths, twelfths, and sixths, the decimal, in hundredths, will be the number of cents in so many threepences, fourpence-halfpennies, sixpences, and shillings, respectively.

Also, any decimal, corresponding with the number of cents in any such part of a dollar, may be reduced to a common fraction, by writing the part instead of the decimal.

262. MODEL OF A RECITATION.

1. Reduce $\frac{5}{16}$ to an equivalent decimal.

$\frac{5}{16} = .3125$

Fourpence-halfpennies being sixteenths of a dollar, the decimal for $\frac{5}{16}$ will be the number of cents in 5 fourpence-halfpennies, which is $31\frac{1}{4}$; consequently, the decimal required is .3125.

2. Reduce $.418\dot{3}$ to an equivalent common fraction.

$.418\dot{3} = \frac{5}{12}$

The hundredths, in this decimal, being the same as the number of cents in 2s. 6d. = 5 sixpences, and sixpences being twelfths (**206**) of a dollar, this decimal is equivalent to $\frac{5}{12}$, which is the answer required.

263. EXERCISES IN REDUCING FRACTIONS BY INSPECTION.

In like manner, solve and explain the following problems.

1. Reduce $\frac{1}{24}$ to a decimal.
2. Reduce $.208\dot{3}$ to a common fraction.
3. What are the decimals equivalent to $\frac{7}{24}$, $\frac{11}{24}$, and $\frac{13}{24}$?
4. What are the common fractions equivalent to $.708\dot{3}$, $.791\dot{6}$, and $.958\dot{3}$?

5. Reduce $\frac{1}{16}$ to a decimal.
6. Reduce .1875 to a common fraction.
7. What are the decimals equivalent to $\frac{7}{16}$, $\frac{9}{16}$, and $\frac{11}{16}$?
8. What are the common fractions equivalent to .8125 and .9375?
9. Reduce $\frac{1}{12}$ to a decimal.
10. Reduce $.58\dot{3}$ to a common fraction.
11. What is the decimal equivalent to $\frac{11}{12}$?
12. What are the common fractions equivalent to .375, .625, and .875?
13. What are the decimals equivalent to $\frac{1}{6}$ and $\frac{5}{6}$?
14. What are the decimals equivalent to $\frac{1}{4}$ and $\frac{3}{4}$?
15. Reduce $.33\frac{1}{3}$ and $.\dot{6}\dot{6}$ to common fractions.
16. Reduce 5 inches to the decimal of a foot.
17. Reduce 5 ounces to the decimal of a pound avoirdupois.
18. How many ounces in $.58\dot{3}$ of a pound Troy?
19. Reduce 7 grains to the decimal of a pennyweight.
20. How many furlongs in .625 of a mile?

IX. PROPORTION.

264. Illustration of Ratios.

When two quantities of the same kind are compared with regard to their relative value, one of them will be less than, equal to, or greater than, the other; and will contain the other less than once, exactly once, or more than once.

The *Ratio* of one quantity to another of the same kind, is the quotient resulting from the division of the latter by the former; the division being *expressed* in a fractional form, or, more frequently, with the dividend following the divisor with this sign (:) between.

: is the *sign* for the ratio of two quantities. It indicates the ratio of the *antecedent*, or the quantity preceding the sign, to the *consequent*, or the quantity which follows the sign.

The ratio of two numbers shows *what part* the dividend is of the divisor. Thus, in comparing 7 dollars with 12 dollars, 4 fathoms with 8 yards, and 11 with 3, we find that 7 dollars is $\frac{7}{12}$ of 12 dollars, and contains 12 dollars $\frac{7}{12}$ of one time,

and that their ratio is 12 : 7; that 4 fathoms, being equal to 8 yards, is $\frac{8}{8}$ of 8 yards, and contains 8 yards $\frac{8}{8}$ of one time, or exactly once, and that their ratio is 8 : 8, which is called the *ratio of equality*, since the two terms of the ratio are equal; and, finally, that 11 is $\frac{11}{3}$ of 3, and contains 3 $\frac{11}{3}$ of one time, or $3\frac{2}{3}$ times, and that their ratio is 3 : 11.

Hence, (**87,**) to ask what *part* of 12 dollars is 7 dollars, is the same as to ask what is the *ratio* of 12 dollars to 7 dollars, or of 12 to 7, since $\frac{7}{12}$ is the *part* of 12 that 7 is, and also the *ratio* of 12 to 7.

Consequently, any fraction is the ratio of its denominator to its numerator; and in writing a ratio fractionally, the first number is made the denominator, or divisor, and the second the numerator, or dividend. Thus, 12 : 7 is read, the ratio of 12 to 7, and is the same as $\frac{7}{12}$.

265. EXERCISES IN FINDING THE RATIOS OF NUMBERS.

In like manner, solve and explain the following problems.

1. What part of 7 is 3, and what is the ratio of 7 to 3?
2. What part of 5 is 12, (**87,**) and what is the ratio of 5 to 12?
3. What part of 8 is $\frac{4}{5}$, (**112,**) and what is the ratio of 8 to $\frac{4}{5}$?
4. What part of 10 is $3\frac{1}{2}$, (**112,**) and what is 10 : $3\frac{1}{2}$?
5. What part of $\frac{2}{3}$ is 4, (**153,**) and what is $\frac{2}{3}$: 4?
6. What part of $11\frac{3}{4}$ is 5, (**152,**) and what is $11\frac{3}{4}$: 5?
7. What part of $\frac{7}{8}$ is $\frac{3}{7}$, (**155,**) and what is $\frac{7}{8}$: $\frac{3}{7}$?
8. What part of $12\frac{1}{2}$ is $6\frac{1}{4}$, (**155,**) and what is $12\frac{1}{2}$: $6\frac{1}{4}$?
9. What part of .1875 is .125, (**189,**) and what is .1875 : .125?
10. What part of 6.25 is .625, (**188,**) and what is 6.25 : .625?
11. What part of $1.1\dot{6}$ is $.8\dot{3}$, (**184,**) and what is $1.1\dot{6}$: $.8\dot{3}$?
12. What part of 12 hours is 5 h. 15 m., (**228,**) and what is 12 h. : 5 h. 15 m.?
13. What part of 1£ 10s. is 13s. 6d., (**230,**) and what is 1£ 10s. : 13s. 6d.?
14. What part of 16s. 6d. is \$2.50, (**254,**) and what is 16s. 6d. : \$2.50?

266. Model of a Recitation.

1. Multiply 25 by the ratio of 7 to 3.

$$\frac{25\times 3}{7}=\frac{75}{7}=10\tfrac{5}{7}$$

The ratio of 7 to 3 being $\frac{3}{7}$, to multiply 25 by the ratio of 7 to 3 is the same as to multiply it by $\frac{3}{7}$, that is, (**144**,) to take $\frac{3}{7}$ of 25, making $10\frac{5}{7}$, which is the answer required.

2. Multiply $8\frac{7}{16}$ by 63 : 40.

$$\frac{\overset{15}{\cancel{135}}\times\overset{5}{\cancel{40}}}{\underset{2}{\cancel{16}}\times\underset{7}{\cancel{63}}}=\frac{75}{14}=5\tfrac{5}{14}$$

$8\frac{7}{16}=\frac{135}{16}$, and 63 : 40 $=\frac{40}{63}$; therefore, multiply the denominator by 63, to obtain $\frac{1}{63}$, (**114**,) and multiply the numerator by 40, to obtain $\frac{40}{63}$. But both terms of this fraction having the common factors 9 and 8, reduce the fraction to its lowest terms, (**121**,) before performing the operations indicated by the signs, (**252**.)

3. If a man travel 30 miles in 7 hours, what distance would he travel in 12 hours?

$$\frac{30\times 12}{7}=\frac{360}{7}=51\tfrac{3}{7}\text{ miles.}$$

If in 7 hours he travel 30 miles, in 1 hour he would travel $\frac{1}{7}$ of 30 miles, or $\frac{30}{7}$ miles, (**94**,) and in 12 hours he would travel 12 times as far, or $\frac{30\times 12}{7}=\frac{360}{7}=51\frac{3}{7}$ miles.

A shorter explanation. 12 hours being $\frac{12}{7}$ of 7 hours, he would travel in 12 hours $\frac{12}{7}$ of the distance that he would in 7 hours. $\frac{12}{7}$ of 30 miles is $\frac{30\times 12}{7}=\frac{360}{7}=51\frac{3}{7}$ miles, (**148**.)

4. If 5 tons of hay keep 60 sheep through the winter, how much would keep 75 sheep the same time?

$$\frac{5\times 5}{4}=\frac{25}{4}=6\tfrac{1}{4}\text{ tons.}$$

75 sheep being $\frac{75}{60}$ of 60 sheep, they would require $\frac{75}{60}$, or $\frac{5}{4}$, as much hay. $\frac{5}{4}$ of 5 tons is $\frac{5\times 5}{4}=\frac{25}{4}=6\frac{1}{4}$ tons.

267. Exercises in multiplying by Ratios.

In like manner, solve and explain the following problems.

1. If a piece of linen cost \$24, what would $\frac{1}{2}$ of a piece cost?

2. If 3 chaldrons of coal cost \$36, what part of \$36, and how much, would 1 chaldron cost?

3. At $4.20 per box of lemons, what part of $4.20, and how much, would $\frac{3}{4}$ of a box cost?

4. At $7.50 per cord, what part of $7.50, and how much, would $\frac{2}{3}$ of a cord of wood cost?

5. At $.75 per bushel, what part of $.75, and how much, would $4\frac{3}{5}$ bushels of corn cost?

6. If 6 horses eat 18 bushels of oats in a week, what part of 18 bushels, and how much, would 5 horses eat?

7. If 25 lbs. of sugar cost $2.25, what would be the cost of 60 lbs.?

8. If 5 tons of hay cost $87.50, what part of $87.50, and how much, would 12 tons cost?

9. At $54 for 9 barrels of flour, what part of 9 barrels, and how much, could be purchased for $186?

10. If a vessel sail 480 miles in 5 days, how long would it take her to sail 3000 miles?

11. If 30 cords of wood cost $200, what part of $200, and how much, would 75 cords cost?

12. If 3 books cost $\frac{7}{8}$ of a dollar, what part of $\frac{7}{8}$ of a dollar and how much, would 8 books cost?

268. Reduction of Complex Fractions.

A complex fraction is a fraction in which either term, or both terms are fractions, or mixed numbers. It may be reduced to a simple fraction by multiplying both terms (**121**) by the denominators of the terms. Thus, $\frac{28\frac{3}{5}}{8\frac{1}{8}}$, or $\frac{\frac{143}{5}}{\frac{65}{8}}$, are complex fractions, and by multiplying both terms by 5 and 8, or 40, we have $\frac{143\times8}{65\times5}=\frac{1144}{325}$.

If the denominators of the terms of a complex fraction have a common multiple (**131**) less than their product, multiply both terms by that least common multiple. Thus, in $\frac{\frac{7}{8}}{\frac{5}{12}}$, by multiplying both terms by 24, we have the simple fraction $\frac{7\times3}{5\times2}=\frac{21}{10}$.

269. Model of a Recitation.

16. If $8\frac{1}{8}$ lbs. of butter cost $$1\frac{4}{11}$, what would be the price of $28\frac{3}{5}$ lbs. at that rate?

$$\$\frac{\overset{3}{\cancel{15}}\times 8\times \overset{13}{\cancel{143}}}{\cancel{11}\times \underset{\cancel{5}}{\cancel{65}}\times 5}=\$\frac{24}{5}=\$4.80.$$

Since $\$1\frac{4}{11}$, or $\$\frac{15}{11}$ is the price of $8\frac{1}{8}$ lbs. or $\frac{65}{8}$ lbs., divide it by 65 for the price of $\frac{1}{8}$ lb. and multiply that quotient, $\$\frac{15}{11\times 65}$, by 8 for the price of $\frac{8}{8}$lb. or 1 lb. (**156**); then, since the price of $28\frac{3}{5}$ lb. or $\frac{143}{5}$ lb. is required, divide the price of 1 lb. $\$\frac{15\times 8}{11\times 65}$, by 5 for the price of $\frac{1}{5}$ lb. and multiply that quotient, $\$\frac{15\times 8}{11\times 65\times 5}$, by 143 for the price of $\frac{143}{5}$ lb. or $28\frac{3}{5}$ lb. (**149**,) making $\$\frac{15\times 8\times 143}{11\times 65\times 5}=\$\frac{24}{5}=\$4.80$, which is the answer required.

In reducing this fraction, $\frac{15\times 8\times 143}{11\times 65\times 5}$, to its lowest terms, (**121**) we divide both terms by 11 by canceling the 11 in the denominator by the 11 which is a factor of 143 in the numerator; 13, the other factor of 143, we cancel by the 13 which is a factor of 65 in the denominator; and 5, the other factor of 65, we cancel by the 5 which is a factor of 15 in the numerator, giving $\frac{3\times 8}{5}$, or $\frac{24}{5}$.

Note. In canceling equal factors, there will be less liability to mistake, and greater facility in reviewing the process, if one continued line be drawn through the two numbers containing the factor to be canceled.

A shorter explanation. If $\$1\frac{4}{11}$ or, $\$\frac{15}{11}$, the price of $8\frac{1}{8}$ lb. or $\frac{65}{8}$ lb. be divided by $\frac{65}{8}$, (**156**,) the quotient, $\$\frac{15\times 8}{11\times 65}$, will be the price of 1 lb.; and if this price of 1 lb. be multiplied by $28\frac{3}{5}$, or $\frac{143}{5}$, the number of pounds whose price is required, (**149**,) the product $\$\frac{15\times 8\times 143}{11\times 65\times 5}, =\$\frac{24}{5}, =\$4.80$, must be the answer required.

Or shorter still. $28\frac{3}{5}$ lb. being $\frac{28\frac{3}{5}}{8\frac{1}{8}}$, or $\frac{\frac{143}{5}}{\frac{65}{8}}$ of $8\frac{1}{8}$ lb. or $\frac{65}{8}$ lb. would cost $\frac{\frac{143}{5}}{\frac{65}{8}}$ of the price of $\frac{65}{8}$ lb. $\frac{\frac{143}{5}}{\frac{65}{8}}$ of $\$1\frac{4}{11}$, or $\$\frac{15}{11}$, is $\$\frac{15\times 8\times 143}{11\times 65\times 5}, =\$\frac{24}{5}, =\$4.80$, as before.

270. OBSERVATION.

OBSERVE, *that the process, by either explanation,* (**269**,) *is the same, and consists of multiplying the given price of a given quantity* BY THE RATIO OF THE GIVEN QUANTITY TO THE REQUIRED QUANTITY, *or by the* PART *of the given quantity that*

15*

the required quantity must be. Labor often may be saved by mentally reducing the ratio to simpler terms, (**121**,) *before writing it.*

271. EXERCISES IN MULTIPLYING BY COMPLEX RATIOS.

In like manner, solve and explain the following problems.

1. If 37 yards of broad cloth cost \$185, what would $5\frac{3}{4}$ yards cost?
2. If $\frac{25}{36}$ of a cask of wine cost \$12.50, what would 6 such casks cost?
3. At $\$3\frac{1}{4}$ for $4\frac{3}{4}$ yards of satinet, what would be the cost of 25 yards?
4. If 12 days' work cost $\$16\frac{2}{3}$, what would $5\frac{2}{3}$ days' work cost?
5. If $\frac{7}{8}$ of a bushel of corn cost $\$\frac{5}{8}$, what is that a bushel?
6. If $1\frac{7}{8}$ barrels of flour serve a family $1\frac{3}{4}$ weeks, how long would $7\frac{1}{2}$ barrels serve them?
7. If a company of workmen mow $72\frac{3}{4}$ acres in $12\frac{3}{8}$ days, how many acres would they mow, at the same rate, in $8\frac{1}{4}$ days?
8. If 3 yds. 3 qrs. of cassimere cost \$10, what would 5 yards cost?
9. If 18 gals. 3 qts. of wine cost \$33.75, what would 43 gals. 3 qts. cost?
10. If 2 roods, 25 rods of land cost \$42, what would 5 acres, 3 roods cost, at that rate?
11. If £$\frac{9}{20}$ sterling money make \$2, how much sterling money is equal to $\$12\frac{1}{2}$?

272. ILLUSTRATION OF THE INVERSE RATIO.

If 5 men could build a wall in 32 days, in how long time could 9 men build it?

It would take 1 man 5 times as long as it would 5 men, or 5 times 32 days; but it would take 9 men only $\frac{1}{9}$ as long as it would 1 man, or $\frac{1}{9}$ of 5 times 32 days, which is $\frac{5}{9}$ of 32 days, or $\frac{32\times5}{9}=\frac{160}{9}=17\frac{7}{9}$ days, the answer required.

273. OBSERVATION.

OBSERVE, (**272**,) *that, though 9 men are $\frac{9}{5}$ of 5 men, it would* NOT *take them $\frac{9}{5}$ of the time that it would take 5 men to build the wall, but rather, $\frac{5}{9}$ of that time; but $\frac{5}{9}$ is $\frac{1}{9}$*

INVERTED. *Hence,* $\frac{6}{5}$ *in this example, and the ratio corresponding to it in similar cases, being called a* DIRECT RATIO, $\frac{5}{6}$ *and the ratio corresponding to it in similar cases, is called an* INVERSE RATIO.

274. Model of a Recitation.

If 12 cows consume a quantity of hay in 90 days, how many cows would consume the same hay in 30 days?

To consume the same hay in 30 days would require $\frac{90}{30}$, or (**121**) 3 times as many cows as would consume it in 90 days. 3 times 12 cows are 36 cows, the answer required.

275. Exercises in Multiplying by Inverse Ratios.

In like manner, solve and explain the following problems.

1. If 9 horses consume a ton of hay in 32 days, how long would it take 12 horses to consume the same hay?
2. If 72 men could do a job of work in 15 months, how many men, working at the same rate, would do the same job in 2 years?
3. If a barrel of flour last a family of 6 persons 12 weeks, how long would a barrel last them if the family be increased to 8 persons?
4. If a pail holding 10 quarts be emptied 200 times to fill a cistern, how much would that vessel hold which must be emptied 75 times to fill the same cistern?
5. 4s. 6d. sterling money being equal to 5s. Canada currency, how much sterling money would cancel a debt of £18 in Quebec?
6. How much Canada currency would cancel a debt of £36 in London?
7. 5s. Canada currency being equal to 6s. N. E. currency, how much Canada currency is equal to £45 N. E. currency?
8. How much sterling money is equal to £36 N. E. currency?
9. 6s. N. E. currency being equal to 8s. N. Y. currency, how many N. E. pounds are equal to 72 N. Y. pounds?
10. A piece of land 8 rods wide and 20 rods long is an acre; then how long must that acre be which is 12 rods wide?
11. A board 9 in. wide and 16 in. long being a square foot, how wide must that board be which contains 1 sq. foot, and is 16 feet long?

12. If a stick of timber, the end of which contains 216 sq. inches, must be 37½ feet long to be a ton, how long must a stick be to measure a ton, the end of which contains 288 sq. inches?

13. If the contents of a cylindrical tube, which measures 18 inches in length and 144 sq. inches on one end, be emptied into another tube the end of which should measure 16 sq. inches, how high would the water rise?

14. How many yards of cloth ¾ of a yard wide would be equal to 12½ yards 1½ yards wide?

15. How many yards of cloth, 1¾ yds. wide, would be equal to 91½ yds. ⅞ of a yard wide?

16. What quantity of wheat, at $1.25 a bushel, should be given for 10 barrels of flour, at $5.25 a barrel?

17. What quantity of sugar, at 8⅓cts. a pound, would pay for board 12 weeks, at $2.75 a week?

18. In how many weeks, at $1000 per annum, could a man earn as much as another man could in 13 weeks, at $700 per annum?

276. ILLUSTRATION OF THE PRINCIPLES OF PROPORTION.

A large and a small map of the U. S. in order to be correct representations of the country, must be of the *same shape*; and the states, mountains, lakes, rivers, cities, towns, &c., must have the *same relative distances* on each map; that is, all the distances on each map, must be *in proportion* to the corresponding distances on the other map. Thus; if New York is ½ as far from Washington as Boston is on one map, it must also be ½ as far from W. as B. is on the other map.

If then, on the larger map, the two distances of B. and N. Y. from W. be 12 inches, and ½ of 12 inches, or 6 inches, and on the smaller map, the distance of B. from W. be 4 in., the distance of N. Y. from W. *must be ½ of 4 inches, or 2 in.* That is, the ratio (**264**) of the two distances of B. and N. Y. from W. on one map, must be equal to the ratio of the corresponding distances on the other map. Thus, 12 : 6 = 4 : 2. This expression constitutes what is called a *proportion*.

Observe, then, that a proportion is composed of two equal ratios, and that a ratio is the relation of two quantities of the *same kind*, in regard to what part (**87**) of the first, the second is, or how many times the first is contained *by* the

second Thus, in the proportion $12 : 6 = 4 : 2$, the first ratio $12 : 6$, or $\frac{6}{12}$, is equal to the second ratio $4 : 2$, or $\frac{2}{4}$, since each is equal to $\frac{1}{2}$. This proportion is read: The ratio of 12 to 6 equals the ratio of 4 to 2; or 12 is to 6 as 4 is to 2; or, as 12 is to 6 so is 4 to 2.

The four quantities forming a proportion are called *proportionals*, or, *the terms of the proportion*.

The first and fourth terms of a proportion are called *the extremes*, and the second and third, *the means*.

Also, the first terms, or divisors in *ratios*, are called *the antecedents*, and the second terms, or dividends, are called *the consequents*.

Two equal fractions may become a proportion, by placing the denominators for antecedents, and the numerators for consequents. And, *any four numbers, arranged like proportionals, form a correct proportion, if the product of the means be equal to the product of the extremes*, since these products, in a correct proportion, will always be equal; for, in a correct proportion, the two ratios, or fractions being equal, if they be reduced to a common denominator, (**139**,) by multiplying both terms of each by the denominator of the other, *the numerators will be equal also*. But one of these numerators is the product of the means, and the other is the product of the extremes.

$$12 : 6 = 4 : 2$$
$$\frac{6}{12} = \frac{2}{4}$$
$$\frac{6 \times 4}{12 \times 4} = \frac{2 \times 12}{4 \times 12}$$
$$\frac{24}{48} = \frac{24}{48}$$

This truth is of great practical utility in the solution of problems which involve proportion; since, by its application, any three terms of a proportion are sufficient for ascertaining the remaining term. For, if the term wanting be an extreme, it may be ascertained by dividing the product of the means, (which is also the product of the extremes, one of which is known,) by the known extreme, (**117**); or, if the term wanting be a mean, it may be ascertained by dividing the product of the extremes, (which is also the product of the means, one of which is known,) by the known mean. Thus, in the proportion, $12 : 6 = 4 : 2$, the product of the means divided by the *first* extreme, is $\frac{4 \times 6}{12} = 2$, the *second* extreme; or divided by the *second* extreme, is $\frac{6 \times 4}{2} = 12$, the *first* extreme; and the product of the extremes divided by the first mean, is $\frac{2 \times 12}{6} = 4$, the second mean; or, divided by the second mean, is $\frac{12 \times 2}{4} = 6$, the first mean.

But in practice, the FOURTH *term will always be that which is wanting, or required, if, of the three given terms, the one which is of the same nature as the required term be put in the* THIRD *place, and* THE OTHER TWO, WHICH WILL BE ALIKE IN KIND, *and compose one of the ratios, be arranged in the first and second places, so as to show* WHAT PART *of the given odd term the required term must be.*

277. MODEL OF A RECITATION.

1. If, on the larger of the above named maps, (**276**,) New Orleans be 27 inches, and Augusta, Me., be 18 inches from Washington; and on the smaller map, N. O. be 9 inches from W., how far, on the smaller map, would A. be from W.?

Since the distance of A. is $\frac{18}{27}$ of the distance of N. O. from W., on the larger map, the distance of A. must be $\frac{18}{27}$ of the distance of N. O. from W. on the smaller map. The distance of N. O. from W. on the smaller map is 9 inches; and $\frac{18}{27}$ of 9 inches is $\frac{9\times18}{27}=6$ inches, which is the answer required.

Or by Proportion. The ratio of the distances of N. O. and A. from W. on the larger map is 27 : 18, which must equal 9 : Ans., the ratio of the corresponding distances on the smaller map. Hence, the proportion would be, 27 : 18 = 9 : Ans. The product of the means divided by the given extreme, is $\frac{9\times18}{27}=6$ inches, as before.

$$27 : 18 = 9 : \frac{\cancel{9}\times18}{\underset{3}{\cancel{27}}}, \text{ or } 6.$$

OBSERVE, *that, in multiplying the means together, and dividing the product by the first extreme, the process is precisely the same as it is in multiplying the odd term by the first ratio,* (**266**,) *as explained in the analysis of the question; and that the process may be much abridged by mentally reducing the ratio* $\frac{18}{27}$ *to lower terms,* $\frac{2}{3}$, *before using it.*

2. If 6 barrels of apples cost \$15, what would be the cost of 32 barrels, at the same rate?

$$6\text{ B.} : 32\text{ B.} = \$15 : \$\frac{\overset{5}{\cancel{15}}\times\overset{16}{\cancel{32}}}{\cancel{6}}, \text{ or } \$80.$$

In this, and similar problems, the ratio of the *prices* is equal to the ratio of the corresponding *quantities*

Since the *quantity* whose price is *required*, is $\frac{32}{6}$ of the *quantity* whose price is *given*, the *required price* will be equal to $\frac{32}{6}$ of the *given price;* that is, 6 barrels : 32 barrels = \$15, the *price* of 6 barrels, : \$, the *price* of 32 barrels. $\frac{32}{6}$ of \$15, or the product of the means divided by the known extreme, is $\$\frac{15\times32}{6}=\$80$, which is the answer required.

Note. If $\frac{32}{6}$ had been reduced mentally to $\frac{16}{3}$, it would have abridged the process.

278. Exercises in solving Problems by Proportion.

In like manner, solve and explain the following problems.

1. If 15 yards of cloth cost \$48, what would 25 yards of the same cloth cost?

2. If 16 men earn \$176 in a month, how much would 24 men earn in the same time?

3. If 12 horses consume 42 bushels of oats in a given time, how much would 20 horses consume in the same time?

4. What is the value of 18 cords of wood, if 45 cords are worth \$225?

5. How much sugar may be bought for \$6, if 9 lbs. cost \$.75?

6. What is the value of 56 gallons of wine, at \$30 for 12 gallons?

7. At \$375.75 for 5 acres of land, what would be the cost of 35 acres?

8. At \$10.50 for 7 pairs of shoes, how many pairs would come to \$52.50?

9. If \$4.25 be paid for 19 lbs. of salmon, how many pounds may be bought for \$25½?

10. If 8 lbs. of rice cost \$$\frac{3}{8}$, what would be the price of 28 lbs.?

11. If 1¾ yds. of calico cost \$.42, what would be the cost of 25 yards?

12. If 8¾ yards of brown linen cost 4⅕ dollars, what would be the cost of 10 yards?

13. At \$6 for $\frac{5}{8}$ of a cord of wood, what would $\frac{7}{8}$ of a cord cost?

14. If $\frac{3}{5}$ of a yard cost \$$\frac{7}{8}$, what will 12⅛ yards cost?

15. If $\frac{7}{16}$ of a ship be worth \$2520, what is the worth of $\frac{3}{32}$ of the ship?

16. At 5s. 4d. per ounce, what is the weight of a tankard that is sold for 10£ 12s.

17. If a staff 5 ft. 8 in. in length, cast a shadow of 6 ft. how high is that steeple whose shadow measures 153 feet?

18. At $791.18 for 17 acres, what would be the price of 12 acres 3 roods 10 rods?

19. At $30 for 12 gallons of wine, what would a barrel, holding 31 gals. 2 qts., cost?

20. What would be the cost of 2 cords 5 cord-ft. of wood, if 25 cords, 8 cubic ft. should cost $100.25?

21. If the interest on a note be $12.50 in 1 year 6 months, what would be the interest in 5 yrs. 3 mos.?

22. If $100 gain $6 in a year, how much would it gain in 4 yrs. 9 months?

23. If $17.50 gain $1.05 in a year, how much should $12.50 gain in the same time?

24. What would be the interest of $175.25 for 1 year, if $1 gain $.06 in the same time?

279. Model of a Recitation.

1. If 6 men could reap a field of rye in 21 hours, how many hours would it take 9 men to reap the same field?

By analysis.— It would take 1 man 6 times as long as it would take 6 men, or 6 times 21 hours; but it would take 9 men only $\frac{1}{9}$ as long as it would 1 man, or $\frac{1}{9}$ of 6 times 21 hours, which is $\frac{6}{9}$, or $\frac{2}{3}$ of 21 hours, or $\frac{21 \times 2}{3} = 14$ hours, which is the answer required.

By proportion.— The ratio of the times is equal to the *inverse* ratio (**272**) of the numbers of men. Thus:

$$9 : 6 = 21 : \frac{\overset{7}{\cancel{21}} \times \overset{2}{\cancel{6}}}{\cancel{9}}, \text{ or 14 hours, as before.}$$

280. Exercises in solving Problems by Inverse Proportion.

In like manner, solve and explain the following problems.

1. If 4 men could build a wall in 20 days, in how many days could 8 men do the same work?

2. If 6 men could reap a field in 21 hours, how many men would it take to reap the same field in 14 hours?

3. If, by travelling 16 hours per day, a man could perform a certain journey in 6 days, how many days would it take him at 12 hours per day?

4. If 1500 lbs. could be transported 150 miles for a guinea, how far could a ton be transported for the same money?

5. How long would a quantity of provisions, sufficient to last 800 men 2 months, last 300 men?

6. If a field would feed 6 cows 91 days, how long would it feed 21 cows?

7. If 5 men mow a meadow in $11\frac{2}{3}$ days, in how many days could 8 men do the same?

8. If, by working 12 hours per day, it would take $7\frac{2}{5}$ days to do a certain job, how many days, at $9\frac{1}{2}$ hours per day, would it take to do the same?

9. How many days would that sum of money pay a laborer, at $\$1\frac{1}{4}$ per day, which would pay him for $8\frac{4}{7}$ days at $\$\frac{5}{6}$ per day?

10. If by paying a teacher \$3.25 per week, a certain school would be kept $11\frac{1}{2}$ weeks, how long might it be kept, if she be paid only \$2.875 per week?

11. How many bushels of potatoes at \$.25 per bushel, would come to as much as 40 bushels of corn, at \$.625 per bushel?

12. How many six-penny loaves may be made from the flour that would make 25 fourpence-halfpenny loaves?

13. How much tea, at 64 cents per pound, must be given for 200 lbs. of chocolate, at 24 cents per pound?

14. How many pounds of lead, at \$.09 per pound, must be given for 783 pounds of iron, at \$.06 per pound?

15. How much broad-cloth, at 16s. 6d. per yard, must A give to B for 165 yds. of linen, at 2s. 9d. per yard?

16. What would be the cost of a pound of pepper, if 500 lbs. be received for 370 yds. of calico, at $\$.3\dot{3}$ per yard.

17. If it would take 43 bushels of wheat to pay a debt, when wheat is 7 shillings per bushel, how much would it take, when it is 9s. per bushel?

18. How much coffee at $\$.1\dot{6}$ per pound, must be given for 18 bushels of apples, at $\$.3\dot{3}$ per bushel?

19. How many bottles of 1 pt. 2 gills each, may be filled from a cask holding 13 pailfuls, at 2 gals. 2 qts. each?

20. How many gallons of milk could be put into a cask that would contain $31\frac{1}{2}$ gallons of water?

21. How many spoons, each weighing 10 dwt. 12 grs., may be made of 10 silver dollars, each weighing 17 dwt. 8 grs. allowing nothing for waste?

16

22. How many yards of lining, $\frac{1}{2}$ yard wide, will line 7 yards, 1 yard wide?

23. How many yards of flannel 4 ft. wide, will line a cloak containing 9 yards that is 3 ft. wide?

24. How many yards of cloth that is $\frac{3}{4}$ of a yard wide, are equal to 12 yds. that is $\frac{5}{8}$ of a yard wide?

25. How many boards, 6 inches wide, would cover the floor that requires 20 boards of the same length, and 15 inches wide?

26. If a street 4 rods wide must be 40 rods long to contain an acre, how long must it be to contain an acre if it be only $2\frac{1}{2}$ rods wide?

27. How long must a piece of land be, that is 8 rods wide, to contain one acre?

28. How long must a piece of board be, that is 9 inches wide, to contain a square foot?

29. If it take $1\frac{7}{8}$ yards of cloth, that is $1\frac{1}{2}$ yards wide, to make a coat, how many yards would it take, of cloth only $\frac{3}{4}$ of a yard wide?

30. How long would it take $1 to gain as much interest as $8 would gain in 3 months?

31. How long would it take $3 to gain as much interest as $1 would gain in 24 months?

32. How long would it take $12 to gain as much interest as $30 would gain in 6 months?

33. John borrows of Job, $25 for 12 months, and afterwards lends him $75, how long may Job keep the money to cancel the favor?

34. How long may John keep $858 of Job's money to compensate him for letting Job have $286 for 12 months?

35. If it take 12 months for $100 to gain $6, how long would it take $150 to gain the same sum?

36. If the interest of $100 for 1 year be $6, in what time would the interest of $33\frac{1}{3}$ be the same?

37. In what time would $75 gain $15 interest, if $500 would gain it in 6 months?

281. ILLUSTRATION OF COMPOUND RATIO AND PROPORTION.

1. If 7 men can build 36 rods of wall in 3 days, how many rods can 20 men build in 14 days?

By analysis **— If 7 men can build 36 rods, 1 man, in the** ***same time***, can build $\frac{1}{7}$ as much, or $\frac{36}{7}$ rods, and in *one day* he can

$$\frac{\overset{12}{\cancel{36}} \times 20 \times \overset{2}{\cancel{14}}}{\cancel{7} \times \cancel{3}} = 12 \times 20 \times 2 = 480 \text{ rods.}$$

build $\frac{1}{3}$ as much as in 3 days, or $\frac{36}{7 \times 3}$ rods; but 20 men can build 20 times as much in 1 day as 1 man can in the *same time*, or $\frac{36 \times 20}{7 \times 3}$ rods, and in 14 days they can build 14 times as much as they can in 1 day, or $\frac{36 \times 20 \times 14}{7 \times 3} = 480$ rods, which is the answer required.

By proportion.— If it were required how many rods 20 men could build in a time *equal* to that in which 7 men could build 36 rods, the answer would be $\frac{20}{7}$ of 36 rods, or $7 : 20 = 36 : \frac{36 \times 20}{7}$, or $102\frac{6}{7}$ rods; but, since the times of working are *unequal*, this proportion cannot give the true answer, for 20 men could build $\frac{14}{3}$ as much in 14 days as they could in 3 days, or $\frac{14}{3}$ of $\frac{20}{7}$ of 36 rods, which is $\frac{20 \times 14}{7 \times 3}$ of 36 rods, (**264**,) or $7 \times 3 : 20 \times 14 = 36 : \frac{36 \times 20 \times 14}{7 \times 3}$, or 480 rods, as by analysis.

Observe that $\frac{20}{7}$, or $7 : 20$, is the ratio of the two numbers of men, that $\frac{14}{3}$, or $3 : 14$ is the ratio of the two times, and that $\frac{20 \times 14}{7 \times 3}$, or $7 \times 3 : 20 \times 14$ is a ratio composed of these two ratios by multiplying them together, and making the product of the antecedents a new antecedent, and the product of the consequents a new consequent.

Hence, also, *observe that the ratio of the given work to the required work is equal to the product of the two ratios, that of the numbers of* MEN, *and that of the* TIMES.

The product of two or more ratios is called a *compound ratio*, and a proportion in which there occurs a compound ratio, is called a *compound proportion.*

The following is a better arrangement of the ratios that form the compound ratio in a compound proportion.

$$\left.\begin{array}{l} \cancel{7} \text{ men} : 20 \text{ men}, \\ \cancel{3} \text{ days} : \underset{2}{\cancel{14}} \text{ days}, \end{array}\right\} = \overset{12}{\cancel{36}} \text{ rods} : 20 \times 2 \times 12, \text{ or } 480 \text{ rods.}$$

The common factors may be canceled (**121**) as they now stand, observing that the second terms and the third **term are the factors in the product of the means, or in the**

dividend, and that the first terms are the factors in the given extreme, or in the divisor. The 7 may be canceled by the 7 in the 14 of the dividend, and the 3 by the 3 in the 36 of the dividend.

282. OBSERVATION.

OBSERVE, *that, as in problems involving proportion,* (**276**,) *so also in those which involve compound proportion, there is one given odd term, which is of the same nature as the required answer, and with it forms the last ratio; and of the other given numbers, there are two of each kind mentioned, and* EACH TWO OF A KIND FORM A RATIO; *that the product of these ratios will form a compound ratio that will show* WHAT PART *of the given odd term, occupying the third place, the fourth, or required term must be,* IF CARE BE TAKEN *that each ratio be arranged in the first and second places, so as to show* WHAT PART OF THE THIRD TERM *the required term* WOULD BE, *without regard to the other ratios with which it is to be compounded.*

283. MODEL OF A RECITATION.

1. If 18 men can build a wall, 40 rods long, 5 feet high, and 4 feet thick, in 15 days; what time would 20 men require to build a wall, 87 rods long, 8 feet high, and 5 feet thick?

Solution by Analysis.

$$\frac{\overset{3}{\cancel{15}}\times\overset{9}{\cancel{18}}\times 87\times\cancel{8}\times\cancel{5}}{\underset{10}{\cancel{20}}\times\cancel{40}\times\cancel{5}\times 4}=\frac{3\times 9\times 87}{10\times 4}=58\tfrac{29}{40}\text{ days.}$$

To build the *same wall*, 20 men would require $\frac{18}{20}$ of 15 days; but, regarding the lengths of the walls, they would require $\frac{87}{40}$ of this second time; regarding the heights, they would require $\frac{8}{5}$ of this third time; and regarding the thicknesses, they would require $\frac{5}{4}$ of this fourth time, or $58\frac{29}{40}$ days, which is the answer required.

By Compound Proportion.

$$\left.\begin{array}{lrcl}\text{Men,} & \overset{10}{\cancel{20}} & : & \overset{9}{\cancel{18}} \\ \text{Lengths,} & \cancel{40} & : & 87 \\ \text{Heights,} & \cancel{5} & : & \cancel{8} \\ \text{Thicknesses,} & 4 & : & \cancel{5}\end{array}\right\} = \overset{3}{\cancel{15}} : \frac{3\times 9\times 87}{10\times 4},\text{ or } 58\tfrac{29}{40}\text{ days}$$

In reducing this expression of the result, the common factor, 2, in the 20 and 18, may be canceled, the 8 may be canceled by the 8 in 40; the other factor, 5, of 40, may be canceled by the 5 in 15, and the two 5s will cancel each other.

2. If it take 2 years for the interest of \$150 to amount to \$18, how long would it take for the interest of \$675 to amount to \$162?

$$\left.\begin{array}{ll}\text{Principals,} & \cancel{9} : 2 \\ \text{Interests,} & 1 : \cancel{9}\end{array}\right\} = 2 : 2 \times 2 = 4 \text{ years.}$$

To gain \$18 interest, it would take \$675 only $\frac{150}{675}$, or $\frac{2}{9}$ as long as it would take \$150. But to gain \$162 interest, it would take $\frac{162}{18}$, or $\frac{9}{1}$, or 9 times this time, or 4 years.

284. Exercises in solving Problems by Compound Proportion.

In like manner, solve and explain the following problems.

1. If 7 men can reap 84 acres of wheat in 12 days, how many men would be required to reap 100 acres, in 5 days?

2. If 20 bushels of wheat are sufficient for a family of 15 persons, 3 months, how much would be sufficient for 4 persons, 11 months?

3. If a family of 9 persons spend \$305 in 4 months, how many dollars would maintain 14 persons 8 months?

4. If 1878 soldiers consume 702 quarters of wheat in 336 days, how many quarters would an army of 22536 soldiers consume in 112 days?

5. If 6 men, in 16 days, build a wall 20 feet long, 6 feet high, and 4 feet thick, in how many days could 24 men build a wall 200 feet long, 8 feet high, and 6 feet thick?

6. If 12 men, in 15 days, of 12 hours each, can build a wall 30 feet long, 6 feet high, and 3 feet thick, in how many days, of 8 hours each, could 60 men build a wall, 300 feet long, 8 feet high, and 6 feet thick?

7. If 3 men, in 24 days, 9 hours long, can dig 328 rods of trench, 6 ft. wide, and 4 ft. deep; how many men would it take to dig a trench 984 rods long, 9 ft. wide, and 8 ft. deep, in 27 days, of 12 hours each?

8. If 12 tailors can make 13 suits of clothes in 7 days;

how many tailors would be required to clothe a regiment of 494 soldiers, in 19 days of the same length?

9. If it be worth $12.50 to transport 2.5 tons 20 miles; what would it be worth to transport 1500 lbs. 50 miles?

10. If a bar of iron, 4 ft. long, .25 ft. broad, and .125 ft. thick, weigh 36 lbs.; what would be the weight of a bar, 12.5 ft. long, .375 ft. broad, and .2 ft. thick?

11. If 12 ounces of wool be sufficient for 2½ yards of cloth, 6 quarters wide; how much would be sufficient for 150 yds., 4 quarters wide?

12. How many pounds of thread would be sufficient to make 50 yds. of linen, 3 quarters wide; if 2 lbs. of thread make 4 yds., 5 quarters wide?

13. If 36 yards of cloth 7 quarters wide be worth $180.72; what would be the value of 98 yards of similar cloth, but 5 quarters wide?

14. If 3 pounds of wool make 15 skeins of 12 knots each; how much wool would make 50 skeins of 16 knots each?

15. How many barrels of water would fill a cistern that is 18 ft. long, 7 ft. broad, and 15 ft. deep; if a cistern, 17½ ft. long, 10½ ft. broad, and 13 ft. deep, hold 546 barrels?

16. If it take 324 tiles, each 12 inches square, to pave a cellar; how many tiles, 9 inches long, and 8 inches wide, would it take to pave the same cellar?

17. How many bricks, 8 in. long, 4½ in. wide, and 2 in. thick, would occupy as much space as 1200 stones, 16 in. long, 13½ in. wide, and 12 in. thick?

18. How many shingles, each covering a space 6 in. long, and 4 in. wide, would cover a roof 40 ft. long, and each of the two sides 30 ft. wide?

19. How many slates, covering a space 10 in. long, and 8 in. wide, would cover a roof which is covered by 160 boards 12 ft. long, and 15 in. wide?

20. How many bricks, ¾ of a foot long, ⅜ of a foot wide, and ⅙ of a foot thick, would build a wall, 6 ft. high, 1½ ft. thick, round a square garden, each side of which is 280 ft. on the outside of the wall?

21. If 2 lbs. 8 oz. of bread be worth 9 pence, when wheat is worth 7s. 6d. per bushel; what weight of bread would be worth 4s. 6d., when wheat is worth 9s. per bushel?

22. If the rent of a house worth $1500, be $45, for 6 months; what would be the rent of a house worth $2500, for a year?

23. If 5 shares in a bank yield their owner \$12.50, in $\frac{1}{2}$ of a year; how much would 18 shares yield, in a year?

24. If \$100 gain \$6 in 1 year; what would \$25 gain in 6 months?

25. If \$100 gain \$6 in 1 year; in what time would \$25 gain \$.75?

26. If \$25 gain \$.75 in 6 months; how much would \$100 gain in 1 year?

27. If it take \$100 1 year to gain \$6; how much would it take to gain \$.75 in 6 months?

28. If the interest of \$100 be \$6 per year; what would be the interest of \$850 for 10 months?

29. If the interest on \$347, for $3\frac{1}{2}$ years, be \$72.87; what would be the interest on \$537, for $2\frac{1}{2}$ years?

285. Model of a Recitation.

If 3 cows require as much pasture as 10 sheep, 4 horses require as much pasture as 7 cows, and 5 acres would pasture 3 horses; how many sheep might be pastured on 30 acres?

$$\left.\begin{array}{ll} \text{Cows,} & \cancel{3} : 7 \\ \text{Horses,} & \cancel{4} : \cancel{3} \\ \text{Acres,} & \cancel{5} : \underset{3}{\cancel{30}} \end{array}\right\} = \overset{5}{\cancel{10}} : 5 \times 7 \times 3, \text{ or } 105 \text{ sheep.}$$

That which would pasture 7 cows, or 4 horses, would pasture $\frac{7}{3}$ of 10 sheep; and the 5 acres, which would pasture 3 horses, would pasture $\frac{3}{4}$ of $\frac{7}{3}$ of 10 sheep; and 30 acres would pasture $\frac{30}{5}$ as many as 5 acres, or $\frac{30}{5}$ of $\frac{3}{4}$ of $\frac{7}{3}$ of 10 sheep, or 105 sheep, which is the answer required.

In reducing this expression of the result, the 3s may cancel each other, one of the factors of 4 may be canceled by the 2 in 30, the other by the 2 in 10, and the 5 by the 5 in 30.

286. Exercises in Conjoined Proportion.

In like manner, solve and explain the following problems.

1. How many pounds could 3 horses draw, if 5 horses could draw as much as 8 oxen, and 2 oxen could draw 2400 pounds?

2. If 3 lbs. of tea be worth 4 lbs. of coffee, and 6 lbs. of coffee be worth 20 lbs. of sugar; how many pounds of sugar may be had for 9 lbs. of tea?

3. How many days' work of D would be equal to 10 days' work of A; if A could do as much work in 5 days as B could do in 6 days, and B as much in 3 days as C in 4 days, and C as much in 8 days as D in 9 days?

4. Suppose 100 lb. at Venice to be equal to 70 lb. at Lyons, and 60 lb. at Lyons to 50 lb. at Rouen, and 20 lb. at Rouen to 25 lb. at Toulouse, and 50 lb. at Toulouse to 36 at Geneva; then how many at Geneva would be equal to 25 lb. at Venice?

5. How much New England currency would be equal to £100 sterling, if £27 sterling equal £30 Canada currency, and £5 Canada equal £8 New York, and £4 New York equal £3 New England?

6. How many bushels of wheat may be bought for £40, if 1 bushel of wheat be worth 2 bushels of rye, and 4 bushels of rye be worth 5 bushels of corn, and 8 bushels of corn be worth 16 bushels of oats, and 1 bushel of oats be worth 2 shillings?

7. If 24s. New England currency equal 32s. New York, and 48s. New York equal 45s. Pennsylvania, and 15s. Pennsylvania equal 10s. Canada; how many Canada shillings would be equal to 100s. in Massachusetts?

8. If 11 barrels of cider would buy 4 barrels of flour, and 7 barrels of flour would buy 40 barrels of apples; what would a barrel of apples be worth, when cider is $2.50 per barrel?

287. Barter defined.

Barter is the exchanging of one commodity for another of equal value, according to prices agreed upon by the parties.

288. Model of a Recitation.

If a farmer should exchange butter, at 20 cents a pound, for sugar, at 8 cents a pound; how many pounds of butter must he give for $37\frac{1}{2}$ lbs. of sugar?

$37\frac{1}{2}$ lbs. are equal to $\frac{75}{2}$ lbs.; and as the price of the butter per pound is $\frac{20}{8}$ or $\frac{5}{2}$ of the price of the sugar per pound, there will be only $\frac{2}{5}$ as many pounds of butter as pounds of sugar, (272); and $\frac{2}{5}$ of $\frac{75}{2}$ is 15 pounds.

$\frac{75\times2}{2\times5} = 15$ lbs. of butter.

$5 : 2 :: \frac{75}{2} : \frac{75\times2}{2\times5} = 15$ lbs.

289. OBSERVATION.

OBSERVE, *that, the ratio of the quantities will be equal to the* INVERSE RATIO, (**272**,) *of the prices, and therefore, such problems may be solved by proportion.*

290. EXERCISES IN BARTER.

In like manner, solve and explain the following problems.

1. How many bushels of potatoes, at $.25 per bush., must be given for 15 gals. of oil, at $.75 per gal.?

2. How much corn, at $.75 per bushel, must be given in barter for 10 yds. of cassimere, at $1.25 per yard?

3. How much wood, at $5.50 per cord, would be equivalent to 11 barrels of flour, at $6.00 per barrel?

4. How much cotton cloth, at 12½ cts. per yard, should be given for 100 lbs. of beef, at 5 cts. per pound?

5. What quantity of wheat, at $1.83⅓ per bushel, should be given for 8 yds. of broad cloth, at $5.00 per yard?

6. How many days, at 83⅓cts. per day, must a laborer work to pay for 5 cords of wood, at $4.16⅔ per cord?

7. A supplied B with 365 qts. of milk, at 4 cts. a qt., and received in payment for one half, molasses at 28 cts. per gal., and for the other half, sugar at 8 cts. per pound; how much molasses and sugar did he receive?

8. How much land, at 6¼cts. a foot, is equivalent to 6000 feet, at 8⅓cts. per foot?

9. If in one place I can buy a house-lot of 5200 ft. at 9d. a foot, but in another place, I can obtain 6500 ft. for the same sum; what price per foot is the second lot?

10. If a merchant buy $10000 worth of cotton, at $50 per bale, and sell it immediately for $60 a bale; how much would he gain by the trades?

11. How many Bibles, at 87½ cts. each, should a bookseller give in exchange for 7 dozen arithmetics, at 62½ cts. each?

12. How much N. E. currency is equal to £175 N. Y. currency?

291. MODEL OF A RECITATION.

1. Three men hired a pasture for $42. A pastured in it 4 horses, B, 6 horses, and C, 8 horses; what ought each to pay.

They all had pastured $4+6+8=18$ horses, and $\frac{4}{18}$, or $\frac{2}{9}$ of them (87) belonging to A, he should pay $\frac{2}{9}$ of the cost, which is 9\frac{1}{3}$; $\frac{6}{18}$, or $\frac{1}{3}$ of them belonging to B, he should pay $\frac{1}{3}$ of the cost, or $14; and $\frac{8}{18}$, or $\frac{4}{9}$ of them belonging to C, he should pay $\frac{4}{9}$ of the cost, or 18\frac{2}{3}$.

$$9:2=42:\frac{\overset{14}{\cancel{42}}\times 2}{\underset{3}{\cancel{9}}},\text{ or } \$9\tfrac{1}{3}\text{, A's share.}$$

$$3:1=42:\frac{42}{3},\text{ or } \$14\text{, B's share}$$

$$9:4=42:\frac{\overset{14}{\cancel{42}}\times 4}{\underset{3}{\cancel{9}}},\text{ or } \$18\tfrac{2}{3}\text{, C's share.}$$

$$\overline{\$42.}$$

The sum of the three shares being $42, what they all were to pay, is sufficient proof that the answers are correct.

222. Exercises in Fellowship.

In like manner, solve and explain the following problems.

1. Two men hired a pew for $28, A's family occupied 5 seats, and B's family occupied the other 3 seats; what ought each to pay?

2. Two men own a house together, which they let for $270 per year; if A's part be worth $2500, and B's $2000 what would be each man's share of the rent per year?

3. George picked 12 quarts of berries; Charles 8 qts., and John 4 qts.; they sold them all together for $1.50; what was each boy's share of the money?

4. If the stock of a rail-road company be divided into 500 shares, and the company make a dividend of $6000 in one year; how much of the dividend should a person receive who owns 5 shares?

5. A and B bought a lot of wood together for $500; A paying $300, and B, $200; they sold it so as to gain $100; how much did each one gain?

6. C and D buy a farm together, for $2500; C paying $1500, and D, $1000, but they sell it for $3000; what is the gain of each?

7. E and F buy goods and trade in company; E furnishes $750, and F, $1000; they gain $700 in one year; how much of the $700 ought each to have?

8. A, B, and C bought a lot of land to speculate upon, A paying $500, B, $300, and C, $700; but they were obliged to sell it for $1200; how much did each lose?

9. A man dying left only $1000 to pay his debts with, which amounted to $1500; to A he owed $450, to B, $510, and to C, $540; what would each receive?

10. A person left in his will $560 to his son Isaac, $600 to George, and $840 to Nathan; but on settling the estate, it was found that there was only $1500 to be divided among them; what was the share of each?

11. Messrs. White, Black, and Co. traded together one year, and cleared $2652; what would be the share of each partner in the gain, supposing Mr. White furnished $1200 of the capital, Mr. Black $568, and Mr. Green the remaining $3536?

12. A, B, and C freight a ship with wine; A put on board 500 tons, B, 356, and C, 94; in a storm they were obliged to cast 190 tons overboard; what loss does each sustain?

13. The ship Grampus, valued at $50000, was lost at sea but was insured for $37500. A's part of the ship was worth $18000, B's $24000, and C owned the rest of it; what should each owner receive of the insurance?

14. Three men took a job of work together for $75; A worked upon it 24 days, B, 16 days, and C, 10 days; what should each receive?

15. Three persons put equal shares into a joint stock to trade upon; A continues his stock in trade 4 months, B, 6 months, and C, 10 months; what was each man's share of the $480 which they gained?

293. Model of a Recitation.

1. Two men undertook a job for $50; A kept 3 men at work upon it 5 days, and B, 7 men, 3 days; how should they divide the money?

Here the *number of men* they kept at work, as well as the *time* that the men worked, being *different*, both of these circumstances must be considered in dividing the money; thus, A's 3 men working 5 days, was equivalent to five times as many men, or 15 men working 1 day; and B's 7 men working 3 days, was equivalent to three times as many men, or 21 men working 1 day; hence, the job took the same as 15

$+21=36$ men 1 day, of which, A furnished $\frac{15}{36}$, and should have $\frac{15}{36}$ of the money; and B furnished the other $\frac{21}{36}$ of the men, and should receive the other $\frac{21}{36}$ of the money.

$$36 : 15 = 50 : \frac{\overset{25}{\cancel{50}} \times \overset{5}{\cancel{15}}}{\underset{6}{\cancel{36}}}, \text{ or } \$20.83\tfrac{1}{3}, \text{ A's share.}$$

$$36 : 21 = 50 : \frac{\overset{25}{\cancel{50}} \times \overset{7}{\cancel{21}}}{\underset{6}{\cancel{36}}}, \text{ or } \underline{\$29.16\tfrac{2}{3}}, \text{ B's share.}$$

$$\$50.00.$$

2. A, B, and C, traded in company; A furnished \$200 for 3 months, B, \$180 for 5 months, and C, \$70 for 10 months; how should they divide the \$132 which they gained?

The partners having furnished *unequal sums* for *unequal times*, each partner's ratio of the dividend will *not* be equal to his ratio of the capital; but the times may be made *equal* as in the preceding example, by multiplying what each partner furnished by the time it was employed; the sum of the products will form a new capital, for a *certain time*, equivalent to the real capital for the *unequal times;* therefore, *each partner's ratio of the dividend will be equal to the ratio that his product is of the sum of the products.* Thus; A's \$200 for three months is equivalent to 3 times as much, or \$600 for 1 month; B's \$180 for five months is equivalent to 5 times as much, or \$900 for 1 month, and C's \$70 for 10 months is equivalent

$$11 : 3 = 132 : \frac{\overset{12}{\cancel{132}} \times 3}{\cancel{11}}, \text{ or } \$36, \text{ A's share.}$$

$$22 : 9 = 132 : \frac{\overset{6}{\cancel{132}} \times 9}{\cancel{22}}, \text{ or } \$54, \text{ B's share.}$$

$$22 : 7 = 132 : \frac{\overset{6}{\cancel{132}} \times 7}{\cancel{22}}, \text{ or } \underline{\$42}, \text{ C's share.}$$

$$\$132.$$

so 10 times as much, or $700 for 1 month; consequently, the whole capital is the same as $600 + 900 + 700 = \$2200$ for 1 month, of which A furnished $\frac{600}{2200}$, or $\frac{3}{11}$, B, $\frac{900}{2200}$, or $\frac{9}{22}$, and C, $\frac{700}{2200}$, or $\frac{7}{22}$; therefore, these fractions show the partners' respective ratios of the dividend.

294. Exercises in Compound Fellowship.

In like manner, solve and explain the following problems.

1. A and B received $33 for teaming flour; how should they divide the money, if A teamed 8 loads of 6 barrels each, and B, 5 loads of 8 barrels each?

2. Two men gave $36 for the use of a pasture; in it A pastured 3 horses for 4 months, and B, 5 horses for 3 months; what ought each to pay?

3. What should A, B, C, and D each pay for a pasture, the whole expense of which is $100.50, and in which A pastured 6 cows 15 weeks, B, 5 cows 16 weeks, C, 10 cows 12 weeks, and D, 8 cows 14 weeks?

4. A, B, and C traded in company; A put in $400 for 9 months, B, $300 for 6 months, and C, $200 for 12 months; they gained $390; what was the gain of each?

5. E, F, and G lost, while in partnership, $720; ascertain the loss of each on the supposition, that E put in $800 for 12 months, F, $1200 for 9 months, and G, $1200 for 3 months?

6. H sold three lots of shoes for $850; he took $50 to pay him for his trouble, and returned the remainder to L, M, and P, the owners; how should they divide the money, if L's lot contained 256 pairs, costing $\$\frac{3}{4}$ per pair, M's lot 300 pairs, at $\$\frac{7}{8}$ per pair, and P's lot 300 pairs, at $\$\frac{5}{8}$ per pair?

7. R and S traded in company 16 months; R put in $1200 at first, and 9 months after, put in $200 more; S put in $1500 at first, and 6 months after, he took out $500; how should the $772.20 which they gained be divided?

8. A mason, carpenter, and painter, take a job for $2500. The mason paid out $200 for stock and team, and employed 4 hands 26 days; the carpenter paid out $700 for stock and teaming, and employed 5 hands 65 days; and the painter paid out $100 for stock, and employed 3 hands 13 days; how should they divide the money?

X. PERCENTAGE.

295. Definition and Illustration of Per Cent.

Per cent., literally signifying *by the hundred*, is a term applied to the *part*, (**87**,) or *ratio*, (**264**,) that one quantity is of another quantity of the same kind, *expressed in hundredths*. The per cent., then, in any case, is *a certain number of hundredths* of what is under consideration; thus, 3 per cent. of any thing or number, is 3 hundredths, (.03) of it.

The decimal figure, or figures, expressing a part of one per cent., will occupy one or more places below the hundredths' place, (**163**,) the thousandths being tenths per cent., the ten-thousandths hundredths per cent., &c.

Thus, $\frac{1}{2}$ per cent. of any thing or number, is .005 of it, or 5 tenths per cent. of it.

$\frac{1}{4}$ per cent. of any thing or number, is .0025 of it, or .25 per cent. of it.

$\frac{1}{8}$ per cent. of any thing or number, is .00125 of it, or .125 per cent. of it.

$12\frac{1}{2}$ per cent. of any thing or number, is .125 of it, or 12.5 per cent. of it.

125 per cent. of any thing or number, is 1.25 times it.

100 per cent. of any thing or number is exactly the whole of it.

.005 of any thing or number is $\frac{1}{2}$ per cent. of it.

.027 of any thing or number is 2.7 per cent. of it.

$2\frac{1}{4}$ times any thing or number is 225 per cent. of it.

$\frac{1}{8}$ of any thing or number, is 12.5 per cent. of it.

$\frac{1}{6}$ of any thing or number, is $16\frac{2}{3}$ per cent. of it.

296. Model of a Recitation.

1. What is 6 per cent. of $125.25?

```
$125.25
    .06
-------
$7.5150
```

6 per cent. of $125.25 is obtained by multiplying it by .06 as any decimal numbers are multiplied together, (**175**); that is, multiplying by 6 as units; then, in consideration of the 6 being hundredths, dividing this product by 100 by removing the decimal point (**181**) two places farther towards the left.

2. What is $\frac{1}{8}$ per cent. of 1728 cords of wood?

8)17.28
―――
2.16 cords.

$\frac{1}{8}$ per cent. of 1728 may be obtained by multiplying it by .00125, (**295**); but it can be more conveniently obtained by dividing by 100 to get one per cent., which divided by 8 will give $\frac{1}{8}$ per cent. as required.

297. Exercises in Percentage.

In like manner, solve and explain the following problems.

1. What is 12 per cent. of 14 casks of sugar, each weighing 734 lbs.?

2. How much is 2 per cent. of 10 bags of West India coffee, each weighing 112 lbs.?

3. What is 8 per cent. of 9 casks of nails, each weighing 217 lbs.?

4. How much is 2 per cent. of 25 bales of cotton, each weighing 532 lbs.?

5. If a man, whose salary is $700 per annum, spend annually 19 per cent. of his salary for house rent; 25 per cent. for stores consumed in his family; 15 per cent. for clothes; 13 per cent. for all other purposes, and save the remainder; what would be each of these items?

6. If a person should hire $375, agreeing to pay it in one year, together with 6 per cent. of it for interest; what would be due at the end of the year?

7. A person, going on a journey of 120 miles, traveled 25 per cent. of the distance the first day, 30 per cent. the second day, 35 per cent. the third day, and 10 per cent. on the fourth day; what was each day's travel?

8. A person, weighing 136.5 lbs., lost by sickness $22\frac{2}{3}$ per cent. of his weight; what did he then weigh?

9. A butcher sold a fat ox for 107 per cent. of the cost; what did he receive for the ox, the cost being $55?

10. What is $\frac{1}{4}$ per cent. of $3000?

11. How much is 5.5 per cent. of $1836?

12. In a certain school of 144 scholars, 75 per cent. study arithmetic, $18\frac{3}{4}$ per cent. study algebra, $16\frac{2}{3}$ per cent. study geometry, $87\frac{1}{2}$ per cent. study grammar, $31\frac{1}{4}$ per cent. study geography, $12\frac{1}{2}$ per cent. study philosophy, $8\frac{1}{3}$ per cent. study astronomy; what is the number of scholars in each of these studies?

13. What is 100 per cent of $.125?

298. Commission defined.

Commission is an allowance made to an agent, sometimes called a factor, or broker, for doing business for his employer; such as buying and selling goods, &c. It is usually reckoned at a *certain per cent.* (**295**) of the price of the goods bought or sold.

299. Model of a Recitation.

An agent sold a quantity of goods for $850, on commission at 3 per cent.; what was his commission?

$850 .03 ——— $25.50	To ascertain the commission, it is only necessary to take 3 per cent., or .03 of the sum for which the goods were sold, (**175**,) which gives $25.50, the answer required.

300. Exercises in reckoning Commission.

In like manner, solve and explain the following problems.

1. An auctioneer sold a house and other real estate, to the amount of $1837.50; what did his commission amount to, at 2½ per cent.?

2. The publishers of a periodical allowed their agents 20 per cent. on all subscriptions procured by them; what was that agent's commission who procured 193 cash subscribers for a periodical, the price of which is $2.50 per annum?

3. If a broker negotiate a loan of $2350 for a man, at ¼ per cent. commission, what would be his commission?

4. What commission should my agent in New Orleans charge me for purchasing on my account 625 bales of cotton, at $48 per bale, commission reckoned at 1¾ per cent.?

5. My agent, after selling property for me to the amount of $1575.25, deducted his commission at 2 per cent., and remitted to me the remainder, which was how much?

6. If a collector should receive .9 per cent. for collecting the taxes of a certain town, amounting to $1776.17, what would be his commission?

301. Stocks defined and illustrated.

Men of wealth often associate into companies, and, having obtained from the government acts of incorporation, combine their wealth into what is called *Capital Stock*, to be employed

according to the acts of incorporation. Such companies take different names, as *Banking* companies, *Insurance* companies, *Manufacturing* companies, &c., according to the business in which their capital stock is employed.

Such stocks, being divided into equal shares, are bought and sold like other property, at various prices; and, according as the market price becomes equal to, above, or below the original value, the stock is said to be *at par*, a certain per cent. *above par*, or *below par*. Sometimes, also, as the market price exceeds, or falls below the par value, the stock is said to be at such a per cent. *advance*, or *discount*.

Acknowledged claims upon the government, and notes issued by government in case the ordinary revenues prove insufficient to meet the demands upon the treasury, are also called stocks, and pass from hand to hand like other stocks.

The profits arising from stocks are, at regular periods, made a *dividend*, and distributed among the stock-holders, each receiving a certain per cent. of the par value of his shares.

202. Exercises in reckoning Stocks.

1. The semi-annual dividend of the Lowell Bank, Oct. 1844, was 3 per cent. of the capital stock; what should a person receive who owned 6 shares, the par value being $100 per share?

2. A person paid 108 per cent., or 8 per cent. advance, for a share of the Merrimack Manufacturing stock; what was the cost, the par value being $1000 per share?

3. A person bought 6 shares of the Lowell Bank Stock at 93 per cent., or 7 per cent. discount, and sold them at par, which is $100 per share; what was his gain?

4. At an auction sale of 5 shares of the Boston and Lowell Rail Road stock, A bid 25 per cent. advance, and B bid $25\frac{3}{8}$ per cent. advance; how much more money did B bid for the 5 shares than A did, the par value being $500 per share?

5. In 1844 the Rail Road Bank reduced its capital stock, by refunding to the stockholders 25 per cent. of the par value of the stock, the par value then being $100 per share; what did he receive who owned 3 shares?

6. The reduction of the capital stock of the Rail Road Bank, mentioned in the last problem, established the par value at $75; what is now the par value of 8 shares?

303. INSURANCE DEFINED AND ILLUSTRATED.

Insurance companies assume the risks of property exposed to accident or danger, upon the conditions recorded in their policies.

A Policy is the written agreement of the parties.

Property, the risk of which has been assumed by an insurance company is said to be insured.

Insurance is the exemption of the owners of insured property from a specified portion of the loss, in case such property becomes damaged, or destroyed.

Premium is the sum paid to obtain insurance. It is estimated according to the value of the property, the degree of its exposure, and the time for which insurance is effected, and is usually a certain *per cent.* of the amount insured. The sum insured is generally less than the full value of the property; but the company is obliged to sustain all losses not exceeding the sum insured, except such as do not exceed a certain per cent. of the sum insured, which are sustained by the owner.

304. EXERCISES IN RECKONING INSURANCE.

1. The estimated value of my house being $2500, and having effected insurance, at 1 per cent., upon $\frac{4}{5}$ of its value, what is the premium?

2. If the house mentioned in the preceding problem, should, within the time specified in the policy, be destroyed by fire, what would be the amount of my claim upon the insurance company, or underwriters, as they are sometimes called?

3. What premium must be paid for the insurance of $7250, upon a ship, and $2750 upon the cargo, during a voyage from Boston to Liverpool, the premium for the ship being 2 per cent., and for the cargo $2\frac{1}{2}$ per cent.?

4. If a person should obtain insurance to the amount of $1275 upon his furniture, at a premium of $\frac{1}{2}$ per cent., what would be the premium?

5. A merchant generally having about $10000 worth of goods in his store, obtained insurance upon $7125 worth of them; what was the premium at $\frac{3}{8}$ per cent.?

6. If $\frac{3}{4}$ of the goods mentioned in the preceding problem, should be destroyed by fire while insured, what would the merchant save by having his goods insured?

305. Exercises in the Assessment of Taxes.

The owners of property are liable to pay *taxes* to the state, county, town, and parish to which they severally belong.

Tax assessed upon property is usually reckoned at a certain *per cent.* of the value of the property taxed.

1. If in the city of Lowell, in 1845, the city, state, and county tax was $\frac{1}{3}$ per cent. of the valuation of the property, what would be the tax upon the property of Mr. A. of Lowell, his property being valued at $4755?

2. In 1844 the tax upon property in the city of Lowell was .58 per cent.; what should have been the tax upon the property of Mr. B. of that city, his property being valued at $7350?

3. What would be the parish tax of Mr. C., the property in his parish being taxed .2 per cent. and his property being valued at $19000?

4. If Mr. D. lived in a town where the tax upon the property for the state was .1 per cent., for the county .2 per cent., for the town .3 per cent., and for the parish $\frac{1}{4}$ per cent.; what would be the tax upon his property which was valued at $2945?

306. Duties defined and illustrated.

Duty is a term applied to the tax which is levied by the government of a country upon goods imported into that country.

Duties are either specific or ad valorem.

Duty is *specific* when it is specified at *a certain sum* per ton, gallon, square yard, &c., without regard to the cost.

Duty is *ad valorem* when it is specified at *a certain per cent. of the cost* of the goods in the country whence they were imported.

307. Allowances in reckoning Duties.

In reckoning duties certain *allowances* or deductions from the gross weight, or measure, are made. These allowances are called draft, tare, and leakage. The same allowances are also made to wholesale merchants when they purchase, and by them when they sell.

Draft is made that the quantity may hold out when retailed. The draft on each parcel weighing not over 112 lbs. is 1 lb.

Over	112 lbs. and not exceeding			224 lbs.	is	2 lbs.
"	224	"	"	336	"	3 "
"	336	"	"	1120	"	4 "
"	1120	"	"	2016	"	7 "
"	2016	"			"	9 "

Tare is an allowance *after* the draft has been deducted, for the weight of the box, cask, bag, &c., containing the goods.

Gross weight is the weight before the deductions of draft and tare.

Neat weight is the weight after the deductions of draft and tare.

Leakage is an allowance of 2 per cent. deducted from the measure of liquids to insure full measure at retail.

In making these allowances a fraction is disregarded, unless it exceed one half, when it is considered a unit.

Duties are reckoned on what remains after all allowances have been made.

308. MODEL OF A RECITATION.

What is the duty on a hogshead of sugar weighing 818 lbs., the tare being 12 per cent. and the duty $2\frac{1}{2}$ cents a pound?

818	814
4	98
814	716
.12	$2\frac{1}{2}$
1628	1432
814	358
97.68	$17.90

The draft is 4 pounds, (**307**,) which deducted from the gross weight, 818 pounds, leaves 814 pounds. The tare is 12 per cent. of 814 pounds, equal to 98 pounds, which deducted from 814 pounds leaves the neat weight, 716 pounds. The duty is $2\frac{1}{2}$ cents a pound, or $2\frac{1}{2}$ times as many cents as pounds (**38**) in the neat weight, equal to $17.90; which is the answer required.

309. EXERCISES IN RECKONING DUTIES.

In like manner, solve and explain the following problems.

1. What is the duty on 5 boxes of sugar weighing severally, 500 lbs., 496 lbs., 487 lbs., 514 lbs., and 508 lbs., the tare being 15 per cent. and the duty $2\frac{1}{2}$ cents a pound?

2. What is the duty, at 30 per cent. ad valorem, on 125

bales of wool that cost in Saxony 30 cents a pound, the bales averaging 500 lbs. each, and the tare being 3 per cent.?

3. The ad valorem duty on feathers being 25 per cent., and tare 3 per cent., what would be the duty on 75 bags, averaging 325 lbs. a bag, and costing in Russia 25 cents a pound?

4. What duty, at 25 cents a gallon, must be paid on 25 casks of linseed oil, averaging 54 gallons a cask, the leakage being 2 per cent.?

5. What duty, at $1 a gallon, must be paid on 18 casks Rochelle brandy, averaging 112 gallons each, the leakage being 2 per cent.?

6. The duty on oranges being 20 per cent. ad valorem, what would be the duty on 64 boxes which cost in Cuba $1.125 per box?

7. What would be the duty, at 20 per cent. ad valorem, on 5000 pairs of India rubber shoes which cost 20 cents a pair in Brazil?

310. INTEREST AND OTHER TERMS DEFINED.

Interest is the compensation to which a creditor is entitled from his debtor for the use of money.

A Promissory Note is a written promise of one person, for value received, to pay to another person a certain sum of money.

The face of a note is the sum of money mentioned in the note.

Principal is the debt, or sum upon which interest is reckoned.

Simple Rate is the per cent. of the principal to which the *annual* interest is equal. (**295. 264.**)

Compound Rate is the per cent. of the principal to which the interest for a *time greater*, or *less than one year*, is equal. (**281. 295.**)

The rate of interest is usually expressed in *hundredths;* hence it is called *per cent.* The legal rate of interest varies in different states and countries. In New England it may not exceed 6 per cent. of the principal per year; and 6 per cent. is implied in all cases where no other rate is specified.

N. B. By the term rate, simple rate is to be understood, unless otherwise specified.

The Amount is the sum of the principal and interest.

311. MODE OF RECKONING COMPOUND RATES.

As 6 per cent. is the common simple rate, it is thought best to obtain the compound rate from that, which is done by taking such part (**93**) of 6 per cent. as the given time is of one year; but, if the given simple rate is other than 6 per cent., it is further necessary to take such part of this result as the given simple rate is of 6 per cent.

The time generally must be obtained from dates by subtracting the earlier from the later date, as directed, (**246**). But should the time be required in days, it may be obtained by adding the remaining days of the month mentioned in the earlier date, and the days of each succeeding month till the day of the month mentioned in the later date, with 365 days for each year, and one extra day for each leap year in the time. The greater convenience, notwithstanding the slight error, has established the custom of considering calendar months, though differing in length, twelfths of a year, and days, not exceeding 30, thirtieths of a month, (**202**).

At the simple rate of 6 per cent. for a year, or 12 months, the compound rate will be 1 per cent. (.01) for every sixth of 12 months, or 2 months; and $\frac{1}{2}$ per cent. (.005) for every half of 2 months, or 1 month; also, $\frac{1}{10}$ per cent. (.001) for every fifth of 1 month, or 6 days, and $\frac{1}{6}$ of $\frac{1}{10}$ per cent. (.000$\frac{1}{6}$) for every sixth of 6 days, or 1 day.

312. MODEL OF A RECITATION.

1. Required the interest upon $1350, for 1 year.

$$\begin{array}{r} 1350 \\ .06 \\ \hline \$81.00 \end{array}$$

In this problem no rate is specified; consequently, 6 per cent. is implied, (**310**,) and 6 per cent. of $1350 is $81, which is the result required.

2. Required the interest upon $1350, for 9 months.

$$\begin{array}{r} 1350 \\ .045 \\ \hline 6750 \\ 5400 \\ \hline \$60.750 \end{array}$$

As the compound rate is 1 per cent. (.01) for every 2 months, for 8 months it will be 4 per cent., (.04,) and for 1 month it being $\frac{1}{2}$ per cent., (.005,) for 9 months it will be 4$\frac{1}{2}$ per cent. (.045); and 45 thousandths (**175**) of $1350 is $60.75, which is the result required.

3. Required the interest upon $1350, for 22 days.

1350.
.003$\frac{2}{3}$
4050
450
450
$4.950

As the compound rate is $\frac{1}{10}$ per cent. (.001) for every 6 days, for 18 days it will be $\frac{3}{10}$ per cent. (.003), and for 4 days it will be $\frac{4}{6}$ or $\frac{2}{3}$ of $\frac{1}{10}$ per cent. (.000$\frac{2}{3}$); consequently, for 22 days it will be .003$\frac{2}{3}$; and 3$\frac{2}{3}$ thousandths of $1350, is $4.95, which is the answer required.

In multiplying by $\frac{2}{3}$, divide by 3 to obtain $\frac{1}{3}$, (**144**,) which written twice, will give $\frac{2}{3}$ of the multiplicand as desired.

4. Required the interest upon $1350, from July 25th, 1844, to Nov. 12th, 1845, (**246**).

1845	11	12	.06	is the simple rate for	1 year.
1844	7	25	.01	" the compound "	2 months.
			.005	" " "	1 "
1y.	3m.	17d.	.002	" " "	12 days.
			.000$\frac{5}{6}$	" " "	5 "
			.077$\frac{5}{6}$	" " "	1 y. 3 m. 17 d.

$1350
.077$\frac{5}{6}$
9450
9450
225
900
$105.075.

Therefore, multiply the principal by .077$\frac{5}{6}$, which gives $105.075, the answer required.

In multiplying by $\frac{5}{6}$, divide by 6 for $\frac{1}{6}$, and multiply that by 4 for $\frac{4}{6}$, which added, gives $\frac{5}{6}$ of the multiplicand as desired.

5. Required the amount of $1350 for 2 years, 6 months, and 16 days, at 8 per cent.

$1350	.12	is the compound rate for 2 years, at 6 per cent.
.203$\frac{5}{9}$	.03	" 6 months "
	.002	" 12 days "
4050	.000$\frac{2}{3}$	" 4 " "
2700		
150	3).152$\frac{2}{3}$	" 2y. 6m. 16d. "
600	.050$\frac{8}{9}$	" " " " 2 per cent.
274.800	.203$\frac{5}{9}$	" " " " 8 "
1350.		
$1624.80		

Therefore, multiply the principal by .203$\frac{5}{9}$, which gives the interest $274.80, to which add the principal for the amount, which gives $1624.80, the answer required.

313. Exercises in reckoning Interest.

In like manner, solve and explain the following problems.

1. Required the interest of \$1350 for 1 year, at 6 per cent.
2. What is the interest of \$250 for 1 year, at 8 per cent.?
3. What is the interest of \$125.50 for 2 years, at 5 per cent.?
4. What is the interest of \$25.375 for 1 year and 6 months, at 6 per cent.?
5. What is the interest of \$144.36 for 5 years and 8 months, at 6 per cent.?
6. What is the interest of \$187.50 for 2 years and 5 months, at 6 per cent.?
7. What is the interest of \$500 for 9 months, at 6 per cent.?
8. What is the interest of \$175 for 12 days, at 6 per cent.?
9. What is the interest of \$248 for 25 days, at 6 per cent.?
10. What is the interest of \$17.28 for 10 months and 24 days, at 7 per cent.?
11. What is the interest of \$18.36 for 1 year, 6 months, and 18 days, at 6 per cent.?
12. What is the interest of \$75.625 for 3 months and 6 days, at 6 per cent.?
13. What is the interest of \$1000 for 7 months and 15 days, at 6 per cent.?
14. What is the interest of \$625 for 4 months; at 7½ per cent.?
15. What is the interest of \$48.25 for 2 years, 2 months, and 15 days, at 4½ per cent.?
16. What is the interest of \$55.40 for 1 year, 4 months, and 10 days, at 5 per cent.?
17. What would be the interest of \$75.60, from April 10th, 1842, to June 10th, 1843, at 6 per cent.?
18. What would be the interest of \$50.75, from May 4th, 1843, to April 19th, 1845, at 6 per cent.?
19. What would be the interest of \$375, from June 6th, 1840, to August 18th, 1844, at 4 per cent.?
20. Required the interest of \$125.50, from July 4th, 1841, to January 8th, 1844, at 6 per cent.

21. Required the interest on the following note.

$250.

Lowell, April 1st, 1844.

For value received, I promise to pay A. B. two hundred and fifty dollars in one year from date, with interest. C. D.

22. $1000.

Boston, September 12th, 1842.

For value received, I promise to pay E. F. one thousand dollars on demand, with interest. G. C

What was due on this note August 24th, 1845?

23. $1296.45.

Providence, July 10th, 1844.

For value received, I promise to pay J. K. one thousand two hundred ninety-six dollars and forty-five cents on demand, with interest at 5½ per cent. L. M.

What was due J. K. at the time of settlement, July 1st, 1845?

24. $333.33⅓.

New Haven, October 25th, 1842.

For value received, I promise to pay N. O. three hundred thirty-three dollars and thirty-three cents on demand, with interest at 5 per cent. Q. P.

What was due N. O. at settlement, Jan. 1st, 1845?

25. $625.

New York, November 19th, 1843.

For value received, I promise to pay R. S. six hundred and twenty-five dollars on demand, with interest at 7 per cent. T. U.

What must T. U. pay R. S. to settle this note, April 19th, 1845?

314. Mode of reckoning Interest in case of Partial Payments.

If partial payments be made upon a note or bond, before the final settlement, such payments must be applied towards canceling the interest due when the payment is made; but, whenever such payment or payments exceed such interest, the excess must be applied to diminish the principal at the same time.

315. Model of a Recitation.

1. $800.

Lowell, June 6th, 1841.

For value received, I promise to pay A. B. on demand, eight hundred dollars, with interest. M. N.

On the back of this note are the following endorsements.

April 24th, 1842, received $75.
October 12th, 1842, received $20.
May 18th, 1843, received $64.
July 12th, 1844, received $100.

What was due at the time of settlement, Jan. 1st, 1845?

Principal, ...		$800.00
Payment, April 24th, 1842, ...	$75.00	
Interest to 1st payment, (10 m. 18 d.,) .	42.40	
Excess of the payment over the interest,		32.60
Principal, or sum due after 1st payment,		$767.40
Payment, Oct. 12th, 1842, . . . $20. / Payment May 18th, 1843, . . . $64.	$84.00	
Interest from April 24th, 1842, to Oct. 12th, 1842, (5 m. 18 d.) $21.49		
Interest from Oct. 12th, 1842, to May 18th, 1843, (7 m. 6 d.) $27.62		
	$49.11	
Excess of the payments over the interest,		34.89
Principal, or sum due after the 3rd payment,		$732.51
Payment July 12th, 1844, . .	$100.00	
Interest from May 18th, 1843, to July 12th, 1844, (1 y. 1 m. 24 d.)	50.54	
Excess of the payment over the interest,		49.46
Principal, or sum due after the 4th payment,		$683.05
Interest from July 12th, 1844, to January 1st, 1845, (5 m. 19 d.)		19.24
Sum due January 1st, 1845, ...		$702.29

316. EXERCISES IN RECKONING INTEREST IN CASE OF PARTIAL PAYMENTS.

In like manner, solve and explain the following problems.

1. $625.

Lowell, October 4th, 1840.

For value received, I promise to pay C. D. on demand, six hundred and twenty-five dollars, with interest. P. Q.

If this note be endorsed as follows; what would be due October 4th, 1845?

August 10th, 1843, received $75.
December 15th, 1843, received $225.
November 18th, 1844, received $150.

2. $375.50.

Worcester, January 1st, 1840.

For value received, we, jointly and severally, promise to pay J. R. three hundred seventy-five dollars and fifty cents, on demand, with interest. S. H.
M. E.

April 10th, 1841, received on this note, $50.
Jan. 6th, 1842, received on this note, $37.50.
March 12th, 1843, received on this note, $20.
Feb. 1st, 1844, received on this note, $125.50.

What must be paid to settle this note April 18th, 1845?

3. What would be due May 1st, 1845, on a note given May 1st, 1840, for $1000, at 6 per cent. interest, on which were paid $200 at the end of each year from the date of the note?

4. If I should give my bond, July 1st, 1844, for $2500, agreeing to pay $300 on the first day of every month, except the first, and settle the bond on the 1st January, 1845; what would be the last payment, interest at 5 per cent.?

5. Suppose I should give my note for $900, at 4½ per cent. interest, and should pay the interest annually for 4 years; what would be due at the end of the 5th year?

6. Required the sum due Oct. 7th, 1846, on a note for $1350, dated Oct. 7th, 1842, on which $81 was paid Oct. 4th, 1843; $100 July 8th, 1844; and $100 July 14th, 1845.

317. The Discounting of Notes explained.

The discounting of notes, or loaning money on notes, is an important branch of the *banking* business.

Notes received at banks are substantially the following.

1. A has a note in which he as principal, together with B and C, either as sureties, or endorsers, promise to pay the bank, at a certain specified future time, a certain sum of money, without interest.

2. A has a note, in which B promises to pay him, at a certain specified future time, a certain sum of money, either with or without interest.

The bank discounts No. 1, by taking it and paying A the sum specified in the note, minus the interest upon the note to three days after the time specified for the payment of the note.

No. 2, is discounted like No. 1, unless it be upon interest, when the bank pays A the *amount* of the note to three days after the time specified for its payment, minus the interest upon that amount to the same time.

Hence, *the bank discount of any sum is the interest of that sum for the time.*

The Avails of a note discounted at a bank, is the sum of money which is received from the bank for the note, or what remains after the discount is paid.

Before discounting No. 2, the bank requires the *endorsement* of A, which is simply his name written upon the back of the note. It obligates the endorser to pay the note, should the promiser fail to do so at its maturity.

The maturity of a note occurs at the time when the note should be paid.

Interest is taken for three days extra, called "*days of grace*," as payment may lawfully be delayed that time.

318. Mode of reckoning Interest for Days.

In reckoning interest, when the time is given in days, every 30 days is considered a month, or $\frac{1}{12}$ of a year. And the simple rate being 6 per cent., the compound rate for $\frac{1}{6}$ of a year, or 60 days, is $\frac{1}{6}$ of 6 per cent., or 1 per cent, (.01); and for $\frac{1}{10}$ of 60 days, or 6 days, it is $\frac{1}{10}$ of 1 per cent., (.001).

Therefore, when the interest for any number of days is required, the simple rate being 6 per cent., multiply the principal by 1 per cent., which is done by removing the point (**10**) two places farther to the left, to obtain the interest for 60 days, and take such part of that as the given days are of 60 days; or, multiply the principal by .001, which is done by removing the point (**10**) three places farther to the left, to obtain the interest for 6 days, and take as many times that as the given days are times 6 days. But if the simple rate is other than 6 per cent., first obtain the interest as though the simple rate were 6 per cent., and then take such part of that as the given rate is of 6 per cent.

319. Model of a Recitation.

$564.25.

Lowell, April 4th, 1846.

In ninety days, for value received, we, A. B. as principal, and C. D. and E. F. as sureties, jointly and severally promise to pay the President, Directors, & Co., of the Lowell Bank, or order, at said bank, five hundred and sixty-four dollars, and twenty-five cents. A. B. C. D. E. F.

$564.25	Removing the point (**318**) two places farther to the left, gives $5.64, the interest for 60 days, $\frac{1}{2}$ of which is $2.82, the interest for 30 days; and $\frac{1}{10}$ of the interest for 30 days is $.28, the interest for 3 days; all added, give $8.74, the bank discount, which subtracted from the principal, gives $555.51, the avails of the note.
5.64	
2.82	
.28	
8.74	
$555.51	

320. Exercises in reckoning the Discount and Avails of Notes.

In like manner, solve and explain the following problems.

What are the avails of the following notes, severally discounted at a bank?

1. $342.

Lowell, April 4th, 1846.

In four months, for value received, I promise to pay A. B., or order, three hundred and forty-two dollars.

Signed by C. D., and endorsed by A. B.

2. $912.

Lowell, May 12th, 1846.

In sixty days, for value received, I promise to pay the President, Directors & Co. of the Rail Road Bank, or order, at said bank, nine hundred and twelve dollars.

Signed by A. B., and endorsed by C. D. and E. F.

3. $510.50.

Lowell, June 20th, 1846.

In thirty days, with interest, for value received, I promise to pay A. B., or order, five hundred ten dollars and fifty cents.

Signed by C. D. and endorsed by A. B.

4. $1000.

Lowell, February 1st, 1846.

In four months, with interest, I promise to pay A. B., or order, one thousand dollars.

Signed by C. D. and endorsed by A. B.

What would be the avails of this note discounted April 4th, 1846?

5. $1250.

Lowell, August 10th, 1846.

In two months, for value received, we A. B. as principal, and C. D. & E. F. as sureties, jointly and severally promise to pay the President, Directors & Co., of the Lowell Bank, or order, at said bank, twelve hundred and fifty dollars, at 4½ per cent.

321. Compound Interest defined.

Compound Interest is the interest which accrues when, at the beginning of equal periods, excepting the first, the principal is increased by the interest for the preceding period.

322. Model of a Recitation.

1. What is the compound interest of $432.27 for 5 years and 6 months, the interest being compounded annually?

$432.27 = Principal.
25.9362 = Interest for the 1st year.
———
458.21 = Amount for 1 year.
27.4926 = Interest for the 2d year.
———

Space and labor may be saved, in this example, by not writing the multipliers; but multiplying mentally, and writing

485.70	= Amount for 2 years.	
29.142	= Interest for the 3d year.	
514.84	= Amount for 3 years.	
30.8904	= Interest for the 4th year.	
545.73	= Amount for 4 years.	
32.7438	= Interest for the 5th year.	
578.47	= Amount for 5 years.	
17.3541	= Interest for the last 6 ms.	
595.83	= Amount for 5 ys. 6 ms.	
432.27	= Original principal.	
$163.56	= Com. interest for 5 ys. 6 ms.	

the products so that the units of each order may stand under those of the same order, which will be effected if the products be written two places farther towards the right, (10.)

The inconvenient increase of decimal figures may be obviated by omitting all below the second or third, and compensating for such loss by occasionally increasing the last figure retained.

2. What would be the amount of $625, from June 10th, 1845, to Oct. 3d, 1846, the interest being compounded every 90 days?

From June 10th, 1845, to June 10th, 1846, are 365 days.
" " 10th, 1846, to the end of June, " 20 "
in July, 31 "
in Aug., 31 "
in Sep., 30 "
in Oct., 3 "

90)480 "

$5\frac{1}{3}$ qrs.

$625.	= Principal.	
6.25 3.12	= Interest for 1st quarter.	
634.38	= Amount for 1 quarter.	
6.34 3.17	= Interest for 2d quarter.	
643.89	= Amount for 2 quarters.	
6.44 3.22	= Interest for 3d quarter.	
653.55	= Amount for 3 quarters.	
6.54 3.27	= Interest for 4th quarter.	

The compound rate for one quarter of a year is .015, or $1\frac{1}{2}$ per cent. The interest at 1 per cent. may be obtained by writing the principal under itself, remov-

663.36	= Amount for 4 quarters.	
6.63 3.32	= Interest for 5th quarter.	
673.31	= Amount for 5 quarters.	
3.36	= Interest for ⅓d of a qr.	
$676.67	= Amount for 5⅓ quarters.	

ed two places farther to the right (10); one half of this will be the interest at ½ per cent.; and all added give the amount for one quarter; and so on for the succeeding quarters.

The rate for ⅓ of a quarter being ½ per cent., the interest for that time is ½ of the principal written two places farther to the right.

323. Exercises in reckoning Compound Interest.

In like manner, solve and explain the following problems.

1. What would be the amount of $800 for 4 years, the interest being compounded annually?

2. What would be the interest of $548 for 3 years, at 5 per cent., the interest being compounded annually?

3. What would be the amount of $175.24 for 4 years and 8 months, the interest being compounded annually?

4. What would be the interest of $1000 for 7 years and 4 months, at 4½ per cent., the interest compounded annually?

5. What would be the amount of $12.375 for 3 years and 6 months, the interest being compounded semi-annually?

6. What would be the interest of $500 for 4 years 2 months and 18 days, the interest being compounded annually?

7. If a note of $250, dated April 1st, 1844, with interest compounded annually, be paid April 16th, 1847, what should be the payment?

8. What would be the difference between the simple, and compound interest of $100 for 5 years, the interest being compounded annually?

324. Mode of reckoning Compound Interest in case of Partial Payments.

To obtain the sum due when *partial payments* are made upon a note or bond before the final settlement, the amount of such payments must be subtracted from the amount of the face of the note.

225. Exercises in reckoning Compound Interest in case of Partial Payments.

1. What would be due Aug. 4th, 1850, on a note for $1000, dated Nov. 8th, 1845, on which were endorsed $200, July 4th, 1847, and $500, Sept. 10th, 1848, the interest compounded annually?

2. Find the balance due on the following note Jan. 1st, 1850.

$800.

Boston, November 10th, 1845.

For value received I promise to pay A. B. eight hundred dollars on demand, with interest compounded annually.

M. N.

Nov. 15th, 1847, Received $50. A. B.
Oct. 16th, 1849, Received $100. A. B.

226. Problem for finding the Time.

To find the time when principal, rate, and interest, or amount are known; divide the interest, which, if not given, may be found (**210**) by subtracting the principal from the amount, by the interest for one year, one month, or one day; for there will be as many years, months, or days, in the time as the interest for one year, one month, or one day, is contained times in the interest for the required time.

227. Model of a Recitation.

How long must $75.25 be on interest at 5 per cent. that the interest may become $6.25?

```
$75.25
   .05
-------
3.7625) 6.2500   (1.661+
         37625     12
        ------   -----
        248750   7.933
        225750      30
        ------   ------
         230000  27.990
         225750
         ------
           4250
```

The interest for one year is $3.7625, which is contained in the *whole* interest 1.661+ times; consequently, the time is 1 year and a fraction; that fraction will make 12 times as many months, (**226**,) or 7 months and a fraction; that fraction will make 30 times as many days, or 28 days; in all, 1 year, 7 months, and 28 days.

228. EXERCISES IN FINDING THE TIME FROM THE PRINCIPAL, RATE AND INTEREST.

In like manner, solve and explain the following problems.

1. What time is necessary for the interest of \$50 to become \$.125 at 6 per cent.?

2. In what time will \$1200 amount to \$1278?

3. If \$3.24 interest be paid on a note of \$144, what must have been the time, the rate being $4\frac{1}{2}$ per cent.?

4. In what time would the interest of \$75, be \$75?

5. In what time would the interest of any sum be equal to the principal, the rate being 6 per cent.?

6. In what time would \$100, or any sum, double itself at 5 per cent.?

229. PROBLEM TO FIND WHAT PER CENT. ONE QUANTITY IS OF ANOTHER.

To find what per cent. one quantity is of another of the same kind, express the part that it is of the other quantity by a common fraction (**87**), and reduce that to a decimal fraction (**178**) in hundredths; for the per cent. that one quantity is of another, is the ratio, or part that it is of the other expressed in hundreths, (**295**).

230. MODEL OF A RECITATION.

1. If a miller take 2 quarts from every bushel of grain he grinds for toll, what per cent. of the grain is the toll?

```
  2
 --
 32)2.00(.06¼
    192
    ---
      8
      8
     --
```

As there are 32 quarts in a bushel, the toll, being 2 quarts per bushel, is $\frac{2}{32}$ of the grain. This fraction reduced to a decimal gives $6\frac{1}{4}$ per cent.

2. If I buy $18\frac{3}{4}$ yards of broad cloth at \$4 per yard, and sell the whole for \$83.50, what should I gain per cent.?

```
$18¾     83.50
   4     75.00
 ———     —————
 75.     )8.50(.11⅓
          75
          ——
          100
           75
           ——
           25
           25
           ——
```

The whole cost is $75, which, subtracted from the sum for which the cloth is sold, leaves the gain, $8.50, which is $\frac{8.50}{75}$ (eight and a fraction seventy-fifths,) of the cost; equal to 11⅓ per cent.

231. Exercises in finding what Per Cent. one quantity is of Another.

In like manner, solve and explain the following problems.

1. If hats be bought for $3.75 apiece, and sold for $4, what per cent. would be the profits?

2. If the price of flour be raised from $6.25 to 6.62½ per barrel, what per cent. is the rise?

3. If the price of a passage on the railroad from Lowell to Boston, be reduced from $1 to $.65, what per cent. is the reduction?

4. If a barrel of flour, which should weigh 196 lbs., weigh only 176 lbs., what per cent. is the deficiency? and what is the fraud per barrel when the price is $6?

5. If $7.50 be paid for the insurance of a house valued at $1500, what per cent. is the insurance?

6. If an auctioneer charge $5.25 for selling property to the amount of $1050, what per cent. is his commission?

7. If bank stock, whose par value is $100 per share, be bought for $93.50 per share, at what per cent. discount is it obtained.?

8. If the property of a bankrupt, whose debts amount to $6475.50, is worth only $1295.10, what per cent. of his debts can he pay?

9. If the annual profits of a merchant, whose stock in trade averages $10000, be $1250, what per cent. does his capital yield?

10. What per cent. is the tax in that town where a person whose property is worth $4000 is taxed $8.25 besides his poll tax?

11. If 7 gallons of molasses leak from a cask gauging 63 gallons, what per cent. is the loss?

332. Problem to find the Rate Per Cent.

To find the rate when the principal, time, and interest, are known, express the part that a year's interest is of the principal, by a common fraction, (87,) and reduce it to a decimal fraction in hundredths; for the simple rate is (310) the part of the principal, expressed in hundredths, to which the annual interest is equal.

333. Model of a Recitation.

If \$11.625 be the interest of \$150 for 1 y. 3 m. 15 days; what must be the rate?

$\$11.625 = \$11\frac{5}{8} = \$\frac{93}{8}$.

15 days $= \frac{15}{30} = \frac{1}{2}$ month.

$3\frac{1}{2}$ m. $= \frac{3\frac{1}{2}}{12} = \frac{7}{24}$ a year.

$1\frac{7}{24}$ y. $= \frac{31}{24}$ years.

$\$\frac{\overset{9}{93}\times 24}{8\times 31} = \$9 =$ year's interest.

$$150\,)\,9.00\,(\,.06$$
$$900$$

The whole interest is $\$\frac{93}{8}$; the whole time is $\frac{31}{24}$ years. Divide the interest for the whole time by 31, to obtain the interest for $\frac{1}{24}$ of a year, which multiplied by 24 gives \$9 for one year's interest, which is $\frac{9}{150}$, or .06, or 6 per cent. of the principal.

334. Exercises in finding the Rate Per Cent

In like manner, solve and explain the following problems

1. If \$52.50 be the interest of \$350 for 2 y. 6 m., what is the rate?

2. If I pay \$31 for the use of \$500 for 9 m. 9 days, what is the rate?

3. At what rate would \$400 amount to \$646.40, in 10 y. 3 m. 6 days?

4. At what rate will the interest of $150 be equal to the principal in 12 y. 6 m.?

5. At what rate will $150 amount to $163.125, from May 10th, 1845, to Aug. 10th, 1846?

335. PROBLEM TO FIND THE PRINCIPAL.

To find the principal when the rate, time, and interest are known, divide the interest by the interest of one dollar for the time; for there will be as many dollars in the principal as the interest of one dollar is contained times in the interest of the required principal.

336. MODEL OF A RECITATION.

What principal is that, the interest of which, at 6 per cent. for 1 year, 4 months, is $4.52?

.08) 4.52
$56.50 = Principal.

The interest of $1 for the time is 8 cents; consequently, for every 8 cents in the interest there will be a dollar in the principal.

337. EXERCISES IN FINDING THE PRINCIPAL.

In like manner, solve and explain the following problems.

1. What is the face of a note which, at 7 per cent., would yield $30 interest in 1 y. 8 m.?

2. What is the value of that person's property which, at 6 per cent., affords an income of $750 per annum?

3. What is the face of a note dated July 4th, 1844, the interest of which, at 5 per cent., was $64.75, January 19th, 1846?

4. What principal, at $4\frac{1}{2}$ per cent., would gain $47.88, from March 10th, to July 4th?

338. PROBLEM TO FIND THE PRESENT WORTH OF A DEBT.

The present worth of a debt which is not on interest, and is due at a certain specified future time, is a sum that, on interest for the same time, would *amount* to the debt.

To find the present worth, divide the debt by the amount of one dollar for the time; for, the debt being the amount of the present worth, there will be as many dollars in the present worth, as the amount of one dollar is contained times in the debt.

339. Model of a Recitation.

What is the present worth of \$834, due in 1 y. 7 m. 6 d., without interest, and money being worth 7 per cent.?

```
$1.112)834.000($750
        7784
        ----
         5560
         5560
         ----
            0
```

As the amount of \$1 is \$1.112, for every dollar in the present worth there will be \$1.112 in the amount of the present worth; consequently, for every \$1.112 in the amount there will be one dollar in the present worth, which gives \$750, as required.

340. Exercises in finding the Present Worth.

In like manner, solve and explain the following problems.

1. Suppose I owe a note of \$416, without interest; if I should pay it 4 y. 2 m. before it becomes due, what ought I to pay?

2. If I purchase property for \$1287.50 on 6 months' credit, how much cash down should pay the debt?

3. What principal would amount to \$19.16$\frac{2}{3}$ in 2 y. 6 m., at 6 per cent.?

4. If a man send you \$2050, with which, reserving your commission at 2$\frac{1}{2}$ per cent., to purchase goods for him, what should be the amount of purchases?

5. What is the par value of a share in the Boston and Lowell Railroad, which cost me \$625, at 25 per cent. advance?

6. Sold 700 barrels of flour for \$4550, gaining 4 per cent. on the cost; what was the cost per barrel?

341. Problem to find the Discount.

Discount is an allowance made to a debtor for paying a debt before it is due, the debt not being on interest. It is the difference between the debt and its present worth; it is, also, the interest of the present worth for the time. Hence,

To find the discount, from the debt, subtract its present worth; or, cast the interest upon the present worth for the time.

But, *to find the discount more conveniently*, divide the interest of the debt for the time by the amount of \$1 for the

same time; for the interest of the debt being the amount of the discount, there will be as many dollars in the discount as the amount of $1 is contained times in the interest of the debt.

That the interest of the debt *is* the amount of the discount, appears from the fact that the debt is the sum of the present worth and discount; consequently, the interest of the debt is the interest of the present worth, and the interest of the discount; but the interest of the present worth is the discount itself, which, with the interest of the discount, is the *amount* of the discount.

342. MODEL OF A RECITATION.

What is the discount of $420, due in 1 y. 6 m., without interest?

```
$1.09)420.00($385.32 = present worth.
      327                                   $420
      ---                                     .09
       930                        $1.09)37.80($34.68—dis.
       872                              327
       ---    $420.                     ---
        580    385.32 = p. worth.        510
        545   -------                    436
        ---    $34.68 = discount.        ---
         350                              740
         327                              654
         ---      $385.32 = p. worth.     ---
          230          .09                 860
          218     ---------                872
          ---     $34.6788 = discount.     ---
           12
```

By the first method the present worth is obtained, and subtracted from the debt; by the second method the present worth is obtained, and the interest cast upon that; and by the third method the interest of the debt is obtained, and divided by the amount of $1 for the time, which also gives the discount, but by fewer figures than are required by the other methods.

343. EXERCISES IN FINDING THE DISCOUNT.

In like manner, solve and explain the following problems.

1. If I pay a debt of $460, which is not on interest, 2 y. 6 m. before it is due, what discount ought to be made?

2. What discount should be made me for paying a debt of $660, which is not on interest, 1 y. 8 m. before it becomes due?

3. If there be intrusted to me $1008 to be expended in the purchase of goods, except 5 per cent. of what I do spend, which I am to retain as commission; how much may I retain?

4. A merchant sold a quantity of goods for $270, by which he gained $12\frac{1}{2}$ per cent. of the cost; how much did he gain?

5. How much more than the par value do I pay for a share in the Railroad Bank, which costs me $78.75, at 5 per cent. advance?

6. What is the difference between the interest and discount of $1060, for one year?

7. How much less than its face is a note worth, April 7th, 1846, the face being $172.66, not drawing interest, and becoming due, Nov. 13th of the same year?

344. PROBLEM TO FIND THE FACE OF A NOTE DISCOUNTED AT A BANK.

To find the face of a note discounted at a bank, when the time, rate, and avails, (**317**,) are known, divide the avails of the note by the avails of $1, on the same conditions; for there will be as many dollars in the face of the note as the avails of $1 are contained times in the avails of the proposed note.

345. MODEL OF A RECITATION.

Designing to obtain a loan of $400, for 4 months, at 6 per cent., what should be the face of the note to be offered to the bank for that purpose?

```
$1.00
  .0205
 .9795)400.00($408.37 +
         39180
           82000
           78360
            36400
            29385
             70150
             68565
              1585
```

The interest of $1, for 4 m. 3 d., is $.0205, which subtracted from $1, gives $.9795 for the avails of $1; by which divide $400, which gives $408.37 +, the face of the note; since, for every $.9795 in the *avails* of the note there will be $1 in the *face* of the note.

346. EXERCISES IN FINDING THE FACE OF A NOTE.

In like manner, solve and explain the following problems.

1. If the avails of a note, discounted at a bank for 60 days, at 7 per cent., were $1185.30, what was the face of the note?
2. What is the face of a note which, discounted for 30 days, at 6 per cent., gives for the avails, $358.02?
3. Wishing to obtain from a bank, $600 for 3 months, for what sum must I give my note?
4. If $787.60 be received at a bank for a note payable in 90 days, what is the face of that note?

347. PROBLEM FOR THE EQUATION OF PAYMENTS.

To find the mean time for the payment of several debts which become due at different times, and are not on interest till due; by each debt multiply the number of years, months, or days, as is most convenient, between the time of the calculation, and the maturity of the debt, and divide the sum of the products by the sum of the debts; the quotient will show the number of years, months, or days, to elapse before the single payment of all the debts. But if the maturity of any of the debts has already past, the sum of the products resulting from them must be subtracted from the sum of the other products, before division.

This method, though not perfectly just, in consideration of its convenience, is deemed sufficiently accurate for ordinary cases. It supposes the *interest* of what is not paid till after it has been due a certain time, is canceled by the *discount*, for an equal time, of an equal sum, before it becomes due; but the discount being *less* than the interest, the *creditor* suffers slight injustice.

The exact equated time is the time when the sum of the amount of those debts whose maturity is past, and of the present worths of those whose maturity is to come, would amount to the sum of the debts. The amounts may be found by (**312**), the present worths by (**338**), and the equated time by (**326**).

The student should solve and explain the following problems by both methods; observing, that the pay-day generally comes a little *earlier* by the latter method.

348. MODEL OF A RECITATION.

A merchant has three notes due to him, as follows; one of \$300, due in 2 months, one of \$250, due in 5 months, and one of \$180, due 3 months ago; in what time may he demand the payment of the three notes at once?

```
2 × 300 =  600 months.
5 × 250 = 1250   "
          ----
          1850   "
3 × 180 =  540   "
  ----    ----
 $730   )1310( 1 month.
          730
          ----
          580
           30
          -----
        )17400(24 days.
         1460
         -----
          2800
          2920
```

The interest of \$300 for 2 months equals the interest of \$1, 300 times as long, or 600 months; the interest of \$250 for 5 months equals the interest of \$1, 250 times as long, or 1250 months; the interest of \$180 for 3 months equals the interest of \$1, 180 times as long, or 540 months. On the first two notes the debtor is entitled to what is equal to the interest of \$1, for 1850 months; but, having already received on the third note, what is equal to the interest of \$1, for 540 months, he is now entitled to only what is equal to the interest of \$1, for 1310 months; and this is equal to the interest of 730 times as much money, or \$730, for $\frac{1}{730}$ as many months, or 1 month and a fraction; this fraction is equal to 30 times as many days, or 24 days. Hence, a single payment of all the notes may be demanded in 1 month and 24 days.

349. EXERCISES IN THE EQUATION OF PAYMENTS.

In like manner, solve and explain the following problems.

1. If a man owe me \$100, to be paid in 4 months without interest, and \$500, to be paid in 7 months, in what time may the whole be paid at once, without loss to either party?

2. A owes B, \$200, to be paid in 30 days, \$150, in 40 days, and \$450, in 80 days; what is the equated time for the payment of the whole?

3. If a merchant buy goods to the amount of \$6000, on a credit of 6 months for \$1000, of 4 months for \$1200, of 9

months for the remainder; in what time may the whole be paid at once?

4. If a person, owing $2000 payable in 7 months, pay $600 down, how long after the 7 months may he delay the payment of the remainder?

5. I hold two notes against A, to the amount of $800, one for $300, due 3 months ago, the other for $500, due 5 months hence; in what time will my claim be exactly $800?

6. C holds three notes against D, one of $175.50, due 4 months ago, one of $125.50, due 3 months ago, and the other of $560.25, due 6 months hence; in what time may D pay the three notes together?

7. Lowell, 1846. A. B. to C. D. *Dr.*

Jan. 4.	To 12 bls. flour, @ $7.50 per barrel, . .	$90.
Feb. 10.	" 42 gals. Molasses, @ $.33⅓ per gal. .	14.
Mar. 15.	" 300 lbs. Br. Havana Sugar, @ $.08⅓ per lb.	25.
Apr. 19.	" 7 bls. No. 1 Mackerel, @ $12 per barrel,	84.

When may this account be paid, so that the purchaser may have a credit that is equivalent to 60 days on each charge?

8. If a merchant purchase $200 worth of goods on 6 months' credit, would he gain or lose, by paying $100 in 3 months, and the other $100 in 9 months?

9. A gave B three notes as follows; one of $525.50, dated April 1st, 1845, due in 2 months; one of $350.75, dated April 19th, 1845, due in 4 months; and one of $637.50, dated June 13th, 1845, due in 2 months; what sum should pay these notes July 1st, 1845?

XI. ALLIGATION.

350. Problem for finding the Average Value of Ingredients forming a Compound.

Alligation is the name given to the process of *finding the average* of several ingredients forming a compound, and of *finding the quantities* of the several ingredients necessary to form a compound of a given average.

To find the average *of several ingredients forming a*

compound, by each quantity multiply its price, or rate, and divide the sum of the products by the sum of the quantities; the quotient will be the average sought, (**347**).

351. MODEL OF A RECITATION.

If a grocer mix 40 lbs. of sugar that cost $5\frac{1}{2}$ cents a lb., with 60 lbs. at $6\frac{1}{4}$ cents a lb., and 30 lbs. at $8\frac{1}{3}$ cents a lb.; what would be the average cost of the mixture per pound?

$$5\tfrac{1}{2} \times 40 = \$2.20$$
$$6\tfrac{1}{4} \times 60 = 3.75$$
$$8\tfrac{1}{3} \times 30 = 2.50$$

130lbs.)$8.45($6\frac{1}{2}$ cts.
780
65
65

At $5\frac{1}{2}$cts. a lb. 40 lbs. cost $2.20.
" $6\frac{1}{4}$ " 60 " 3.75.
" $8\frac{1}{3}$ " 30 " 2.50.
130 " 8.45.

If 130 pounds cost $8.45, each pound would cost $\frac{1}{130}$ as much, or $6\frac{1}{2}$ cents, which is the average required.

352. EXERCISES IN AVERAGING THE VALUES OF INGREDIENTS FORMING A COMPOUND.

In like manner, solve and explain the following problems.

1. If 4 lbs. of tea at $1 per lb., 8 lbs. at $.83$\frac{1}{3}$ per lb., and 6 lbs. at $.50 per lb. be mixed, what would be the average price of the mixture per pound?

2. A farmer sold a load of potatoes as follows; 6 bush. at $.33$\frac{1}{3}$ per bush., 8 bush. at $.30 per bush., 10 bush. at $.31 per bush., and 4 bush. at $.37$\frac{1}{2}$ per bush.; what was the average price per bushel?

3. If a grocer should mix 42 gals. of water with 126 gals. of wine that cost $1.50 a gal., what is the cost of a gallon of the mixture?

4. A goldsmith melted together 5 ounces of gold, 19 carats fine, 2 ounces, 22 carats fine, and 3 ounces, 23 carats fine; what was the fineness of the mixture?

Note. 24 carats make one ounce; and the quality of gold is expressed by stating the number of *carats of fine gold in an ounce;* the rest is alloy, which is some other metal considered of no value. The quality of silver is expressed by stating the number of *ounces of fine silver in a pound.*

6. If 1 lb. of silver, 8 ounces fine, 2 lbs. of 9 ounces fine, 3 lbs. of 10 ounces fine, and 4 lbs. of 11 ounces fine, be melted together, what will be the fineness of the mixture?

253. PROBLEM TO FIND THE QUANTITIES OF INGREDIENTS TO FORM A COMPOUND OF A GIVEN AVERAGE RATE.

TO FIND THE QUANTITIES *of the several ingredients necessary to form a compound of a given average rate; find the difference between the given average price and the price of a unit of each ingredient; write it with the sign* +, *or* —, *to show it an* EXCESS *above, or a* DEFICIENCY *below the given average; and take for the mixture such quantities of each ingredient as shall make the* EXCESSES EQUAL THE DEFICIENCIES; *for then the deficiencies will be just canceled by the excesses, and the rate of the compound will not differ from the given average rate.*

254. MODEL OF A RECITATION.

If a farmer should wish to mix rye at $.90 a bush., corn at $.75 a bush., wheat at $1 a bush., barley $.50 a bush., and oats at $.30 a bushel, what quantities of each kind would make a compound at $.80 a bushel?

80	90	+10	1	+10	90
	75	— 5	2	—10	150
	100	+20	4	+80	400
	50	—30	1	—50	50
	30	—50	1	—30	30
				9 bus.)	$7.20
					$.80.

In this process, we 1st, write 80, the given average; 2nd, the price of a unit of each ingredient; 3rd, the differences between these prices and the average, 90 giving an excess of 10, 75 a deficiency of 5, 100 an excess of 20, 50 a deficiency of 30, and 30 a deficiency of 50, the excesses being designated by the sign + and the deficiencies by the sign —; 4th, we write the quantities that are taken of each ingredient. To obtain these quantities, we observe, that by taking 2 bush. of corn and 1 of rye, the excess will just cancel the deficiency, each being 10 cents; and by taking 1 bush. each of barley and oats and 4 bush. of wheat, the excess will just cancel the deficiency, each being 80 cents; 5th, we note these excesses and deficiencies; 6th, we write the values of the whole quantities

mixed, they being the products of the rates by the corresponding quantities; 7th, we divide the sum of the products by the sum of the quantities for the average, which agreeing with the given average, is proof that the mixture is correct.

355. Mode of mixing Ingredients.

Such problems admit of a great variety of answers; but it is desirable to *obtain the answers in small numbers.* This may be done thus; select two differences of opposite signs, find their least common multiple, and take such quantities, at their corresponding prices, as shall cause an excess and deficiency, each equal to this common multiple; in like manner, proceed with the other differences; but should there be more differences of one sign than of the other, they may be compared with differences that already have been considered; or the sum of two or more differences of the same sign, may be compared with one of an opposite sign. Sometimes it will be convenient to compare the sum of two or more differences of one sign with two or more of the opposite sign. Indeed, *the differences may be combined in any way to make the* EXCESSES EQUAL THE DEFICIENCIES, *always being careful that as many units, at the rate corresponding with any difference, enter the compound as that difference is taken times to make the equality of the opposite differences.*

356. Exercises in mixing Ingredients to form a compound of a given average Rate.

In like manner, solve and explain the following problems.

1. What quantities of tea, at \$.50 and \$.80 a pound, may be mixed to form a compound, at \$.75 a pound?

2. What quantities of grain, at \$.50, \$.75, and \$1 per bushel, would make a mixture at \$.90 per bushel?

3. What quantities of wine, at \$1.50, \$2, and water at no value per gallon, may form a mixture, at \$1 per gallon?

4. A grocer would mix sugars at 5 cts., 7 cts., 11 cts., and 14 cts. a pound; how much of each kind may be taken that the compound may be worth 10 cts. a pound?

5. How many ounces of gold, at 15, 20, 22, and 24 carats fine, may be melted together to form a compound, at 21 carats fine?

357. Problem to mix Ingredients, some of which are limited in quantity.

When some of the ingredients are limited in quantity, *write the average prices, and differences as before* (**353**)*; and in the fourth column the quantities of the limited ingredients, and in the fifth column the excesses and deficiencies which they cause; of these add those of like signs, and cancel the less sum by a part of the greater; the rest of this greater sum cancel by the opposite differences belonging to the rates of the unlimited ingredients, and then proceed as before.*

358. Model of a Recitation.

A man has 8 ounces of silver 10 ounces fine, 10 ounces 9 ounces 14 pwt. fine, and 2 ounces 11 ounces 10 pwt. fine, which he wishes to melt with some 11 ounces 4 pwt. fine, and some fine silver, so that the compound may be 11 ounces fine; what quantities of the last two kinds may he take?

11	10	− 1	8	− 8	80
	9.7	− 1.3	10	− 13	97
	11.5	+ .5	2	+ 1	23
	11.2	+ .2	10	+ 2	112
	12	+ 1	18	+ 18	216
			48 oz.		) 528
					11

The three limited ingredients cause a deficiency of 21, and an excess of 1, which together give a deficiency of 20, of which 18 may be canceled by 18 ounces of the pure silver, and the remaining 2 by 10 ounces of that at 11.2 ounces fine.

359. Exercises in mixing Ingredients, some of which are limited.

In like manner, solve and explain the following problems.

1. How much water must be mixed with 100 gallons of molasses, at 32 cents per gallon, to reduce it to 25 cents per gallon?

2. If 10 barrels of flour, at $11 a barrel, be mixed with other kinds, at $6, $5, and $12, per barrel, how many barrels of these other kinds may be taken that the mixture may be worth $10 a barrel?

3. How much tea at \$.50, \$.62½, and \$1 a pound, may be mixed with 25 pounds at \$.72, that the mixture may be worth \$.75 a pound?

4. How many ounces of gold at 18, 19, and 21 carats fine, may be melted with 8 ounces 23½ carats fine, so that the mixture may be 20 carats fine?

360. PROBLEM TO MIX A LIMITED COMPOUND.

When the compound is limited, find a compound as before, without regard to the limitation; and, for the required compound, take such part of each ingredient in this result as this result is of the required compound.

361. MODEL OF A RECITATION.

A grocer has different kinds of wine worth \$3, \$2, and \$1.50 a gallon; how much of each of these kinds and of water may be mixed, so that the compound may fill a 20 gallon keg, and be worth \$1.75 per gallon?

\$1.75	3	+ 1.25	3	+ 3.75	6	18
	2	+ .25	2	+ .50	4	8
	1.50	− .25	3	− .75	6	9
	0	− 1.75	2	− 3.50	4	0
					20)	35
						\$1.75

2 gallons of water cause a deficiency of \$3.50, and 3 gals. of wine at \$3, an excess of \$3.50 to cancel the deficiency, and \$.25 more which, with the \$.50 excess caused by 2 gals. at \$2, is sufficient to cancel the deficiency of \$.75, caused by 3 gals. of wine at \$1.50. The mixture now consists of 10 gallons which only half fills the keg. Therefore, take twice as much of each kind, which will fill the keg at the given rate.

362. EXERCISES IN MIXING INGREDIENTS WHEN THE COMPOUND IS LIMITED.

In like manner, solve and explain the following problems.

1. How much wine at 12s. per gal. and water, would fill a cask of 20 gallons at 9s. per gallon?

2. How much sugar at 8 cts. 10 cts. and 14 cts. per pound, may make a compound of 112 pounds at 12 cents per pound?

3. Required the ingredients necessary to form 12 ounces of gold, 21 carats fine, from kinds 18½, 20, and 23 carats fine.

XII. POWERS AND ROOTS.

363. DEFINITION OF THE POWERS AND ROOTS OF NUMBERS.

The measure of a square is the product of its length by its breadth (**195**); but, as the length and breadth are equal, the measure of a square is the product of two equal factors. Hence, the product of any two equal factors may represent the contents of a square whose side is one of those factors.

One of the *two* equal factors composing a number, is called the *second root*, or *square root* of that number; and the number itself the *second power* (**49.** 40), or *square* of one of the two equal factors composing it.

One of the *three*, *four*, &c., equal factors composing a number, is the *third*, *fourth*, &c. *root* of that number; and the number itself is the *third*, *fourth*, &c. *power* of one of those three, four, &c. equal factors composing it. (**49.** 45.)

364. MODE OF INDICATING THE POWERS AND ROOTS OF NUMBERS.

The power, or root, of a number is *indicated* by an *index* placed over the number (**124**), the denominator of the index showing how many equal factors compose the number over which the index stands, and the numerator showing how many of those factors are taken for the power, or root, of that number. The indices of integral powers are considered as having 1 for a denominator. Thus,

$16^3 = 16 \times 16 \times 16 = 4096 =$ 3d power of 16.
$16^2 = 16 \times 16 = 256 =$ 2d power of 16.
$16^1 = 16 =$ 1st power, or 1st root of 16.
$16^{\frac{1}{2}} = (4 \times 4)^{\frac{1}{2}} = 4 = \frac{1}{2}$ power, or 2d root of 16.
$16^{\frac{1}{3}} = \qquad = \quad = \frac{1}{3}$ power, or 3d root of 16
$16^{\frac{1}{4}} = (2 \times 2 \times 2 \times 2)^{\frac{1}{4}} = 2 = \frac{1}{4}$ power, or 4th root of 16.
$16^{\frac{3}{4}} = (2 \times 2 \times 2 \times 2)^{\frac{3}{4}} = 2 \times 2 \times 2 = 8 = \frac{3}{4}$ power of 16.

The integral power of a number to any degree, may be obtained by taking that number as a factor as many times as there are units in the index; but the fractional powers, or the roots, of most numbers cannot be obtained so exactly as to be expressed by figures, except of 1, every power and root of which is 1, the number itself.

Those roots that cannot be obtained exactly, can be approx-

20

imated to a sufficient degree of exactness, and are called *approximate roots.*

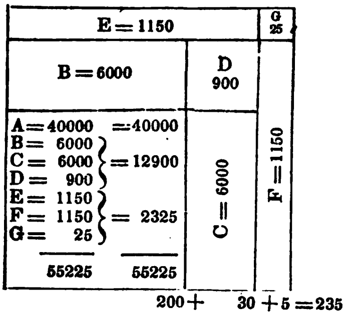
The extraction of the roots of numbers is more appropriately treated in Algebra, but the extraction of the first three roots will be explained here.

The *first root* of a number being the number itself, needs no further explanation.

365. EXTRACTION OF THE SQUARE ROOT ILLUSTRATED.

Let it be required to find the second, or square, root of 55225.

For illustration, we will consider this number as expressing the contents of a square (**363**) whose side is sought — say 55225 square feet of land are to be laid out in a perfect square, what will be the side of that square?




```
55225 (235
4
―――
152 &c.( 43
129
―――
 2325 (465
 2325
 ―――
```

We will first lay out the largest square, A, that can have its side expressed by one digit with ciphers. The second power of a number expressed by one digit can occupy only two places; and the second power of a number having one or more ciphers on the right, will have twice as many ciphers on its right (**47**). Therefore, to ascertain the largest square in this number that can have its side expressed by one digit with ciphers, beginning at the right, count off the figures into periods of two figures each, till you come to the last one, or two figures; the largest square in this figure (5) is 4, or 40000, which is the square sought; the second root of which (2, or 200,) expresses its side, and is the

first figure of the root. Subtract the contents of this square, 4 (ten-thousands,) from the 5 (ten-thousands,) leaving 15225, from which we will enlarge our square by a quantity having the greatest breadth that can be expressed by one digit in the next (tens') place; and, that it may preserve its square form, we will lay out the addition on the two sides, B and C and the corner D. The contents of this addition must be the product of the length of the three pieces, B, C, and D, by their common breadth, (**195**,) which is to be the greatest that can be expressed by one digit in the tens' place in the root. By dividing the contents of this addition by its length, we should obtain its breadth (**117**). The length of B and C, each being equal to the side of A, 2 (hundreds,) the length of both is double the side of A, 4 (hundreds;) the length of D, being equal to the breadth of the addition which is not yet known, cannot, at present, form a part of the divisor; however, as it is short compared with the length of B and C, by using the length of B and C for a *trial* divisor, and making a suitable deduction from the quotient figure, we shall at the first, or second trial, at farthest, obtain the right quotient figure, which will express the breadth of the addition, and be the second figure in the root, and also the figure that is necessary to complete the divisor, or the length of the addition.

As the divisor is hundreds, by omitting the lowest two figures of the dividend, (those not yet brought down,) the divisor and dividend would be of the same denomination, (**189**,) and, consequently, would give units for the quotient; but as tens are required, omit one more figure of the dividend, and the quotient figure will express the tens of the root. 4 (hundreds) are contained in 15 (thousands) 3 (tens) times, and so large a remainder that 3 without any deduction, probably, is the right quotient figure. Therefore, put it in the tens' place, both in the root and the divisor, and then multiply the divisor, which now expresses the length of the addition in tens, by this figure, which expresses the breadth of the addition in tens, and we obtain its contents in hundreds; which subtract from the hundreds of the dividend, or that part of the number that has been brought down, and annex to the remainder the next two figures of the number, making 2325. With this we must enlarge our square, laying out this second addition, as before, on two adjacent sides and the included corner, with the greatest breadth that can be expressed by one digit. To

obtain the breadth of this addition we divide its contents 2325, by its length, (**117**,) except the length of the small square in the corner, whose length is not yet known. The length of both E and F is double the side of the present square, that is, double the hundreds and tens of the root already found, 46 (tens;) but, in dividing, as the divisor is tens, omit the right hand figure of the dividend, and we shall obtain (**189**) units for the quotient as desired. 46 (tens) are contained in 232 (tens) 5 times and a sufficient remainder, probably, to make any deduction from this quotient figure unnecessary. Therefore, put it in the units' place both in the root and divisor, and multiply the divisor, which now expresses the length of the addition, by this figure, which expresses the breadth, and we obtain its contents; which subtracted, nothing remains. We have, therefore, laid out the 55225 square feet of land in a perfect square whose side is $200+30+5=235$ feet; consequently, 235 is the square root of 55225.

366. Observation upon the Diagram.

Observe the construction of the diagram — that it consists,

1st. Of the square, A No. 1, whose side is expressed by the first figure of the root.

2d. Of the two rectangles, B and C, and the square D, whose common breadth is expressed by the second figure of the root; and the whole composing a square, No. 2, whose side is expressed by the first two figures of the root.

3d. Of the two rectangles, E and F, and the square G, whose common breadth is expressed by the third figure of the root; and the whole composing a square, No. 3, whose side is expressed by the first three figures of the root.

4th. That, whatever be the number of figures in the root, the same manner of construction might be continued, adding on two rectangles and a square for each successive digit in the root.

367. Observation upon the Process.

Observe in the process the following particulars — That,

I. We consider a number whose second root is required, as expressing the contents of a square whose side is equal to the root sought; because, the resemblance being so complete, the square serves well for a visible illustration of the process by which the root is extracted.

II. We ascertain, and subtract from the number, the contents of a square, No. 1, whose side is expressed by the first figure of the root. This square is the greatest square contained in the first left hand figure of the number, if the number of figures is odd; otherwise, in the first two figures; because, the second power of one, or more digits having ciphers on the right, will stand at the left of as many times two ciphers as there are ciphers on the right of the digits.

III. We ascertain, and subtract the contents of an addition to square No. 1, that shall increase its side as much as must be expressed by the second figure of the root. To do this, 1st. We annex to the remainder the next two figures of the number, for a dividend; because, the digits expressing the side of square No. 2, having one less cipher on the right than the digit expressing the side of square No. 1, their second power will have two less ciphers, (**47**,) or stand lower by two places. 2d. We double the first figure of the root for a trial divisor; because, the addition is to be made on two sides of the square No. 1, each of which is expressed by this part of the root. 3d. In dividing we omit one figure on the right of the dividend, in order that the dividend may be of the right denomination (**189**) to give only the next figure of the root. 4th. We annex this figure to the divisor, (after some deduction, if necessary, on account of the deficiency of the divisor,) to complete the length of the addition. 5th. We multiply the length, thus completed, by the quotient, or second figure of the root, as it expresses the breadth of the addition, to ascertain its contents.

IV. We ascertain, and subtract, the contents of an addition to square No. 2, that shall increase its side as much as must be expressed by the third figure of the root. To do this, we proceed as we did with the addition to square No. 1, and for precisely the same reasons; annexing two more figures to the remainder for a dividend, doubling the part of the root already found for a trial divisor, omitting the right hand figure of the dividend when dividing, annexing the quotient figure, after a suitable deduction, to the preceding figures of the root, and to the divisor, and multiplying the divisor, thus completed, by this last quotient figure.

V. We repeat these steps for each succeeding figure of the root, till all are obtained. Or, if there be a remainder at last, the operation may be continued into decimals, by annexing

two ciphers to the remainder, (**10**,) or two such decimal figures as may already belong to the number; which will give tenths in the root, and two ciphers, or decimal figures, annexed to this remainder will give hundredths in the root; and so on, till the approximate root is obtained as nearly as desired.

NOTE 1. When a divisor is not contained in its dividend, we annex a cipher to the root and divisor, and proceed as usual to obtain the succeeding figure of the root.

NOTE 2. If, at any time, the remainder be as much as double the root already obtained, plus 1, the last figure of the root should have been larger by 1; for, by increasing the side of a square by 1, the increase in superficial units, as may be seen by the diagram, is a single row on two adjacent sides of the square and one to fill out the corner, or a number equal to double the side of the former square, plus 1.

368. EXERCISES IN THE EXTRACTION OF THE SQUARE ROOT.

In like manner, solve and explain the following problems.

1. What is the square root of 1024?
2. What is the second root of 729?
3. Required the second root of 59049.
4. Extract the second root of 65536.
5. What is the square root of 1048576?
6. What is the second root of 390625?
7. Required the square root of 60466176.
8. Extract the second root of 5764801.
9. What is the side of a square whose contents are 117649 feet?
10. What is the second root of 361?
11. What is the square root of 6561?
12. What is the approximate second root of 1728?
13. What is the approximate second root of 3?
14. What is the side of a square whose contents are 42.25 square inches?
15. What is the side of a square whose contents are 30.25 square feet?
16. How many rods in the side of a square acre?

369. MODE OF EXTRACTING THE SECOND ROOT BY FACTORS.

Any perfect second power being the product of two equal factors, (**363**,) and the product of any two numbers being composed of all the prime factors of both of those numbers (**117**); it follows that any second power has twice as many

prime factors as its root, or each prime factor of its root repeated. Consequently, the square root of any perfect second power is composed of half as many prime factors as the power, or one of each two equal factors of the power.

Therefore, *the square root of any perfect second power may be obtained by separating the number into its prime factors,* (**124**,) *and multiplying together one of each two equal factors.* Thus:

Let it be required to find the second root of 1764.

This number, $1764 = 49 \times 9 \times 4 = 7 \times 7 \times 3 \times 3 \times 2 \times 2$:
the second root
of which is $42 = 7 \times 3 \times 2 = 7 \times 3 \times 2$.

370. EXERCISES IN EXTRACTING THE SECOND ROOT BY FACTORS.

In like manner, solve and explain the following problems.

1. Extract the second root of 25×49.
2. Extract the second root of $49 \times 36 \times 25$.
3. Extract the second root of $2 \times 3 \times 2 \times 3 \times 5 \times 5$.
4. Extract the second root of 7056.
5. Extract the second root of 44100.
6. What is the square root of 20449?
7. What is the second root of 11664?
8. Extract the square root of 1587600.

371. MODE OF EXTRACTING THE SECOND ROOT OF FRACTIONS.

The product of two equal factors, each of which is a fraction (**149**) in its lowest terms, or the second power of such a fraction, is a fraction whose terms are the second powers of the corresponding terms of the original fraction: Thus,

$$\frac{3}{5} \times \frac{3}{5} = \frac{3^2}{5^2} = \frac{9}{25}; \text{ and } .3 \times .3 = \frac{3^2}{10^2} = .09:$$

consequently, *the second root of a fraction must be a fraction whose terms are the second roots of the corresponding terms of the original fraction.* But if both terms of a fraction, whose root is required, be not perfect powers, it will be most convenient to reduce the fraction to a decimal, and extract its approximate root; the first two figures next the decimal point giving tenths in the root, the second two, hundredths, &c.: because, the second power of a decimal fraction having twice as many decimal figures (**176**) as the fraction itself, there will be one decimal figure in the root for every two in the power.

372. EXERCISES IN EXTRACTING THE SECOND ROOTS OF FRACTIONS.

1. Extract the second root of $\frac{9}{25}$.
2. Extract the second root of .09.
3. What is the square root of $\frac{729}{117649}$?
4. What is the second root of $\frac{6561}{15625}$?
5. Required the second root of $\frac{625}{4096}$.
6. Extract the second root of .64.
7. What is the square root of 10.24?
8. Required the second root of 5.9049.
9. Extract the second root of .046656.
10. Extract the second root of 576.4801.
11. What is the approximate second root of $\frac{1}{2}$?
12. Extract the approximate second root of .9.
13. Extract the approximate second root of .016.
14. Extract the approximate second root of $\frac{3}{16}$.
15. Extract the square root of $\frac{45}{125}$.
16. Extract the approximate root of $18\frac{3}{4}$.
17. Extract the approximate root of .036.
18. Extract the approximate root of .00004.
19. How many feet in the side of a square whose contents are 272.25 sq. feet?
20. What is one of the two equal factors whose product is $69\frac{4}{9}$?
21. How many inches in the side of a square whose contents are .0625 of a foot?
22. What is that number whose second power is $156\frac{1}{4}$?
23. What is the side of that square floor which requires $30\frac{1}{4}$ square yards of carpeting?
24. What is one of the two equal factors composing $6\frac{1}{4}$?

373. MEASURE OF A CUBE, AND CUBE ROOT DEFINED.

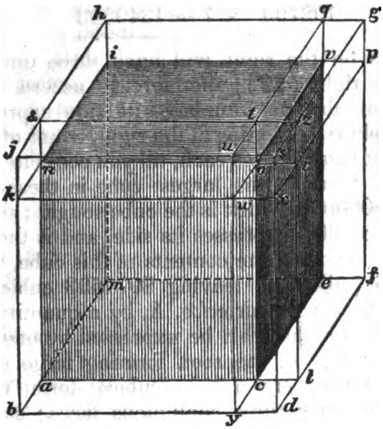
The measure of a cube is the product of its length, breadth, and height (**196**); but as the length, breadth, and height are equal, the measure of a cube is the product of three equal factors; hence, the product of any three equal factors may represent the contents of a cube whose side is one of those factors.

One of the three equal factors composing a number is called *the third root*, or *cube root* of that number, and the number itself, the *third power*, (**363**,) or *cube* of one of the three equal factors composing it.

374. Extraction of the Third or Cube Root Illustrated.

Let it be required to find the third or cube root of 16974593.

For illustration, we will consider this number as expressing the contents of a cube (373) whose side is sought; say, 16974593 cubic feet of stone are to be piled up into a perfect cube; what would be the side of that cube?



This diagram, introduced to illustrate the several steps in the process of extracting the third root, consists of eight parts.

1st. Of the part, *acem-novi*, which we will call the *large* cube No. 1, or A.

2nd. Of the three equal parts, { *acon-bywk*, *cevo-lfpr*, *novi-stqh*, } called the three *flat* pieces, { B. C. D. }

3rd. Of the three equal parts, { *wocy-xrld*, *kwon-juts* *orpv-tzgq*, } called the three *long* " { E. F. G. }

4th. Of the part, *wxro-uszt*, which we will call *small* cube No. 1, or H.

These eight parts are such that, arranged as exhibited in the diagram, they form a perfect cube, which we will call *large* cube No. 2.

We will first construct the largest cube, A, that can have its side expressed by one digit with ciphers.

```
                                   16974593(257
        2×2×2=                     8
   2×2×3=12                      )8974, &c.
   2×5×3=  30
       5×5=   25
                   1525 × 5=7625
25×25×3=1875                     )1349593
25×  7×3=    525
     7×7=       49
              192799  × 7=1349593
```

The third power of a number expressed by one digit, can occupy only three places, and the third power of a number having one or more ciphers on the right, will have three times as many ciphers on its right (**47**); therefore, to ascertain the largest cube in this number that can have its side expressed by one digit with ciphers, beginning at the right, count off the figures into periods of three figures each, till you come to the last one, two, or three figures; the largest cube in these last figures, 16, is 8, or 8000000, which is the cube sought; the third root of which, 2, or 200, expresses its side, and is the first figure in the root. Subtract the contents of this cube, 8 (millions,) from the 16 (millions,) leaving 8974593 cubic feet; from which we will enlarge our cube, A, by a quantity having the greatest thickness that can be expressed by one digit in the next (tens') place in the root, forming large cube No. 2. That our pile may preserve its cubical form, the addition must be made upon three contiguous faces; say, the front, right hand, and upper, faces; thus equally increasing the three dimensions.

These additions will be represented by the three flat pieces, B, C, and D, each of which is as long and wide as a face of A, and, consequently, has a base (**195**) equal to the second power of the side of A, or of the first figure in the root, regarding the order of that figure.. But between these flat pieces are three deficiencies to be filled by additions represented by the three long pieces, E, F, and G, each of which is as long as the side of A, and has a breadth equal to the thickness of the addition to A, and, consequently, has a base equal to the product of the first and second figures of the root, in all cases regarding the order of the figures, (**47**). These deficiencies being filled, there is still a deficiency between the three long pieces, to be filled by an addition represented by the small cube, H, which, having a length and breadth equal to the thickness of the addition to A, has a base equal to the second power of the

second figure of the root. As the several parts of the addition to A have the same thickness, their contents will be the product (**196**) of the sum of their bases by their common thickness; consequently, by dividing the contents of this addition by the sum of these bases, or the base of the addition, we should obtain its thickness, (**117**,) or the second figure of the root. The bases of B, C, and D, each being the second power of the first figure in the root, 2 (hundreds) $\times$ 2 (hundreds) $=$ 4 (ten-thousands), their sum will be three times the second power of the first figure in the root, or 12 (ten-thousands); the bases of the other parts, having the thickness of the addition for a factor, which is not yet ascertained, are unknown, and cannot, at present, form a part of the divisor; however, as they are small, compared with the bases of B, C, and D, by using the bases of B, C, and D, for a trial divisor, and making a suitable deduction from the quotient figure, we shall, at the first or second trial, at farthest, obtain the right quotient figure, or the second figure of the root. As the divisor is ten-thousands, by omitting the lowest four figures of the dividend, the three that are not brought down and one that is, the divisor and dividend would be of the same denomination, and, consequently, would give units for the quotient (**189**); but as tens are required, omit one more figure, or two that are brought down, and the quotient figure will express the tens of the root. 12 is contained 7 times in 89, but, probably, 5 is sufficiently large; therefore, put 5 in the tens' place in the root, and complete the divisor. The base of each of E, F, and G, being the product of the first and second figures of the root, 2 (hundreds) $\times$ 5 (tens) $=$ 10 (thousands), their sum will be three times this product, or 30 (thousands), which, being of a denomination one degree lower (**47**) than 12, (the bases of B, C, and D,) must stand one degree lower. The base of H, being the second power of the second figure of the root, 5 (tens) $\times$ 5 (tens) $=$ 25 (hundreds), is of a denomination one degree lower still, (**47**,) and must stand one degree lower. The sum of these bases is 1525, (hundreds,) the complete divisor, which will give no larger figure than 5 for the quotient, whence we may safely conclude that 5 is sufficiently large. Multiply this whole base, 1525 (hundreds,) by the thickness, 5, (tens,) to obtain the contents (**196**) of the addition, 7625 (thousands,) which substracted from the thousands of the dividend, (those figures already brought down,) leaves 1349593

cubic feet; which we will dispose of by increasing large cube No. 2, by an addition having the greatest thickness that can be expressed by one digit in the next (units') place in the root, forming large cube No. 3. This second addition is not represented in the diagram; but it must be made on three contiguous faces of No. 2, and their included corners, precisely in the same manner as the first addition was made to large cube No. 1, and for precisely the same reasons. Addition No. 2, will consist; 1st, of three flat pieces, each of which has a base equal to a face of large cube No. 2, which is expressed by the second power of the first two figures of the root, (**195**,) and a thickness to be expressed by the third figure of the root; 2nd, of three long pieces, each of which has a base as long as large cube No. 2, and wide as the thickness of the addition, and expressed by the product of the first two figures of the root by the third; and 3rd, of a small cube No. 2, whose side is equal to the thickness of the addition, and consequently, it has a base expressed by the second power of the third figure of the root.

As the several parts of addition No. 2 have the same thickness, their contents will be the product of the sum of their bases by their common thickness (**196**); consequently, by dividing the contents of this addition by the sum of these bases, or the base of the addition, we should obtain its thickness, (**117**,) or the third figure of the root. The bases of the three flat pieces, each being the second power of the first two figures of the root, 25 (tens) $\times$ 25 (tens) = 625 (hundreds), their sum will be three times the second power of the first two figures in the root, or 1875 (hundreds). The bases of the other parts, having the thickness of the addition for a factor, which is not yet ascertained, are unknown, and cannot, at present, form a part of the divisor; however, as they are small compared with the bases of the flat pieces, by using the sum of the bases of the three flat pieces for a trial divisor, and making a suitable deduction from the quotient figure, we shall, in one or two trials, obtain the right quotient figure, or the third figure of the root. As the divisor is hundreds, by omitting the right hand two figures of the dividend, the quotient will be units, as desired. 1875 is contained 7 times in 13495, and a sufficient remainder, probably, to make any deduction from this quotient figure unnecessary; therefore, put 7 in the units' place in the root, and complete the divisor. The base of each of the three long pieces, being the product

of the first two figures of the root by the third, 25 (tens) $\times$ 7 $=$ 175 (tens), their sum will be three times this product, or 525 (tens), which, being of a denomination one degree lower than 1875, the bases of the flat pieces, must stand one degree lower. The base of the small cube No. 2, being the second power of the third figure of the root, $7 \times 7 = 49$, is of a denomination one degree lower still, and must stand one degree lower.

The sum of these bases is 192799, the complete divisor, which gives just 7 for the quotient. Multiply this whole base, 192799, by the thickness, 7, to obtain the contents of addition No. 2, 1349593 cubic feet, which subtracted from the dividend, nothing remains.

We have, therefore, piled up all our stone, and constructed a perfect cube, (large cube No. 3,) whose side is $200 + 50 + 7 = 257$ feet; consequently, 257 is the cube root of 16974593.

375. Observation upon the Diagram.

Observe the construction of the diagram — that it consists;

1st. Of cube No. 1, whose side is expressed by the first figure of the root.

2nd. Of an addition to cube No. 1, the thickness of which is expressed by the second figure of the root: this addition consisting; 1st, of three equal flat pieces; 2nd, of three equal long pieces; and 3rd, of a small cube; the whole composing a cube, No. 2, whose side is expressed by the first two figures of the root.

3rd. That, whatever be the number of figures in the root, the same manner of construction might be continued, adding on three flat pieces, three long pieces, and a small cube, for each successive digit in the root.

Note. *The blocks themselves here described, would make a more efficient illustration.*

376. Observation upon the Process.

Observe in the process the following particulars; that,

I. We consider a number whose third root is required as expressing the contents of a cube whose side is equal to the root sought; because, the resemblance being so complete, the cube serves well for a visible illustration of the process by which the root is extracted.

II. We ascertain, and subtract from the number, the contents of a cube, No. 1, whose side is expressed by the first figure of the root. This cube is the greatest cube contained in the first one, two, or three, figures on the left of the number, according as there may be one, two, or three, figures in the left hand period when the figures are counted off from the right, as far as possible, into periods of three figures each · because, the third power of one or more digits having ciphers on the right, will stand at the left of as many times three ciphers (**47**) as there are ciphers on the right of the digits.

III. We ascertain, and subtract, the contents of an addition to cube No. 1, that shall increase its side as much as must be expressed by the second figure of the root. To do this; 1st, we annex to the remainder the next three figures of the number for a dividend; because, the digits expressing the side of cube No. 2, having one less cipher on the right than the digit expressing the side of cube No. 1, their third power will have three less ciphers, (**47**,) or stand lower by three places; 2nd, we take three times the second power of the first figure of the root for a trial divisor; because the addition is to be made on three faces of cube No. 1, each of which is expressed by the second power of this part of the root, and because the contents of a regular solid divided by its base, will give its thickness (**117**); 3rd, in dividing, we omit two figures on the right of the dividend, in order that the dividend may be of the right denomination (**189**) to give the next figure of the root; 4th, we write under the trial divisor, one degree lower, three times the product of the first and second figures of the root, and under this, one degree lower still, the second power of the second figure of the root, and add these three products together, as they stand, to obtain the complete divisor, or whole base of the addition; the second and third being written each one degree lower than its preceding number, because, each, containing a factor lower by one degree than the corresponding factor in the preceding product, is of a denomination lower by one degree than its preceding number; 5th, we multiply the base, thus completed, by the quotient, or second figure in the root, as it expresses the thickness of the addition, to ascertain its contents (**196**).

IV. We ascertain, and subtract, the contents of an addition to cube No. 2, that shall increase its side as much as must be expressed by the third figure of the root. To do this, we

proceed as we did with the addition to cube No. 1, and for precisely the same reasons; annexing three more figures to the remainder, for a dividend; taking three times the second power of the root already found, for a trial divisor; omitting the right hand two figures of the dividend, when dividing; completing the divisor by adding to it three times the product of the figure last obtained and the preceding figures of the root, and the second power of this last figure of the root; and multiplying the divisor, thus completed, by this last figure.

V. We repeat these steps for each succeeding figure in the root, till all are obtained: or, if there be a remainder at last, the operation may be continued into decimals (**10**) by annexing three ciphers, or three such decimal figures as may already belong to the number, which will give tenths in the root, and three more ciphers, or decimals, annexed to this remainder will give hundredths in the root, and so on, till the approximate root is obtained as nearly as desired.

Remark. When a divisor is not contained in its dividend, we annex a cipher to the root, and proceed as usual to obtain the succeeding figure.

277. Exercises in extracting the Cube Root.

In like manner, solve and explain the following problems.

1. What is the cube root of 512?
2. What is the third root of 729?
3. Extract the cube root of 15625.
4. Required the third root of 125000.
5. What is the third root of 531441?
6. What is that number whose third power is 262144?
7. What is one of the three equal factors which compose 46656?
8. What is the side of that cube whose contents are 117649 cubic feet?
9. What is the depth of a cubical box which would contain 17.64 bushels of wheat?
10. Extract the third root of 1953.125.
11. What is the third root of 10077696?
12. What is the third root of 40353.607?
13. Extract the third root of 134.217728.
14. Extract the third root of 387420.489.
15. What is the approximate third root of 1836?

16. Extract the approximate third root of 12.
17. Extract the third root of 3099363912.
18. What is the side of a cubic reservoir holding 847 hogs heads of water of 63 gallons each?
19. What would be the side of a ton of timber in a cubi cal form?

378. MODE OF EXTRACTING THE THIRD ROOT BY FACTORS

Any perfect third power being the product of three equal factors, (**373**,) and the product of any three numbers being composed of all their prime factors, (**117**,) it follows that any third power has three times as many prime factors as its root, or each prime factor of its root taken three times. Consequently, the cube root of any perfect third power is composed of one third as many prime factors as the power, or one of each three equal factors of the power.

Therefore, *the cube root of any perfect third power may be obtained by separating the number into its prime factors, and multiplying together one of each three equal factors.* Thus;

Let it be required to find the third root of 74088.

This number, $74088 = 343 \times 27 \times 8 = 7 \times 7 \times 7 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2$;
the third root of which is $42 = 7 \times 3 \times 2 = 7 \times 3 \times 2$.

379. EXERCISES IN EXTRACTING THE THIRD ROOT BY FACTORS.

In like manner, solve and explain the following problems.

1. Extract the third root of $8 \times 8 \times 8$.
2. Extract the third root of $3 \times 9 \times 3 \times 9 \times 3 \times 9$.
3. What is the third root of $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$?
4. What is the cube root of 216?
5. Extract the third root of 3375.
6. What is the third root of 2744?
7. Extract the cube root of 9261000.
8. Extract the third root of 15625.
9. What is the third root of 373248?
10. What is the cube root of 4741632?

380. MODE OF EXTRACTING THE THIRD ROOT OF FRACTIONS.

The product of three equal factors, each of which is a fraction in its lowest terms, or the third power of such a fraction, is a fraction whose terms are the third powers of the

corresponding terms of the original fraction (**149**). Thus, $\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}=\frac{3\times3\times3}{5\times5\times5}=\frac{3^3}{5^3}=\frac{27}{125}$; and $.3\times.3\times.3=\frac{3^3}{10^3}=$ .027; consequently, *the third root of a fraction must be a fraction whose terms are the third roots of the corresponding terms of the original fraction.* But if both terms of a fraction whose third root is required be not perfect powers, it will be most convenient to reduce the fraction to a decimal, and extract its approximate root; the first three figures next the decimal point giving tenths in the root, the second three, hundredths, &c.; because, the third power of a decimal fraction having three times as many decimal figures (**176**) as the fraction itself, there will be one decimal figure in the root for every three in the power.

381. EXERCISES IN EXTRACTING THE THIRD ROOT OF FRACTIONS.

1. What is the third root of $\frac{27}{125}$?
2. What is the cube root of .027?
3. Extract the third root of $\frac{64}{512}$.
4. Extract the third root of $\frac{8}{512}$.
5. Extract the third root of $\frac{4096}{19683}$.
6. Required the cube root of $\frac{343}{15625}$.
7. What is the cube root of $\frac{1}{27}$?
8. What is the third root of $\frac{1}{216}$?
9. What is the approximate third root of $\frac{1}{5}$?
10. What is the cube root of .003375?
11. What is the approximate cube root of 2150.4?
12. What is the approximate third root of $\frac{1}{3}$?
13. What is the approximate third root of $\frac{1}{6}$?
14. What is the cube root of $15\frac{5}{8}$?
15. What is the third root of $1953\frac{1}{8}$?
16. What is the approximate third root of $\frac{5}{11}$?
17. What is the depth of a cubical box that would contain 8 bushels?
18. What would be the side of a cubical pile of wood which should be equivalent to another pile 4 ft. wide, 6 ft. high, and $21\frac{1}{3}$ ft. long?
19. What would be the height of a cubical room equivalent to another room 25 ft. long, 15 ft. wide, and 9 ft. high?
20. The side of a cubical pile of wood containing 4 cords being 8 ft., what would be the side of a similar pile containing 8 times as much wood?

XIII. SERIES.

382. Series by Difference defined.

A *Series by Difference* is a series of numbers which progress uniformly from one extreme to the other, the same difference existing between every two adjacent terms. Thus. 5, 8, 11, 14, 17, 20; and 45, 40, 35, 30, 25, 20, 15, are series by difference.

The largest and smallest terms are called the *extremes*, the others, the *means;* and the difference between the adjacent terms, the *common difference.*

Any three of the five following particulars in a series by difference being given, the other two may be ascertained; viz. The *smallest extreme*, *largest extreme*, *number*, *common difference*, and *sum*, of the terms.

A few only of the more important problems under this head will be noticed here.

383. Problem I. To find either Extreme and the Sum of the Series.

The number of the terms, common difference, and either extreme being given, TO FIND THE OTHER EXTREME, *and* THE SUM OF THE TERMS; ADD *to the smallest extreme, or* SUBTRACT *from the largest extreme, the common difference taken as many times less one as the number of the terms,* FOR THE REQUIRED EXTREME; *and multiply the average of the extremes by the number of the terms,* FOR THE SUM *of the series.*

384. Model of a Recitation.

Let there be a series by difference composed of 20 terms, the smallest of which is 4, and the common difference 3; what is the largest extreme, and the sum of the series?

As *once* the common difference added to the smallest extreme gives the *second* term, *twice* the common difference added to the smallest extreme gives the *third* term, *three* times the common difference added gives the *fourth* term, &c., *any term*, consequently, the *largest extreme*, *is equal to the smallest extreme, plus the common difference taken as many*

times less one as the number of the terms; and, conversely, *the smallest extreme is equal to the largest extreme, minus the common difference taken as many times less one as the number of the terms.* Therefore, $4+3\times(20-1)=4+3\times 19=4+57=61$, the largest extreme; and $61-3\times(20-1)=61-57=4$, the smallest extreme.

As the series progresses uniformly from one extreme to the other, the largest extreme must exceed the average of the terms by the same quantity that the smallest extreme falls short of it; hence, the sum of the extremes will be twice the average of the terms, one half of which will be the average, which, multiplied by the number of the terms, will give the sum of the series. Therefore, $\frac{61+4}{2}\times 20=65\times 10=650$, the sum of the series.

385. EXERCISES IN FINDING AN EXTREME AND THE SUM OF THE SERIES.

In like manner, solve and explain the following problems.

1. Required the largest extreme, and sum of a series by difference composed of 25 terms, the smallest of which is 5, and the common difference 2.

2. Falling bodies fall at a rate which increases uniformly per second by a common difference. How far will a body fall in 10 seconds, falling 16 ft. the first second, and increasing uniformly by 32 feet a second?

3. What is the smallest extreme, and sum, of a series by difference composed of 51 terms, the largest of which is 500, and the common difference 5?

386. PROBLEM II. TO FIND THE COMMON DIFFERENCE AND SUM OF THE SERIES.

The extremes and number of terms being given, TO FIND THE COMMON DIFFERENCE, *and* SUM OF THE SERIES; *divide the difference of the extremes by the number of the terms less one, for the common difference; and obtain the sum by problem I.*

387. MODEL OF A RECITATION.

Let there be a series by difference composed of 32 terms, the extremes of which are 7 and 162; what is the common difference, and sum, of the series?

Since, by adding to the smallest extreme the common dif-

ference taken as many times less one as the number of terms, (**384**,) we obtain the largest extreme; the difference of the extremes must be the common difference taken as many times less one as the number of the terms. Therefore, $\frac{162-7}{31}=\frac{155}{31}=5$, the common difference; and $\frac{162+7}{2}\times 32=169\times 16=2704$, the sum of the series.

388. EXERCISES IN FINDING THE COMMON DIFFERENCE, AND SUM OF THE SERIES.

In like manner, solve and explain the following problems.

1. Required the common difference, and sum, of a series by difference composed of 36 terms, the extremes of which are 15 and 85.

2. If a body fall in the first second, 16 ft., and in the thirteenth second, 400 ft., what is the increase per second in the rate; and how far would it fall in 13 seconds?

389. PROBLEM III. TO FIND THE NUMBER OF TERMS, AND SUM OF THE SERIES.

The extremes and common difference being given, TO FIND THE NUMBER OF TERMS, *and the* SUM OF THE SERIES; *divide the difference of the extremes by the common difference, and increase the quotient by* 1, *for the number of terms; and obtain the sum by problem I.*

390. MODEL OF A RECITATION.

Let there be a series by difference in which the extremes are 10 and 100, and the common difference 6; what is the number of the terms, and the sum of the series?

Since the difference of the extremes is the common difference taken as many times less one as the number of terms (**387**); by adding 1 to the quotient obtained by dividing the difference of the extremes by the common difference, we obtain the number of the terms; therefore, $\frac{100-10}{6}+1=\frac{90}{6}+1=16$, the number of terms; and, $\frac{100+10}{2}\times 16=110\times 8=880$, the sum of the series.

391. EXERCISES IN FINDING THE NUMBER OF TERMS AND SUM OF THE SERIES.

In like manner, solve and explain the following problems.

1. Required the number of terms, and sum of a series by difference, in which the extremes are 25 and 124, and the common difference 9.

2. How many seconds was a body falling, which fell 16 ft. the first second, and 368 feet the last second, having increased its rate uniformly by 32 ft. per second?

3. What is the sum of all the numbers spoken in counting five hundred?

4. If I pay \$33.75 for house-rent at the end of each quarter for five years, what would the rent amount to at the end of the time, each payment being on interest at 6 per cent. from the time it was paid?

5. What is the sum of the series, $6\frac{1}{4}$, $12\frac{1}{2}$, $18\frac{3}{4}$, continued to 16 terms?

6. What is the sum of the series, 100, $95\frac{5}{6}$, $91\frac{2}{3}$, continued to 25 terms?

7. If the smallest extreme be a shilling, the largest extreme one dollar, and the common difference a sixpence; what would be the number of terms?

8. If there be 57 terms in a series by difference, the extremes of which are \$.25 and \$3.75, what would be the common difference?

302. Series by Quotient defined.

A series by quotient is a series of numbers which progress uniformly from one extreme to the other, the same *quotient* arising from the division of every term by its next preceding term: Thus, 96, 48, 24, 12, 6, 3; and 7, 14, 28, 56, 112, are series by quotient. The first is a *decreasing series*, in which the common quotient is $\frac{1}{2}$; the second an *increasing series*, in which the common quotient is $\frac{2}{1}$, or 2.

The first and last terms are called the *extremes*, the others the *means*, and the common quotient the *ratio* (**264**).

Any three of the five following particulars in a series by quotient being given, the other two may be ascertained, viz. *The first extreme, last extreme, number, ratio,* and *sum, of the terms.*

A few only of the more important problems under this head will be noticed here.

303. Problem I. To form a Series by Quotient.

Either extreme, the ratio, and the number of the terms, being given, TO FORM THE SERIES, MULTIPLY THE FIRST EXTREME *by the ratio for the second term, which also multiply by the ratio for the third term, &c. Or* DIVIDE THE LAST

EXTREME *by the ratio for the preceding term, which also divide by the ratio for its preceding term, &c., till all the terms are obtained.*

394. Model of a Recitation.

Let it be required to form a series of 6 terms, the first extreme being 5, and the ratio $\frac{3}{1}$, or 3.

Since the same quotient arises from the division of every term by its next preceding term, (**392**,) any term must be the product of its preceding term by the ratio. Therefore, the second term is $5 \times 3 = 15$, the third term is $5 \times 3 \times 3 = 5 \times 3^2 = 45$ (**364**); the fourth term is $5 \times 3^2 \times 3 = 5 \times 3^3 = 135$; the fifth term is $5 \times 3^3 \times 3 = 5 \times 3^4 = 405$; and the sixth term is $5 \times 3^4 \times 3 = 5 \times 3^5 = 1215$. The required series then is 5, 5×3^1, 5×3^2, 5×3^3, 5×3^4, 5×3^5; or 5, 15, 45, 135, 405, 1215.

If the last extreme, 1215, and the ratio, 3, were given, by which to form this same series, we should divide the last extreme by the ratio for the preceding term, and divide this also by the ratio for its preceding term, &c. For, any term being the product of the preceding term by the ratio, when divided by the ratio (**117**) it must give the preceding term Therefore, $\frac{1215}{3} = 405$ the 5th term; $\frac{1215}{3^2} = \frac{405}{3} = 135$ the 4th term; $\frac{1215}{3^3} = \frac{135}{3} = 45$ the 3d term; $\frac{1215}{3^4} = \frac{45}{3} = 15$ the 2d term; and $\frac{1215}{3^5} = \frac{15}{3} = 5$ the 1st term. Hence the required series is 5, 15, 45, 135, 405, 1215, as before.

395. Observation.

OBSERVE, (**394**,) THAT ANY TERM IS THE PRODUCT OF THE FIRST EXTREME AND A POWER OF THE RATIO (**364**) EQUAL TO THE NUMBER OF THE TERM *less one; also that, counting the terms backwards, any term is the quotient of the last extreme by a power of the ratio equal to the number of the term less one.*

396. Exercises in forming Series by Quotient.

In like manner, solve and explain the following problems.

1. Form a series of 7 terms, beginning with 1 and having 5 for a ratio.

2. Form a series of 5 terms, ending with 144, and having 2 for a ratio.

3. Form a series of 7 terms, the first of which shall be 81, and the ratio $\frac{1}{3}$.

4. Form a series of 8 terms, the last of which shall be 5, and the ratio $\frac{1}{2}$.

5. Form a series of 6 terms, the first extreme being 8, and the ratio $\frac{2}{3}$.

6. Form a series of 9 terms, the first extreme being 81, and the ratio $\frac{2}{3}$.

397. PROBLEM II. TO FIND EITHER EXTREME.

Either extreme, the ratio, and the number of terms, being given, TO FIND THE OTHER EXTREME; MULTIPLY *the* FIRST *extreme, or* DIVIDE *the* LAST *extreme, by a power of the ratio equal to the number of terms less one. The truth of this problem follows directly from* (**393**, **394**).

398. PROBLEM III. TO FIND ANY POWER OF THE RATIO.

TO FIND ANY POWER OF THE RATIO, *Multiply together two or more different powers of the ratio, the sum of whose indices is equal to the index of the required power. For, the index of the power shows how many times the ratio is taken as a factor* (**364**) *to compose that power, and the product of two different powers of the ratio gives a power whose index is equal to the sum of the indices of the two powers* (**117**).

399. MODEL OF A RECITATION.

Let it be required to find the last term of a series of 13 terms, the first of which is 3, and the ratio 2.

As the number of terms is 13, the 12th power of the ratio is required. The ratio is 2, and the 12th power of it is the product of $2^6 \times 2^6 = 64 \times 64 = 4096$, which, multiplied by the first term, gives $4096 \times 3 = 12288$, the thirteenth, or last term required.

400. EXERCISES IN FINDING EITHER EXTREME.

In like manner, solve and explain the following problems.

1. What is the last term in a series of 6 terms, the first of which is 5, and the ratio 3?

2. What is the 11th term of the series 7, 14, 28, &c.?

3. What is the seventh term of the series 256, 128, &c.?

4. If the 7th term of a series be 1728, the sixth term 864, &c., what would be the first term?

5. What is the tenth term of the series 5, 50, 500, &c.?

6. What is the 8th term of the series 1728, 172.8, &c.?

7. What is the last term in a series of nine terms, beginning thus; 25, 7.5, 2.25, &c.?

8. What is the sixth term in a series where the first term is $\frac{3}{390625}$, and the ratio 5.?

9. What is the eighth term in the series 27, 9, 3, &c.?

401. PROBLEM IV. TO FIND THE SUM OF THE SERIES.

Either extreme, the ratio, and the number of terms, being given, TO FIND THE SUM OF THE SERIES; *find the other extreme by problem II., then multiply the last extreme by the ratio, and divide the difference between this product and the first extreme by the difference between* 1 *and the ratio.*

402. MODEL OF A RECITATION.

Let it be required to find the sum of the series, 4, 12, 36, 108, 324, 972.

If we multiply each term in this series, or any other series by quotient, by the ratio, each product would be equal to the succeeding term of the series, (**394**,) and we should have a second series, differing from the first only in the first extreme of the first series and the last extreme of the second series, (which is the product of the last extreme of the first series by the ratio.)

Thus the several terms of the series
4, 12, 36, 108, 324, 972, multiplied by the ratio 3, gives
the series 12, 36, 108, 324, 972, 2916.

Hence, the difference between a series and the product of that series by the ratio, is the difference between the first extreme and the product of the last extreme, in the same series, by the ratio; and, as this difference is the difference between *once* the series and *as many times the series as there are units in the ratio*, if it be divided by the difference between 1 and the ratio, the quotient must be the sum of the series.

Therefore, $\frac{972\times3-4}{2}=\frac{2916-4}{2}=\frac{2912}{2}=1456$, the sum of the series required.

403. Exercises in finding the Sum of the Series.

In like manner, solve and explain the following problems.

1. What is the sum of the following series, 7, 21, 63, 189, 567, 1701, 5103?
2. What is the sum of 6 terms of the series, 5, 10, 20, &c.?
3. Find the sum of the following series, 3125, 625, 125, 25, 5.
4. If in a series, $\frac{3}{16807}$ be the first term, 7 the ratio, and 6 the number of terms, what is the sum?
5. What is the sum of 7 terms of a series, where the first term is 4, and the ratio $\frac{3}{2}$?
6. The extremes being 12500 and 4, and the ratio 5, what is the sum of the series?
7. Required the sum of a series in which the extremes are 100, and .01, and the ratio $\frac{2}{5}$.

404. Problem V. To find the Sum of Infinite Series.

To FIND THE SUM *of a series by quotient decreasing indefinitely, divide the first term by the difference between* 1 *and the ratio.*

405. Model of a Recitation.

Let it be required to find the sum of the series, 12, 3, $\frac{3}{4}$ &c.

As in a decreasing series each successive term approaches to 0, by continuing the series without limit, we can conceive of a term which, multiplied by the ratio and subtracted from the first term, would not affect the first term.

Therefore, according to problem IV., if the first term of such a series be divided by the difference between 1 and the ratio, the quotient must be the sum of the series.

Thus, 12 divided by $\frac{3}{4}$, the difference between 1 and the ratio $\frac{1}{4}$, gives $\frac{12\times4}{3}=16$, the sum of the series.

406. Exercises in summing Infinite Series.

In like manner, solve and explain the following problems.

1. What is the sum of the series, 1, $\frac{1}{3}$, $\frac{1}{9}$, &c.?
2. What is the sum of the series, .3, .03, &c., or .333 &c., or $.\dot{3}$?

3. What is the sum of the series $.1\dot{6}$?
4. What is the sum of the series $.\dot{2}\dot{7}$?
5. What is the sum of the series 100, 6, .36, &c.?
6. Required the sum of the series 9, 6, 4, &c.

407. PROBLEM VI. TO FIND COMPOUND INTEREST.

TO FIND THE AMOUNT *of any sum at Compound Interest, Find the last term of a series by quotient,* (**397**,) *in which the principal is the first term, the number of periods plus one in the time is the number of terms, and* 1 *plus the rate for one period is the ratio; and obtain the compound interest by subtracting the principal from the amount* (**310**.)

408. MODEL OF A RECITATION.

Let it be required to find the compound interest of $25, for 5 years, at 6 per cent., the interest compounded annually.

The principal, $25.	is the first term of the series.
$25 \times 1.06 = 26.50$	is the am't. for 1 yr., or the 2d term.
$25 \times 1.06^2 = 28.09$	" " " " 2 " " " 3d "
$25 \times 1.06^3 = 29.7754$	" " " " 3 " " " 4th "
$25 \times 1.06^4 = 31.561924$	" " " " 4 " " " 5th "
$25 \times 1.06^5 = 33.45563944$	" " " " 5 " " " 6th "

The principal, $25, subtracted from the amount, $33.46, gives $8.46, the compound interest required. Multiplying the principal by the rate gives the interest for one period, and multiplying by 1 gives the principal, which together give the amount (**322**) for one period; this amount becomes the principal for the second period, which, multiplied by the same factor, gives the amount for two periods, &c. Therefore, the amount of a sum at compound interest is the last term of a series by quotient, in which the principal is the first term, the number of periods plus one in the time is the number of terms, and 1 plus the rate is the ratio.

409. EXERCISES IN FINDING COMPOUND INTEREST.

In like manner, solve and explain the following problems.

1. Find the amount of $100 for 5 years, at 5 per cent., the interest compounded annually.

2. What is 6 per cent. interest of $50, compounded annually for 4 years?

3. What would $500 amount to in 3 years, at 4 per cent., interest compounded semi-annually?

4. What would $1000 amount to in 2 years, at 8 per cent., the interest compounded quarterly?

5. What would be the compound interest of $2500 for 3 years, at 10 per cent.?

410. PROBLEM VII. TO FIND COMPOUND DISCOUNT.

TO FIND THE PRESENT WORTH *of a debt when money brings compound interest, find by problem II., the first term of a series by quotient, in which the debt is the last term, the number of periods plus one in the time is the number of terms, and 1 plus the rate for one period is the ratio; and obtain the discount by subtracting the present worth from the debt,* (**341.**)

411. MODEL OF A RECITATION.

Let it be required to find the compound discount of $33.45563944, due in 5 years, without interest, when money is worth 6 per cent. compound interest.

A debt which is due at a future time, and not on interest, being the amount of the present worth of that debt for the same time, (**338,**) and the amount of any sum at compound interest being the last term of a series by quotient, (**408,**) the present worth of that sum must be the first term of that series; therefore, the present worth of a debt when money commands compound interest, is the first term of a series by quotient, in which the debt is the last term, the number of periods plus one is the number of terms, and 1 plus the rate is the ratio.

Hence, $\frac{33.45563944}{1.06^5}=\frac{33.4556394400}{1.3382255776}=\$25$, is the present worth, which subtracted from the debt, $33.46, gives $8.46, the discount required. Fewer decimals might be used with sufficient accuracy.

It may be observed, that this problem is simply the converse of the problem in the preceding model.

NOTE. *When the time is not in exact periods, in all cases where the present worth is required, divide the present worth for the exact periods by the amount of* $1 *for the odd time.*

412. EXERCISES IN FINDING COMPOUND DISCOUNT.

In like manner, solve and explain the following problems.

1. What is the present worth of $1000, due in 4 years, without interest, money being worth 6 per cent., compound interest?

2. What principal, at 10 per cent. compound interest, will amount in 4 years to $8.7846?

3. What would be the compound discount of $2500, for 3 years, at 8 per cent.?

4. What would be the compound discount on $800, for 2 years, at 6 per cent.?

413. ANNUITIES DEFINED.

An *annuity* is a sum of money due annually, quarterly, or at any regular periods.

The periodical sums are called *instalments*.

An annuity may be *limited* in time, or *perpetual*.

An annuity is in *possession* when the payments have commenced; and in *reversion* when the payments are to commence at a future time.

An annuity is *certain* when the payments are to commence and cease at specified times; and *contingent* when the payments are to commence or cease at some contingent event.

An annuity is in *arrears* from the time an instalment becomes due until it is paid.

414. PROBLEM VIII. TO FIND THE AMOUNT OF A LIMITED ANNUITY AT SIMPLE INTEREST.

TO FIND THE AMOUNT *of a limited annuity at* SIMPLE INTEREST; *to the last instalment add half of the interest of the first, and multiply the sum by the number of instalments.*

415. MODEL OF A RECITATION.

Let it be required to find the amount of an annuity to be paid in 10 instalments of $100 each.

The amounts of the several instalments form a series by *difference*, (382); thus, $100, 106, 112, &c., to 154, in

which the smallest term is the last instalment, the largest term is the amount of the first instalment, and the common difference is the interest of one instalment for one of the equal periods; since the last instalment, being paid at the time of settlement, has no interest, the preceding instalment has one year's interest, the next preceding has two years' interest, &c. The sum of this series may be obtained by problem II., in series by difference, multiplying the average of the terms by their number; but the average of the terms is obtained by adding to the last instalment one half the interest of the first; for, the average of the extremes being the average of the series, (**384**,) and the interest of the first instalment being the difference of the extremes, if half of this interest be taken from the largest extreme and added to the smallest, the extremes will be equalized; therefore, $(100 + 27) \times 10 = \$1270$ is the answer required.

416. Exercises in finding the Amount of Limited Annuities at Simple Interest.

In like manner, solve and explain the following problems.

1. What is the amount of an annuity of $250 a year, that has been in arrears for 10 years, that is, 10 years have elapsed since the first unpaid instalment became due?

2. What will a quarterly payment of $175 amount to in 5 years from the payment of the first instalment?

3. If a man's wages be $50 a month, 25 of which he regularly saves, at 4 per cent. interest, what would his savings amount to in ten years from the receipt of the first instalment?

4. If the annual rent of a house be $135, and be paid quarterly in advance, this rent, put at interest regularly for 10 years, would amount to sufficient to purchase a house of what value?

417. Problem IX. To find the Present Worth of a Limited Annuity.

To find the present worth *of a limited annuity, divide the amount as found by problem VIII., by the amount of $1 for the whole time* (**338**); *for, the amount of the annuity being the amount of its present worth, there will be as many dollars in the present worth as the amount of $1 is contained times in the amount of the annuity.*

418. Model of a Recitation.

Let it be required to find the present worth of an *annuity*, to be paid in 9 annual instalments of $125 each, money worth 6 per cent., and the annuity being 3 years in reversion.

$125	$125
.24	30
500	155
250	9
$30.00	1.66)1395.00($840.40 —
	1328
	670
	664
	600
	664

The first instalment being on interest 8 years, one half of its interest would be 24 per cent., or $30, which added to one instalment, gives $155, the average of the terms; this multiplied by 9, the number of instalments, gives $1395, the amount of the annuity; which divided by $1.66, the amount of $1 for 11 years, (it being 3 years to the first instalment, and 8 years from the first to the last instalment,) gives $840.40, the present worth of the annuity required.

419. Exercises in finding the Present Worth of Limited Annuities.

In like manner, solve and explain the following problems.

1. What is the present worth of an annuity to be paid in 7 annual instalments of $400 each, the first to be paid in one year from this time?

2. How much should be paid in advance for occupying a house 5 years, to be equivalent to $200 annual rent, the payments being made at the close of each quarter, and the rate of interest being 8 per cent.?

3. What is the present worth of an annual salary of $700 for 5 years, the instalments to be paid quarterly, the first being due at the end of 3 months?

4. What sum of ready money is equivalent to 4 instalments of an annuity of $200 a year, which is 5 years in reversion?

420. PROBLEM X. TO FIND THE AMOUNT OF A LIMITED ANNUITY AT COMPOUND INTEREST.

TO FIND THE AMOUNT *of a limited annuity at* COMPOUND INTEREST, *find the sum of a series by* QUOTIENT, (**392**,) *in which the first term is the last instalment, the last term is the amount of the first instalment, the ratio is* 1 *plus the rate for one period, and the number of terms is the number of instalments.*

421. MODEL OF A RECITATION.

Let it be required to find the amount of an annuity, at compound interest, to be paid in 5 annual instalments of $200 each.

The amounts of the several instalments form a series by quotient; thus, $200, $200 \times 1.06, 200 \times 1.06^2, 200 \times 1.06^3, 200 \times 1.06^4$, in which the first term is the last instalment, the last term is the amount of the first instalment, (**408**,) and the ratio is 1 plus the rate for one period; since the last instalment, being paid at the time of settlement, has no interest, the preceding instalment has one year's interest, the next preceding has two years' interest, &c.

The sum of this series may be obtained by problem IV.; thus,

$$\frac{200 \times 1.06^4 \times 1.06 - 200}{1.06 - 1} = \frac{200 \times 1.06^5 - 200}{.06} =$$

1127.418592, or $1127.42, is the amount required.

422. EXERCISES IN FINDING THE SUM OF LIMITED ANNUITIES.

In like manner, solve and explain the following problems.

1. What is the amount due on an annuity of $100 a year, that has been in arrears 5 years?

2. What is now due on an annuity of $333. a year, that has been in arrears 4 years, allowing 5 per cent., compound interest?

3. What will a quarterly payment of $175 amount to in 5 years from the payment of the first instalment?

4. What is the amount due on a pension of $400 a year, payable semi-annually, which has been in arrears 3 years and 6 months?

423. PROBLEM XI. TO FIND THE PRESENT WORTH OF LIMITED ANNUITIES.

TO FIND THE PRESENT WORTH *of a limited annuity at* COMPOUND INTEREST, *find the amount by problem X.; then find the present worth of that amount by problem VII.*

424. MODEL OF A RECITATION.

Let it be required to find the present worth of an annuity to be paid in 5 annual instalments of $625 each, money being 5 per cent. compound interest, and the annuity being 2 years in reversion.

$$\$\frac{625 \times 1.05^4 \times 1.05 - 625}{1.05 - 1} = \frac{625 \times 1.05^5 - 625}{.05} =$$

$3453.51953125, the amount of the annuity; and

$$\$\frac{3453.51953125}{1.05^6} = \frac{3453.51953125}{1.340095640625} = \$2577.07,$$ is the present worth required.

The annuity being 2 years in reversion, (**413**,) there are 7 terms in the series; hence, the amount of the annuity being divided by the sixth power of the ratio, will give the present worth required.

425. EXERCISES IN FINDING THE PRESENT WORTH OF LIMITED ANNUITIES.

*In like manner, solve and explain the following problems.**

1. What is the present worth of an annuity to be paid in 4 annual instalments of $500 each, the first instalment to be paid in one year from this time?

2. What must be given for an annuity of 5 annual instalments of $400 each, beginning in 2 years from this time?

3. What is the difference between 4 times $500, in one advance payment, and 5 times $500, in 5 equal annual instalments, beginning in one year, and reckoning at 10 per cent. compound interest?

426. PROBLEM XII. TO FIND THE PRESENT WORTH OF A PERPETUAL ANNUITY.

TO FIND THE PRESENT WORTH *of a perpetual annuity, divide the first instalment by the periodical rate of interest.*

427. MODEL OF A RECITATION.

Let it be required to find the present worth of a perpetual annuity of $60 a year, to begin in one year, reckoning compound interest at 6 per cent.?

The present worths of the several instalments form a series by quotient decreasing indefinitely; thus, $\frac{60}{1.06}, \frac{60}{1.06^2}, \frac{60}{1.06^3}$, &c., each term being obtained by problem VII.

The sum of this series, which is the present worth of the annuity, may be obtained by problem V.; but it will be more convenient to invert the series, and obtain the sum by problem IV.; for, the present worth of the first instalment, $\frac{60}{1.06}$, being now the last term, if it be multiplied by the ratio,

$$\$\frac{60}{1.06} \times 1.06 \div (1.06 - 1) = \frac{60}{.06} = \$1000.$$

which is now 1.06, we shall have the first instalment, $60; this divided by the difference between 1 and the ratio, (1.06 — 1) = .06, which is the periodical rate of interest, (**310**,) gives $1000, the sum of the series, or the present worth of the annuity, as required.

428. EXERCISES IN FINDING THE PRESENT WORTH OF PERPETUAL ANNUITIES.

In like manner, solve and explain the following problems.

1. What is the value of a freehold estate of which the annual rent is $100, clear of all expense?

2. What is the present worth of a perpetual annuity of $20 a month, beginning in one month?

3. What sum of money, at 5 per cent., will yield an annual interest of $600?

4. What sum of money at 6 per cent. is equivalent to an annual pension of $700?

XIV. MENSURATION OF SURFACES AND SOLIDS.

429. DEFINITIONS.

1. *A Point* is position without dimensions.

2. *A Line* has only one dimension, *length*.

3. *A straight line* does not change its direction.

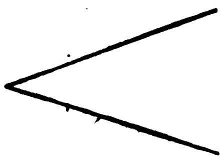
4. *A curve line* constantly changes its direction.

5. *Parallel lines* are lines equally distant in every point.

6. *A Surface* has only two dimensions, *length* and *breadth*.

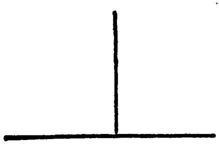
7. *A plane surface* does not change its direction.

8. *A curve surface* constantly changes its direction.



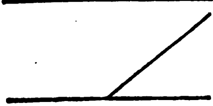
9. *An Angle* is the space comprehended between two lines that meet.

10. The angular point, or *vertex*, is the point where the lines meet.

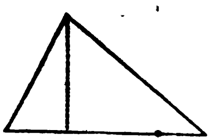


11. *Right angles* are such as are made by two lines meeting so as to form equal angles.

12. *Perpendicular lines* are such as meet at right angles.



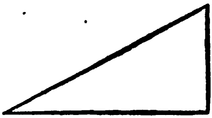
13. *Oblique angles* are *obtuse*, or *acute*, that is, greater or less than right angles.



14. *A Triangle* is a plane surface enclosed by three straight lines.

15. The *base* of a triangle is the side on which it is supposed to stand.

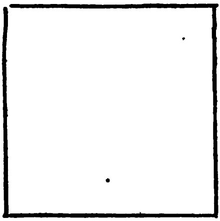
16. The *altitude* of a triangle is a line drawn from a vertex perpendicular to the base.



17. *The Hypothenuse* is the side in a triangle opposite a right angle.

18. *The square of the hypothenuse is equivalent to the sum of the squares of the other two sides.**

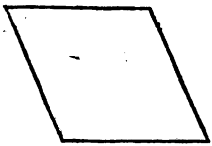
19. *The area of a triangle is half the product of the base and altitude.**



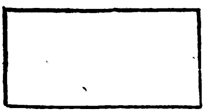
20. *A Quadrilateral* is a plane surface enclosed by four straight lines.

21. *A Parallelogram* is a quadrilateral which has its opposite sides parallel. — Among parallelograms are distinguished;

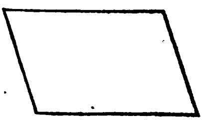
22. *The Square*, which has its sides equal and its angles right;



23. *The Rhombus*, which has its sides equal and its angles oblique;



24. *The Rectangle*, which has its opposite sides equal and its angles right;



25. *The Rhomboid*, which has its opposite sides equal and its angles oblique.

26. *The area of a parallelogram is the product of its base and altitude, which, in case of the square, is the second power of its side*, (**195**.)

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* The demonstration of this principle, and others in this article, may be found in some treatise upon *Geometry*.

Hitherto we have explained everything; but it is thought best not to attempt here an explanation of these principles; since such attempt, in most cases, must necessarily prove a failure, discouraging the scholar, and disgusting him with the whole subject of Geometry.

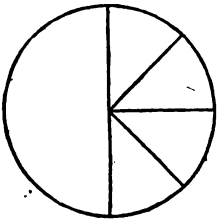
But it is hoped the student, from seeing a limited application of these important principles, will be incited *to investigate them in the proper place.*

27. *Polygon* is the common name for other plane surfaces enclosed by four or more straight lines.

28. *The Perimeter* of a polygon is the sum of the lines enclosing it.

29. *A Diagonal* is a line joining the vertices of two angles not adjacent.

30. *The area of a polygon is found by dividing it into triangles by diagonals, and measuring the triangles separately.**



31. *A Circle* is a plane surface enclosed by a line so curved that all its points are equally distant from the centre.

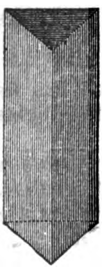
32. The *circumference* of a circle is the curved line enclosing it.

33. *A diameter* of a circle is a straight line passing through the centre and terminated by the circumference.

34. *A radius* is a straight line extending from the centre o the circumference of a circle.

35. *The circumference of a circle is the product of the diameter and* 3.1415926 +, or 3.1416 —*

36. *The area of a circle is half the product of its circumference and radius.**



37. *The area of a circle is the product of the second power of its radius and* 3.1416— *

38. *A Solid* has three dimensions, *length*, *breadth*, and *thickness*.

39. *A Prism* is a solid whose bases are equal parallel planes of any rectilinear figure, and whose convex surface is composed of parallelograms.

40. The *altitude* of a prism is the perpendicular distance between its bases.

41. *A right prism* has its sides perpendicular to its bases.

42. *The contents of a prism is the product of its base and altitude.**

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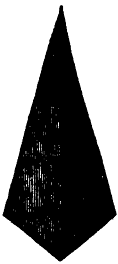
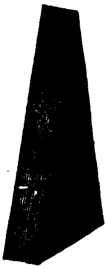
* See note on page 263.

43. *A Cube* is a prism included by six equal square faces.

44. *The contents of a cube is the product of its three dimensions, or the third power of its side*, (**196.**)

45. *A Pyramid* is a solid tapering uniformly to a point called the vertex.





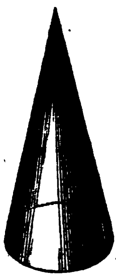
46. The *base* of a pyramid may be any rectilinear figure, and its convex surface is composed of triangles.

47. *The contents of a pyramid is one third the product of its base and altitude.**

48. *A Wedge* is a solid which has a rectangular base, two equal rectangular faces meeting in an edge, and two equal triangular faces.

49. *The contents of a wedge is one half the product of its base and altitude.**

50. *A Cylinder* is a solid of uniform thickness, whose bases are equal and parallel circles.



51. *The convex surface of a cylinder is the product of its circumference and altitude.**

52. *The contents of a cylinder is the product of its base and altitude.**

53. *A Cone* is a solid having a circle for a base, and tapers uniformly to a point called its vertex.

54. *The convex surface of a cone is half the product of the circumference of the base and the slant height.**

55. *The contents of a cone is one third the product of its base and altitude.**

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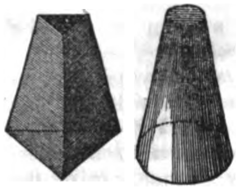
* See note on page 263



56. *A frustum* of a pyramid, or cone, is what remains when the top has been removed by a section parallel with the base.

57. *The convex surface of a frustum of a cone is half the product of the slant height and the sum of the circumferences of its bases.**

58. *The contents of a frustum of a pyramid or cone is one third the product of its altitude and the sum obtained by adding the two bases and the square root of their product, the* MEAN PROPORTIONAL *between the bases.**

59. *A Sphere* is a solid whose surface is so curved that all its points are equally distant from the centre.

60. *A diameter* of a sphere is a straight line passing through the centre and terminated by the surface.

61. *A radius* of a sphere is a straight line extending from the centre to the surface.

62. *A circumference* of a sphere is the circumference of a circle of equal diameter, called a *great circle* of the sphere.

63. *The surface of a sphere is four times the surface of its great circle.**

64. *The contents of a sphere is one third the product of its surface and radius.**

65. *Capacities of vessels having the forms of the regular solids are measured in a similar manner.*

66. The *contents of irregular solids* may be found by immersing them in a vessel of regular form, partly filled with water, and measuring the space through which the water rises.

67. *The capacity of a cask may be found like that of a cylinder of equal length and average diameter*, which is found by adding to the head diameter from .5 to .7 the difference between the head and bung diameters, according as the staves are little or much curved.

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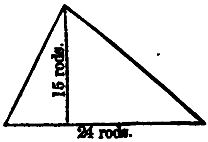
* See note on page 263.

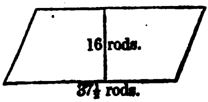
430. MODEL OF A RECITATION.



$$\frac{\overset{3}{\cancel{24}}\times\overset{3}{\cancel{15}}}{2\times\underset{4}{\cancel{160}}}=\frac{9}{8}=1\tfrac{1}{8}\text{ acres.}$$

1. What is the area of a triangle whose base is 24 rods, and altitude 15 rods?

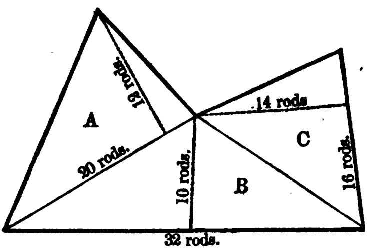
Since the area of a triangle (**429**, 19) is half the product of the base and altitude, we multiply the base by the altitude, and divide the product by 2, for the square rods, which will make $\frac{1}{160}$ as many acres, (**195**,) equal to $\frac{9}{8}$, or $1\frac{1}{8}$ acres, the answer required.



$$\frac{\overset{7.5}{\cancel{37.5}}\times\cancel{16}}{\underset{2}{\cancel{160}}}=3.75\text{ acres.}$$

2. How many acres in a parallelogram whose base is $37\frac{1}{2}$ rods, and altitude 16 rods?

Since the area of a parallelogram (**429**, 26) is the product of its base and altitude, we multiply the base by the altitude for the square rods, and there will be $\frac{1}{160}$ as many acres (**195**), equal to 3.75 acres, the answer required.



$$\frac{\overset{10}{\cancel{20}}\times\overset{3}{\cancel{12}}\times\cancel{40}}{\cancel{2}\times\underset{4}{\cancel{160}}}=\$30.=\text{A.}$$

3. What is the value of the field, the plan of which is annexed, at \$40 per acre?

By the two diagonals the field is divided into three triangles, A, B, and C, (**429** 29, 14). We multiply the base of each by its altitude and divide the products by 2, to obtain the square rods (**429**, 19); then, as there will be $\frac{1}{160}$ as many

$$\frac{\cancel{32}\times\cancel{10}\times 40}{\cancel{2}\times\cancel{160}} = \$40. = \text{B.}$$
$$\cancel{5}$$

$$2$$
$$\frac{\cancel{10}\times 14\times\cancel{40}}{\cancel{2}\times\cancel{160}} = \$28. = \text{C.}$$
$$\cancel{10} \qquad \$98.$$

acres, (**195**,) we divide by 160 to reduce the rods to acres; and at $40 per acre, there will be 40 times as many dollars.

These results added give $98 for the answer.

431. Exercises in Measuring Plane Surfaces.

In like manner, solve and explain the following problems.

1. How many acres in a common 25 rods square?

2. How many square feet in a board 21 ft. long, and 1.5 ft. wide?

3. How much would a triangular piece of land cost at $6\frac{1}{4}$ cents a foot, the base being 64 ft. and the altitude 75 ft.?

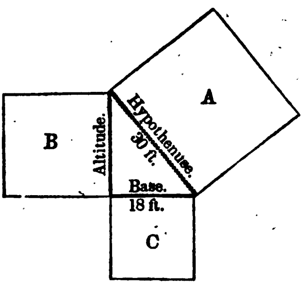
4. How much must be given for paving a yard 64 ft. long, 15 ft. 6 in. wide, at $.40 per 100 ft.?

5. What would be the cost, at 10 cents a sq. yd., of plastering the ceiling of a room 16 ft. 6 in. long, and 15 ft. wide?

6. What is the value of a lot of land, at $.0625 per foot; the lot being irregular, but divisible into four triangles, A, B, C, and D; the base and altitude of A, 130 ft., and 40 ft.; B, 160 ft., and 125 ft. 6 in.; C, 160 ft., and 62 ft. 6 in.; and D, 200 ft., and 65 ft.?

7. What would it cost to carpet a room 16 ft. 6 in. long, and 15 ft. wide, with carpeting a yard wide, at $.83$\frac{1}{3}$ per yard?

432. Model of a Recitation.



How high on the side of a house will a ladder strike which is 30 ft. long, and stands 18 ft. distant from the house?

Since the square of the hypothenuse is equivalent to the sum of the squares of the other two sides of a right-angled triangle (**429** 18), if from the square of the hypothenuse we subtract the square

$30 \times 30 = 900$, sq. of hypoth.
$18 \times 18 = 324$, sq. of base.

Sq. of altitude 576(24, altitude.
4

176(44.
176

of the base, we shall obtain the square of the altitude 576, the square root of which, 24 ft., is the answer required.

433. Exercises upon the Right-angled Triangle.

In like manner, solve and explain the following problems.

1. What is the distance between the opposite corners of a floor which is 16 ft. long, and 12 ft. wide?

2. If the distance between the opposite corners of a room be 20 ft., and the height 12 ft., how far from a corner of the floor to the opposite corner of the ceiling?

3. If the length, breadth, and height of a room be 20 ft., 15 ft., and 14 ft., respectively, what is the distance from a corner of the floor to the opposite corner of the ceiling?

4. The gable-end of a house 30 ft. wide, is 14 ft. high; what must be the length of a rafter?

5. What is the side of the largest square that can be inscribed in a circle 4 ft. in diameter?

6. How wide is a street when it is 56 ft. from one side to the eaves of a house, 25 ft. high, upon the opposite side of the street?

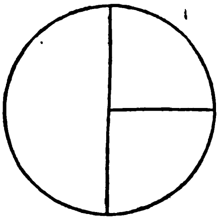
7. How far does a boy swim in crossing a river 100 yards wide, if he land on the opposite shore 60 yds. lower down than his starting point?

8. How much greater is the perimeter than the sum of the two diagonals of a rectangle 20 rods long, and 8 rods wide?

9. How much farther do I walk in crossing a bridge diagonally, than by continuing on the same side, the bridge being 160 ft. long and 40 ft. wide?

10. If in crossing this bridge I pass half way upon one side, then turn at a right angle to the other side, how much farther do I walk than I should by crossing the bridge diagonally from end to end?

434. MODEL OF A RECITATION.



What is the area of a quadrant, or quarter of a circle whose diameter is 40 rods?

Since (**429**, 35) the circumference of a circle is the product of 3.1416 — and the diameter, we multiply 3.1416 by 40, the diameter, to obtain the circumference; and since the area of a circle (**429** 36) is half the product of the circumference by the radius, we multiply the circumference by 20, the radius, and divide by 2, for the area of the circle, which divided by 4 gives the area of the quadrant in square rods, and there will be $\frac{1}{160}$ as many acres, or 1.9635 acres, the answer required.

$$\frac{\overset{.3927}{\cancel{3.1416}} \times \cancel{40} \times \overset{5}{\cancel{20}}}{\underset{8}{\cancel{160}} \times \cancel{2} \times \cancel{4}} = 1.9635 \text{ acres.}$$

This process might be abridged thus; as a quadrant only is required, we multiply 3.1416 by $\frac{1}{4}$ of the diameter 10, by removing the point one place farther to the right (**34**); and, instead of multiplying by 20 and dividing by 2, we multiply by half of radius 10, by removing the point one place farther still, giving 314.16 rods, which divided by 160 gives 1.9635 acres, as before.

$$160)\underline{314.16}$$
$$1.9635 \text{ acres.}$$

435. EXERCISES UPON THE CIRCLE.

In like manner, solve and explain the following problems.

1. What is the area of a circle whose diameter is 4 feet?
2. What is the area of the largest circle that can be made on the floor of a room which is 20 ft. long and 16 ft. wide?
3. How much land can a horse feed over while tied to a stake so that he can reach only 20 ft. from the stake?
4. If two circles have a common centre, how much larger is one than the other, their radii being 8 ft. and 4 ft. respectively?

5. How many times as large is a circle 16 ft. in diameter as another circle of one half the diameter?

6. How much larger is a square circumscribed about a circle 4 ft. in diameter than the circle itself?

436. Model of a Recitation.

1. What is the value, at $12.50 a ton, of a prismatic stick of timber whose length, or altitude, is 30 ft., and whose end, or base, is a triangle having a base of 2 ft. and an altitude of 1 ft. 6 inches?

Since the area of a triangle (**429**, 19) is half the product of its base and altitude, we multiply the base 2 by the altitude 1 ft. 6 in., or $\frac{3}{2}$ ft., and divide by 2, to obtain the base of the prism; and since the contents of a prism (**429**, 42) is the product of its base and altitude, we multiply by 30, the altitude, to obtain the cubic feet.

$$\frac{\cancel{2}\times 3\times 3\cancel{0}\times \overset{5}{\cancel{25}}}{\cancel{2}\times 2\times \cancel{50}\times 2}=\frac{45}{4}=\$11.25$$

There will be $\frac{1}{50}$ as many tons (**196**) and $12\frac{1}{2}$, or $\frac{25}{2}$ times as many dollars as tons; therefore, we divide by 50 and multiply by $\frac{25}{2}$, which gives $11.25, the answer required.

437. Exercises upon the Prism.

In like manner, solve and explain the following problems.

1. How many tons in a stick of timber 44 ft. long, 2 ft. 6 in. wide, and 2 ft. thick?

2. What are the superficial, and cubical, contents of a cube whose side is 10 ft?

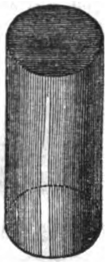
3. How many times as much are the superficial contents of a cube whose side is 6 ft., as of a cube whose side is only half as much?

4. How many times as much are the cubical contents of a cube whose side is 6 ft., as of a cube whose side is only half as much?

5. How many tons of hay may be stowed into the attic of a barn, which is 40 ft. long, 30 ft. wide, and 15 ft. high, 50 pounds occupying a cubic yard?

6. How many bricks in a chimney 35 ft. high, and 2 ft. square, with a flue 1 ft. 4 in. square, the bricks being 8 in. long, 4 in. wide, and 2 in. thick?

438. MODEL OF A RECITATION.



1. What are the cubic contents of a cylinder, whose diameter is 8 inches and altitude 36 inches?

$$\frac{3.1416\times\cancel{8}\times\cancel{2}\times\cancel{36}}{\cancel{1728}} = 1.0472 \text{ cub. ft.}$$
$$\begin{array}{cc} 3 & \cancel{48} \end{array}$$

Since (**429**, 52, 36, 35,) the contents of a cylinder is the product of its base and altitude, and the base is half the product of the circumference and radius, and the circumference is the product of 3.1416 and the diameter; we multiply 3.1416 by 8, the diameter, for the circumference, then by 2, half the radius, for the base, and then by 36, the altitude, for the cubic inches in the cylinder. There will be $\frac{1}{1728}$ as many cubic feet, which gives 1.0472 cubic feet, the answer required.

2. How many square feet in the surface of the cylinder described in the preceding problem?

$$\begin{array}{c} 1.0472 \\ \dfrac{\cancel{3.1416}\times\cancel{8}\times\cancel{2}\times 2}{\cancel{144}} \\ \begin{array}{cc} 3 & \cancel{9} \end{array} \end{array} = \frac{2.0944}{3} = .6981 \text{ sq. ft. in bases.}$$

$$\frac{3.1416\times\cancel{8}\times\overset{2}{\cancel{36}}}{\cancel{144}} = \underline{6.2832} \text{ convex surface}$$

surface of the cylinder 6.9813 square feet.

We multiply 3.1416 by 8, to obtain the circumference of the base, (**429**, 35, 36,) then by 2, half of radius, to obtain one base. In the two bases there will be twice as many square inches, and $\frac{1}{144}$ as many square feet (**195**); therefore, we multiply by 2, and divide by 144, which gives .6981 sq. ft.

The convex surface being the product of the circumference and altitude, (**429**, 51,) we multiply 3.1416 × 8, the circumference, by 36, the altitude, and divide by 144, to obtain the square feet (**195**) in the convex surface; which added to the surface of the bases gives 6.9813 sq. ft., the answer required.

439. EXERCISES UPON THE CYLINDER.

In like manner, solve and explain the following problems.

1. How many gallons of water would a cylindrical tub hold, whose diameter is 2 ft. and height 2 ft.?

2. How many more cubic feet in a stick of timber 2 ft. square and 40 ft. long, than in the greatest cylinder that could be made from the same stick?

3. What is the difference of surface in the two sticks described in the preceding problem?

4. How many gallons would an aqueduct pipe hold, the pipe being 80 rods long, and the hole 2 inches in diameter?

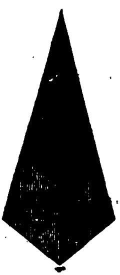
5. How many cubic feet of lead in the pipe described in the preceding problem, the thickness of the lead being .2 of an inch?

6. How many cubic feet of stone work in the stoning of a well which is 25 ft. deep and 3 ft. in diameter, the wall being one foot thick?

7. How many gallons of water would a fire engine throw in 15 minutes, from a pipe of one inch diameter, at the speed of 10 ft. a second?

8. How many gallons of water would fill the reservoir near the school-house in Belvidere, it being 10½ ft. deep, and 20 ft. in diameter?

440. MODEL OF A RECITATION.



1. What are the cubic contents of a triangular pyramid 20 ft. high, the base and altitude of the triangle which forms its base being 6 ft. and 5 ft. respectively?

$$\frac{\cancel{6}\times5\times20}{\cancel{2}\times\cancel{3}}=100 \text{ cub. ft.}$$

We multiply 6 by 5, and divide the product by 2, to obtain the base of the pyramid (**430**); and, since the contents of a pyramid is one third of the product of its base and altitude, (**429**, 47,) we multiply the base by 20, the altitude, and divide by 3, to obtain the cubic contents, which gives 100 cubic feet, the answer required.

2. What are the cubic contents of a cone, the altitude being 18 ft. and the diameter of the base 5 ft.?

$$\frac{\overset{.7854}{\cancel{3.1416}}\times5\times5\times6}{\cancel{4}}=117.81 \text{ cubic feet.}$$

We multiply 3.1416 by the diameter, 5, for the circumference of the base, (**429**, 35, 36,) which we multiply by half the radius, or $\frac{1}{4}$ of the diameter, $\frac{5}{4}$, to obtain the base; and since the contents of a cone is $\frac{1}{3}$ the product of its base and altitude, (**429**, 55,) we multiply the base by one third of the altitude, 6, to obtain the contents, which gives 117.81 cubic feet, the answer required.

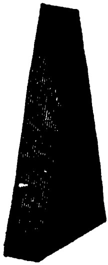
3. What is the convex surface of the cone described in the preceding problem?

$18\times18=324$ sq. of the altitude.
$\frac{5}{2}\times\frac{5}{2}=6.25$ sq. of radius of the base.

Sq. of slant height 330.25(18.17 + slant height.
1
28)230
224
361)625
361
3627)26400
25389

$$\frac{\overset{1.5708}{\cancel{3.1416}}\times5\times18.17}{\cancel{2}}=\left\{\begin{array}{l}142.71\\ \text{sq. ft.}\end{array}\right.$$

Since the altitude of the cone and the radius of the base form a right angle, and the sum of their squares is equal to the square of the slant height, (**429**, 18,) we add the squares of the altitude and radius, and extract the square root of the sum, (**367**,) to obtain the slant height. And as the convex surface of a cone is half the product of the circumference of its base by the slant height, (**429**, 54,) we multiply 3.1416 by the diameter, 5, for the circumference; then multiply by the slant height, 18.17, and divide by 2, to obtain the convex surface, which gives 142.71 square feet, the answer required.



4. How many cubic inches in an iron wedge whose base is 3 inches long and 2 inches wide, and the length 10 inches?

$$\frac{3\times\cancel{2}\times10}{\cancel{2}}=30 \text{ cub. in.}$$

Since the contents of a wedge is one half the product of its base and length, (**429**, 49,) we multiply 3 by 2 for the base, then multiply by the length, 10, and divide by 2, to obtain the cubic contents, which gives 30 cubic inches, the answer required.

441. Exercises upon the Pyramid, Cone and Wedge.

In like manner, solve and explain the following problems.

1. How may cubic feet in a pyramid whose base is 25 feet square, and height 300 feet?

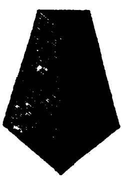
2. How many cubic feet in a conical steeple 60 ft. high, with a base 12 ft. in diameter?

3. What would it cost, at $12\frac{1}{2}$ cts. a sq. yd., to paint the steeple described in the preceding problem?

4. How many cords of wood in a tree which has a conical trunk 75 ft. high and 3 ft. in diameter at the ground, the branches affording as much wood as the trunk?

5. How many cubic inches in a wedge whose base is $2\frac{1}{2}$ inches by 4 inches, and height 1 ft. 3 inches?

442. Model of a Recitation.



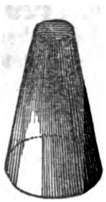
1. If the frustum of a triangular pyramid be 15 ft. high, the lower base having a base and altitude of 8 ft. and 6 ft., and the upper base a base and altitude of 5 ft. and 4 ft.; how many cubic feet in the frustum?

$8\times3=24$ lower base.
$5\times2=10$ upper base.
$(24\times10)^{\frac{1}{2}}=15.5$ — mean proportional.

$49.5\times5=247.5$ cub. ft

Since the contents of the frustum of a pyramid is $\frac{1}{3}$ the product of the altitude of the frustum, and a sum obtained by adding the two bases and their mean proportional, (**429**, 58,) we multiply the base, 8, by half the altitude, 3, of the lower base;

and the base, 5, by half the altitude, 2, of the upper base, to obtain the two bases of the frustum (**430**); then we multiply the two bases together, and extract the square root (**367**) of their product, for their mean proportional. These added, and their sum multiplied by one third of the altitude, 5, gives 247.5 cubic feet, the answer required.



2. How many gallons in a conical firkin, which is 16 inches in diameter at the bottom, 12 inches in diameter at the top, and 3 ft. deep?

Since the contents of the frustum of a cone is $\frac{1}{3}$ the product of the altitude of the frustum, and a sum obtained by adding the two bases and their mean proportional (**429**, 58); we multiply 3.1416 by the diameter, 16, then by half of radius, 4, for the lower base; also, we multiply 3.1416 by the diameter, 12, then by half of radius, 3, for the upper base (**429**, **36**); but the mean proportional between these bases is the square root of their product, which we obtain by multiplying together one of each two equal factors (**370**) in the product; 3.1416 occurring twice, 4 twice in 16, 2 twice in 4, and 6 twice in $12 \times 3 = 36$. The lower base is $16 \times 4 = 64$ times, the upper base $12 \times 3 = 36$ times, and the mean proportional $4 \times 2 \times 6 = 48$ times 3.1416, which added gives 148 times 3.1416; this multiplied by the number of inches in $\frac{1}{3}$ of the altitude, 12, gives the cubic inches; and there will be $\frac{1}{231}$ as many gallons, (**198**,) which is 24.1516 gallons, the answer required.

$$3.1416 \times 16 \times 4 = \text{lower base.}$$
$$3.1416 \times 12 \times 3 = \text{upper base.}$$
$$3.1416 \times 4 \times 2 \times 6 = \text{mean prop.}$$
$$\frac{3.1416 \times 148 \times 12}{231} = 24.1536 \text{ gals.}$$

443. Exercises upon the Frustums of Pyramids and Cones.

In like manner, solve and explain the following problems.

1. How many cubic feet in a monument 25 ft. square at the bottom, 12 ft. square at the top, and 200 feet high?

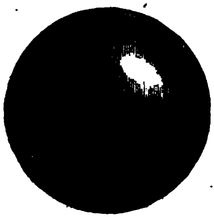
2. How many gallons in a tub 1 ft. 8 in. diameter at the bottom, 2 ft. diameter at the top, and 1 ft. 6 in. deep?

3. How many square inches of tin in a pail, including the bottom, (429, 57,) which is 8 inches in diameter, the top being 10 in. in diameter, and the depth 12 inches?

4. How many cubic feet of 3 inch plank in a round cistern, not including the top and bottom, the diameter at the bottom being 3 ft., at the top 4 ft., and the depth 8 ft., the dimensions taken on the outside?

5. How many cubic feet in a log 3 ft. in diameter at one end, 2 ft. at the other, and 30 ft. long?

444. Model of a Recitation.



What are the cubic contents of a sphere whose diameter is 12 inches?

$3.1416 \times 12 \times 3 \times 4 \times 2 =$ $904.78 +$ cubic inches.

We first multiply 3.1416 by the diameter, 12, for the circumference, (429, 35, 62); then multiply by half of radius, 3, for the surface of a great circle (429, 37); that product by 4, for the surface of the sphere (429, 63); which multiplied by one third of radius, 2, gives the contents, 904.78 cubic inches, the answer required.

445. Exercises upon the Sphere.

In like manner, solve and explain the following problems.

1. How many gallons of water would a hollow sphere 18 inches in diameter hold?

2. What would it cost, at $.75 a square foot, to gild a sphere 2 ft. in diameter?

3. What weight of brass .4 of an inch thick, at 4 oz. 14 dr. per cubic inch, would make a hollow sphere 3 feet in diameter?

4. How many gallons would a hollow sphere, 43.9824 inches in circumference, hold, if the shell be one inch thick?

446. Model of a Recitation.

How many wine gallons will a cask hold whose head and bung diameters are 20 and 24 inches, and length 3 feet?

$$24 - 20 = 4$$
$$4 \times .7 = 2.8$$
$$20 + 2.8 = 22.8$$
$$\frac{22.8 \times 3.1416 \times 5.7 \times 36}{231} = 63.62 \text{ galls.}$$

The difference between the head and bung diameters is 4 inches, .7 of which is 2.8; this added to the head diameter gives the average diameter, 22.8 inches, (**429**, 67.) We multiply 3.1416 by 22.8, the diameter, for the circumference; then by 5.7, half of radius, for the surface of the end; and that product by 36, the inches in the length, for the cubic inches (**429**, 35, 36, 52); and there will be $\frac{1}{231}$ as many gallons, which gives 63.62 gallons, the answer required.

447. Exercises in the Gauging of Casks.

In like manner, solve and explain the following problems.

1. How many gallons of water would a cask hold, whose head and bung diameters are 18 and 24 inches, and length 30 inches?
2. How many bushels of wheat would a hogshead hold whose end, and middle diameters are 40, and 50 inches, and depth 60 inches?
3. How many gallons of milk would a keg hold, whose head and bung diameters are 7 and 8 inches, and length 15 inches?

XV. REVIEW.

448. Exercises in reviewing Fractions.

1. Add $\frac{3}{14}$ and $\frac{5}{21}$.
2. Add $\frac{7}{8}$, $\frac{5}{16}$ and $\frac{9}{24}$.
3. Subtract $\frac{9}{35}$ from $\frac{25}{42}$.
4. Subtract $\frac{18}{72}$ from $\frac{11}{28}$.
5. Multiply $\frac{9}{25}$ by $\frac{2}{3}$.
6. Multiply $\frac{5}{28}$ by $\frac{4}{7}$.
7. Divide $\frac{16}{35}$ by $\frac{4}{7}$.
8. Divide $\frac{7}{60}$ by $\frac{5}{12}$.
9. What is the sum of two numbers, the least of which is $7\frac{5}{18}$, and their difference $5\frac{7}{12}$?
10. What fraction is that, to which if you add $\frac{5}{12}$ the sum will be $\frac{41}{48}$?
11. What number is that, from which if you take $\frac{7}{20}$ the remainder will be $\frac{17}{30}$?
12. How much greater is the sum of $2\frac{5}{14}$ and $9\frac{8}{21}$ than their difference?

13. Reduce $\frac{12\frac{1}{2}}{20\frac{1}{4}}$ and $\frac{1}{1\frac{1}{5}}$ to their least common denominator.

14. What number is that, to which if $\frac{2}{7}$ and $\frac{5}{8}$ be added the sum will be 1?

15. What is the value of $19\frac{7}{8}$ barrels of flour at \$$6\frac{3}{8}$ a barrel?

16. What cost $\frac{7}{8}$ of a cord of wood at \$5.75 a cord?

17. What will $9\frac{3}{4}$ tons of hay come to at \$$11\frac{3}{8}$ a ton?

18. If $\frac{4}{5}$ of a ship cost \$12580, what is the whole ship worth?

19. If $\frac{7}{8}$ of a yard cost \$$\frac{7}{12}$ what is the cost of 1 yard?

20. What number is that, which, being increased by $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{5}{6}$ of itself, becomes $234\frac{3}{4}$?

21. What number must be multiplied by $3\frac{7}{8}$, and the product divided by 7.365, to give $8\frac{3}{4}$?

22. Sold one dozen of chickens at \$.375 each, 9 pounds of butter at \$.16$\frac{2}{3}$ each, 3 dozen eggs at \$.125 a dozen, and 7 quarts of berries at \$.04$\frac{1}{6}$ each; what did they come to?

23. If 16.25 barrels of apples cost \$40.625, what is that a barrel?

24. If $77\frac{3}{4}$ yards cost \$26.435, what will one yard cost?

25. Bought a piece of cloth containing 85 yards for \$191.25, and sold it at \$2.81 a yard; how much did I gain?

449. EXERCISES IN REVIEWING COMPOUND NUMBERS.

1. Reduce 11 h. 6 m. to the fraction of a day.

2. Reduce $\frac{37}{80}$ to a decimal fraction.

3. If .72 of a ton of lead cost \$50, what would a ton cost?

4. What is the difference between 10 miles, 5 fur. 16 rods, and 25 rods?

5. Subtract 27 m. 7 f. 39 r. 5 yd. 1 ft. 1.9 in. from 28 m.

6. How many raisins, at $8\frac{1}{2}$ cents a pound, must be given for 163 gals. 2 qts. of wine, at \$1.12$\frac{1}{2}$ a gallon?

7. At £1 11s. a yard, what would be the price of 3 qr. 3 na. of cloth?

8. How many square feet in a surface whose length is 5 ft. 6 in. and breadth 1 ft. 8 in.?

9. How many cords of wood in a pile 20 ft. 6 in. long, 4 ft. wide, and 6 ft. 3 in. high?

10. What is the value, at \$5.50 a cord, of a load of wood measuring 12 ft. in length, 3 ft. 6 in. in height, and 3 ft. 8 in. in breadth?

11. At $\$12\frac{1}{2}$ for 56 gals. 3 qts. of molasses, what is the cost per gallon?

12. How many pairs of shoes, at 1s. 6d., may be made for £1 1s.?

13. How much would 25 yards of cloth come to at 1s. 6d. per yard?

14. Reduce $.16\frac{2}{3}$ to a common fraction.

15. What decimal fraction is equivalent to $\frac{103}{85}$?

16. If the difference between the longitude of Boston and Cincinnati be 13° 30′, what time will it be at C. when it is noon at B.?

17. If a stone be put into a vessel whose capacity is 14 wine quarts, and besides the stone it requires $2\frac{1}{2}$ qts. of water to fill the vessel, what is the contents of the stone?

18. What would 78455 bricks cost at $8.25 per thousand?

19. What would 24750 ft. of boards cost at $18.75 per thousand?

20. If an ounce of silver be worth $1.10, what is the value of 10 spoons, each weighing 1 oz. 4 pwt?

21. If the snow be uniformly 6 inches deep, how many cubic ft. on an acre of land?

22. If a piece of land 40 rods in length and 4 in breadth make an acre, how wide must it be when it is but 25 rods long?

23. Boston being situated about 6° 40′ E. from Washington, has what o'clock when it is 2 o'clock at Washington?

24. If sound move 1142 ft. in a second, and 12 seconds elapse between perceiving the lightning and the thunder, at what distance was the explosion?

25. How many minutes of a degree does the earth turn in a minute of time?

450. Exercises in reviewing Proportion.

1. When a post $11\frac{1}{4}$ ft. high casts a shadow $7\frac{3}{4}$ ft., what must be the height of that tree whose shadow is 62 feet?

2. What will be the price of $37\frac{7}{8}$ bushels of salt, if $13\frac{5}{8}$ bushels cost $11.75?

3. If $29\frac{1}{8}$ cwt. of beef cost $87.50, how many cwt. may be bought for $31.75?

4. How many men, in 6 months, could build a wall that 36 men would build in 8 months?

5. If $10\frac{3}{5}$ bushels of grain supply 11 horses 5.5 days, how long will 16 bushels supply 9 horses?

6. If the rent of 19 acres 3 roods of land is £4 10s. what would be the rent of $24\frac{1}{2}$ acres?

7. If a cistern discharge $83\frac{1}{3}$ gallons of water in 1.3 hours, how much will it discharge in $6\frac{5}{7}$ hours?

8. If 93 barrels of flour cost \$481.50, what will 465 barrels cost?

9. If 7 men reap 84 acres of wheat in 24 days, how many men would reap 96 acres in 16 days?

10. How many day's travel at 6 hours a day, will be required to perform a journey of 1250 miles, if 900 miles may be traveled in 12 days, at 9 hours per day?

11. If a coat and vest be made from 3 yds. 3 qrs. of cloth that is $1\frac{1}{2}$ yds. wide, how much will it require to make the same from cloth only 2 qrs. 3 nails wide?

12. If $3\frac{1}{3}$ tons are carried $87\frac{1}{2}$ miles for \$$15\frac{1}{3}$, how far may $2\frac{7}{8}$ tons be carried for \$27.75?

13. If in 9 days 24 men make 80 rods of road, in how many days will 50 men make $\frac{5}{8}$ of a mile?

14. A pile of wood 60 ft. long, 10 ft. high, and 4 ft. wide, costing \$$238\frac{1}{3}$, what would be the price of a pile 20 ft. long, 12 ft. high, and 4 ft. wide?

15. How many yards of flannel $\frac{3}{4}$ of a yard wide, will line $3\frac{1}{2}$ yds. of broadcloth that is $1\frac{1}{2}$ yards wide?

16. How many oxen can be pastured $13\frac{3}{5}$ weeks for \$$43\frac{1}{2}$, if it cost \$266.50 to pasture 26 oxen $6\frac{5}{7}$ weeks?

17. If 20 men perform a piece of work in 12 days, how many men would perform another three times as large in $\frac{1}{4}$ of a day?

18. If 2500 slates, each 8 in. long, 5 in. wide, cover a roof, how many would be required that are 6 in. long, and 4 in. wide?

19. If 180 men, in 6 days, of 10 hours each, can dig a trench 200 yds. long, 3 wide, and 2 deep, in how many days, of 8 hours each, would 100 men dig a trench 360 yards long, 4 wide, and 3 deep?

20. If 12 oxen eat $7\frac{3}{5}$ tons of hay in 22 weeks, how many oxen would eat $13\frac{3}{5}$ tons in $43\frac{2}{7}$ weeks?

21. How many stones 10 in. long, 9 broad, and 4 thick, would be needed in a wall 80 ft. long, 20 high, and $2\frac{1}{4}$ thick?

22. If a cellar 22.5 ft. long, 17.3 ft. wide, and 10.25 ft. deep, be dug in 2.5 days, by 6 men, working 12.3 hours a day, in how many days of 8.1 hours, would 9 men dig a cellar 56.25 ft. long, 27.9 ft. wide, and 12.5 ft. deep?

23. How many days of 8½ hours, will 42 men require to build a wall 98.75 ft. long, 7.5 ft. high, and 2.75 ft. thick, if 63 men would build a wall 45.375 ft. long, 6.125 ft. high, and 3.25 thick, in 68 days of 11⅓ hours?

24. If 2½ tons of merchandise be carried ¼ of 96⅓ miles for ⅓ of $11.25, what would be the freight of 1½ cwt. for 7⅓ miles less than 72⅔ miles?

25. A and B speculated; A furnished $120, and took ⅗ of the gain; what must B have furnished?

26. A bankrupt is indebted to A, $780, to B, $460, and to C, $760; his estate is worth $600; how should it be divided?

27. A, B and C traded in company; A put in $750, B, $400, and C, a sum unknown; they gained $640, of which C took $180 for his share; what did C put in?

28. Two men hired a pasture for $27; A put in a number of horses for 4 months, B put in 5 horses for 3 months, and paid ⅝ of the rent; how many horses did A put in?

29. Three partners gained $1230, of which A received $400, B $350, and C $480; A's stock was in 6 months, B's 5 months, and C's stock, which was $1800, was in trade 8 months. Required the stock of A and B.

30. How much land, at $25 per acre, must be given in exchange for 360 acres, at $37.50 per acre?

31. If eggs be bought for 12½ cts., and sold for 15 cts. per dozen, to gain in the same ratio how should wood be sold that cost $4.20 per cord?

32. If flour be bought for $5.62½, and sold for $6 per barrel, how should molasses be sold that cost 25 cents per gallon?

451. Exercises in reviewing Percentage.

1. How much shall I receive for my real estate, the commission being $318, at 3⅓ per cent. on the price?

2. A draper sold cloth at 12½ cts. per yard less than it cost him, losing 2 per cent. of the cost; how much was the cost of 25 yards?

3. Bought 39½ reams of paper for $162.93¾; for how much per ream must I sell it to gain 33⅓ per cent. of the cost?

4. Bought in Jamaica, 17 hogsheads of molasses, each

holding 118 gals. 2 qts., at 15 cents a gallon; paid for freight $1.25 per hogshead, for insurance $8.06, and for duties $35.25, the leakage being 2 per cent.; at what price per gallon must I sell it to gain 25 per cent.?

5. What must I pay for 25 shares of bank stock, at an advance of $7\frac{3}{8}$ per cent., the par value being $250 per share?

6. The par value of bank stock being $100, how much might be received for 31 shares, at $9\frac{1}{4}$ per cent. discount?

7. What premium must I pay for the insurance of $\frac{3}{4}$ of the value of my house against loss by fire, at the rate of $\frac{1}{2}$ per cent., the house valued at $1750?

8. What is the value of that property, the premium for insuring which takes $65.30, at $\frac{3}{8}$ per cent.?

9. When taxes are rated at $\frac{1}{4}$ per cent., what amount should be paid by a man whose property is valued at $5750, and who pays for 3 polls, at $1.25?

10. The poll-tax being $.75, and the property tax $.45, on every $100, how much should I pay for $4675 real estate, $963 personal property, and 2 polls?

11. What is my property, if I pay $10.75 taxes, at $\frac{3}{4}$ per cent., including one poll, at $1.25?

12. What is the duty on 1560 lbs. of chocolate, tare 10 per cent., and duty 4 cts. a pound?

13. What is the duty on 4 casks of wine, gauging severally, 118 gals. 2 qts., 126 gals. 1 qt., 97 gals. 3 qts., and 117 gals. 2 qts., at 35 cts. a gallon?

14. What is the interest on $942.50, from Feb. 11th, 1842, to Dec. 3d, 1844, at 6 per cent.?

15. What is the amount of $281.50 for 3 y. 7 m. 18 d., at $5\frac{1}{2}$ per cent.?

16. What is the amount of $2500, from Sept. 30, 1839, to March 27, 1848, at $4\frac{1}{2}$ per cent.?

17. What is due July 15, 1846, on the following note?

$765.50.

Lowell, July 4th, 1841.

For value received, I promise to pay John Tyler, or order, seven hundred sixty-five dollars and fifty cents, on demand, with interest at 5 per cent. ANDREW JACKSON.

Endorsements.

Sep. 5, 1842,	received	$50.	Aug. 18, 1844,	received	$60.25.
June 10, 1843,	"	75.	July 4, 1845,	"	212.50.

18. John Smith gave a note for $2000 to the bank, payable in 60 days; what were the avails of the note?

19. How much can I receive at a bank on a note payable in 90 days, for $639.50?

20. What is the difference between the simple and compound interest of $1000 for 3 y. 9 m. 12 d.?

21. To what would a note for $631, dated June 17th, 1844, amount, Aug. 23d, 1847, the interest compounded semi-annually?

22. In what time will $700 amount to $1000?

23. The interest on a note of $564.10, at 5 per cent., was $75.15; what was the time?

24. In what time will the interest on any principal, at $3\frac{1}{2}$ per cent., be equal to the principal?

25. What is the rate of interest when $625 amounts to $875 in 8 ys.?

26. At what rate per cent. will $700 amount to $1300.60 in 11 ys?

27. At what rate will $913 amount to 1095.50 in 1 y. 3 m.?

28. What principal will gain $49.875 in 1 year 9 m.?

29. A merchant sold a quantity of goods for $983, by which he lost 12 per cent. of the cost; what was the cost?

30. What sum of money will gain $12.50 in 1 y. 4 m. 12 d., at 3 per cent?

31. A merchant sold a quantity of goods for $243, by which he gained $\frac{1}{8}$ of the cost; how much did they cost?

32. Sold a quantity of flour for $147, by which I gained $\frac{3}{4}$ of the cost; what was the cost?

33. Sold goods for $187, by which I lost $\frac{1}{4}$ of their cost; what was the cost?

34. Sold goods so as to gain $\frac{2}{7}$ of the cost, or $43; what was the cost?

35. How much do I gain by purchasing molasses for $275 on 6 months credit, and selling it immediately for the same, cash down?

36. What is the present worth of a note at 30 days, for $2072.25, discounting at 5 per cent.?

37. How much less than its face should a note of $840, payable in 4 months, be sold?

38. What is the bank discount of a note of $540, at 90 days?

39. Bought goods for $874.90, and hired the money to pay for them, at the rate of 8 per cent. a year, and having kept the goods on hand 3 months and 18 days, I sold them for $950, on 4 months' credit; how much did I gain, reckoning at the time of sale?

40. A owes B $400, of which $100 is to be paid in 6 months, $150 in 8 months, and the remainder in 12 months; in what time may one payment of the whole be made?

41. A owes B $800, due May 16th, and B owes A $600, due the 1st of the same month; when may the accounts be settled by A paying B $200?

42. If $\frac{1}{6}$ of a sum of money be due in 2 months, $\frac{1}{3}$ in 3 months, and the rest in 6 months, in what time may all be paid at once?

43. If $950 be due me 2 months ago, $450 in 3 months hence, and $875 in 6 months, what sum would pay all in 1 month hence?

44. If I owe $1000, to be paid in 6 months, but pay $350 down and $350 more in 3 months, when would $300 settle the account?

452. Exercises in reviewing Alligation.

1. If a farmer mix 10 bushels of wheat at $1.25 per bushel, 18 bushels of rye at 0.83\frac{1}{3}$ per bushel, and 20 bushels of barley at 0.62\frac{1}{2}$ per bushel, what would the mixture be worth per bushel?

2. Mix ingredients at 12$\frac{1}{2}$, 15, 24, and 30 cents a pound, that the average price may be 25 cents a pound.

3. Mix 32 pounds of ingredients at 40, 51, and 62$\frac{1}{2}$ cents a pound, that the average price may be 54 cents.

4. Fill a hogshead of 63 gallons with ingredients at 18$\frac{3}{4}$, 37$\frac{1}{2}$, and 50 cents a gallon, and averaging 31$\frac{1}{4}$ cents per gallon.

453. Exercises in reviewing Powers and Roots.

1. What number multiplied into itself will produce 42$\frac{1}{4}$?

2. What is the length of one side of a square field containing 15 acres and 1 rod?

3. What is the side of a square field equivalent to two other fields, one containing 40, and the other 50 acres?

4. What is the square of that number, which, if multiplied by $\frac{2}{3}$ of $\frac{4}{5}$ of 1$\frac{1}{2}$, will produce 1?

5. If three times the square of my age you multiply by 5 times my age, and obtain 491520, what is my age?

6. What is the side of a cubical pile of wood that contains 256 cords?

7. What is the surface of one face of a cubic block, which contains 110592 cubic feet?

8. What is the cube root of $8 \times 27 \times 64 \times 343$?

454. EXERCISES IN REVIEWING SERIES.

1. What would be the compound interest of $1163 for 5 years, at 5 per cent.?

2. What principal at 10 per cent. compound interest will amount in 4 years to $8.7846?

3. At 5 per cent. compound interest, in what time will $40 amount to $68.41?

4. What ready money will purchase the reversion of a lease of $60 per annum, the first instalment to be paid at the end of 3 years, reckoning the interest at 6 per cent., compounded annually?

5. What is the present worth of an annuity of $100 a year, for 6 years, the first instalment to be paid in 2 years, and reckoning simple interest at 4 per cent.?

6. What sum of money must a man lay up annually to amount to $10000.50, in 20 years, the investments being made at the end of each year, at 5 per cent., simple interest?

7. What sum will build a fence worth $1000, and renew it every 15 years, reckoning compound interest at 5 per cent.?

455. EXERCISES IN REVIEWING MENSURATION.

1. What is the value, at 10 cts. a square foot, of a quadrilateral piece of land, having two sides parallel and 100 ft. distant, one 40 ft. long, and the other 60 ft. long?

2. What is the value, at $8\frac{1}{3}$ cts. a sq. foot, of a lot of land in the form of a right-angled triangle, the base being 15 yds., and the hypothenuse 25 yards?

3. What is the value, at $6\frac{1}{4}$ cts. a sq. foot, of a walk 6 ft. wide, round a circular pond 30 ft. in diameter?

4. If I purchase a load of wood, which is 8 ft. long, $4\frac{1}{2}$ ft. wide, and $6\frac{1}{2}$ ft. high, at $$5\frac{1}{2}$ per cord, and pay for sawing and splitting, 2 shillings a cut per cord, the splitting reckoned as one cut, each stick 4 ft. long, and cut into sticks a foot long; what does the wood cost me?

5. How many cubic feet in a hexagonal prism 3 ft. long, whose base may be divided into six equal triangles, each with a base of 8 inches, and an altitude of 6.9282 inches?

6. How many cubic feet of copper in two lines of wire $\frac{1}{5}$ of an inch in diameter, extending from Boston to Liverpool, a distance of 3000 miles?

7. How many cubic feet of wood is taken from a log 30 ft. long, in boring it for a pump with an auger 4 inches in diameter?

8. How many cubic inches in a pentagonal pyramid 2 ft. high, whose base may be divided into three triangles, the first with a base of 8, and an altitude of 4 inches, the second a base of 6, and altitude of 9 inches, and the third a base of 5, and altitude of 10 inches?

9. How many square yards of tin would cover a conical steeple 51 ft. high, and 16 ft. in diameter at the base?

10. How many bricks, 8 in. long, 4 in. wide, and 2 in. thick, in a chimney 100 ft. high, and 10 ft. square on the outside, at the bottom, 3 ft. square at the top, and the walls 8 inches thick?

11. How many bricks, 8 in. long, 4 in. wide, and 2 in. thick, in a conical chimney, 96 ft. high, 31.416 ft. in circumference at the bottom, 15.708 ft. at the top, and the wall 1 ft thick?

12. If the earth be considered a sphere 8000 miles in diameter, how many times as many cubic miles in the outside to the depth of 1000 miles as in all the remainder?

456. Exercises upon Curious Problems.

1. If now a boy's age is double his brother's, but in 9 years it will be only $\frac{5}{4}$ of his brother's age, how old are they?

2. If your sister is $\frac{3}{4}$ as old as you are, and 3 years ago she was only $\frac{2}{3}$ as old as you, how old must she be now?

3. A person having spent $10 more than $\frac{1}{3}$ of his yearly income, had $15 more than $\frac{1}{2}$ of it left; what was his income?

4. At what time between 3 and 4 o'clock, will the hour and minute hands of a clock be together?

5. What is the hour of the day, when the time past from midnight is equal to $\frac{5}{11}$ of the time to noon?

6. Two men talking of their ages, one says $\frac{2}{3}$ of my age is equal to $\frac{4}{5}$ of yours, and the sum of our ages is 55 years; what were their ages?

7. What are the ages of John and Henry, if $\frac{2}{5}$ of John's age is equal to $\frac{3}{4}$ of Henry's, and the difference of their ages is 7 years?

8. If \$12500 be willed to two persons, so that the second shall have $\frac{7}{8}$ as much as the first, what will be the share of each?

9. A and B can do a job of work in 18 days; but with C's assistance they can do it in 11 days; in what time would C do it alone?

10. If A could do a piece of work in 8 hours, B in 10, and C in 12, how long would it take them all together to finish it?

11. What is the value of \$21.08$\frac{1}{3}$ in sterling money?

12. If the ice on a pond, a mile square, be one foot thick, what would be the size of a cubical ice-house such that 6806$\frac{1}{4}$ like it would be capable of containing the ice?

13. A starts from Lowell and B from Boston at the same time, and meet at the end of 2$\frac{1}{2}$ hours; when it appeared that A travelled 3 miles per hour faster than B, and both had travelled 25 miles; what were their rates of traveling?

14. Purchased a harness for \$25, together with a horse and chaise; the chaise cost 9 times as much as the harness, and the horse $\frac{2}{3}$ as much as the chaise; what was the price of the horse, and of the chaise?

15. A, B and C, built a house, which cost \$35000; A paid \$500 more, and C \$300 less than B; what did each pay?

16. A man bought some calves and sheep, for \$108; the calves at \$5, and the sheep at \$8 apiece; how many of each sort, there being twice as many calves as sheep?

17. A farmer sold some oxen at \$28 apiece, twice as many cows at \$17 apiece, and three times as many sheep as cows at \$7.50 apiece, receiving for them \$749; required the number of each sort.

18. A certain number of men at \$.75 each per day, and $\frac{1}{2}$ as many boys at \$.25 each, earned \$5.25 daily; how many were there of each?

19. Divide \$.62$\frac{1}{2}$ into two such parts that one may be as much above, as the other is below \$.31$\frac{1}{4}$.

20. Required the ages of three men, the first of whom is 5¼ years older than the second, and twice as old as the third, who is 9¼ years younger than the second.

21. If one man can do 4/7 of a piece of work in 3 days, and another can do 11/14 of it in 3 days, how long would it take both of them to do the whole?

22. If A can do a piece of work in 6 days, B twice as much in 8 days, and C 3 times as much in 9 days, in what time would they finish it by working together?

23. If a boy buy lemons at 2 cents each, and ¾ as many at 3 cents each, and gain 25 cents by selling all at the rate of 5 cents for 2 lemons, how many did he buy?

24. How many apples were bought, at 5 cents a dozen, if half of them were exchanged for pears at the rate of 8 apples for 5 pears, and then the remaining apples and pears sold at a cent apiece; thus gaining 19 cents on the whole?

25. If eggs be bought at the rate of 5 for 4 cents, how must they be sold per dozen, to gain 25 per cent.?

26. A man having $100, spent part of it, then received five times as much as he spent, making his money double what it was at first; what was the sum spent?

27. A hare starts 50 leaps before a grey-hound, and takes 4 leaps to the hound's three; but 2 of the hound's leaps are equal to 3 of the hare's; in how many leaps will he overtake the hare?

28. A lady has a silver cup weighing 12 ounces, and if it be covered it will weigh twice as much as another cup, which receiving the same cover, would weigh three times as much as the first cup; what is the weight of the second cup, and of the cover?

29. A man bought some oranges for 25 cents; but if he had bought 3 less for the same money, the price of an orange would have been three halves of what he gave; what did the oranges cost apiece?

30. A gentleman divided his fortune among his three sons, giving A $7500 more than C, and B 5/6 as much as A, or 7/6 as much as C; what was the share of each?

31. A laborer cleared $75 in 60 working days, by receiving $1.50 a day, when he worked, and spending $.50 a day when idle; how many days did he work?

KEY.

I. NUMERATION.

14.

1. Nineteen.
2. Seventy.
3. Ninety-eight.
4. Five hundred and two.
5. Six hundred and ten.
6. Eight hundred and forty-seven.
7. One thousand and five.
8. Five thousand and forty-nine.
9. Nine thousand one hundred and fifty-three.
10. Twenty thousand nine hundred and seven.
11. Four hundred and seventeen thousand and sixteen.
12. Five million eight thousand eight hundred and forty.
13. Forty million nine hundred and ten thousand and eight.
14. One hundred and thirty-six million and two hundred.
15. Six million five hundred thousand and four.
16. One billion one hundred and forty-seven million eight hundred and sixty-five thousand four hundred and seventy-nine.
17. Four hundred and sixteen thousand.
18. Nine hundred billion one million three hundred and seventeen thousand six hundred and one.
19. One hundred and twenty trillion six hundred and forty-three billion seven hundred and ninety million eight thousand and seventy-four.
20. Nine trillion sixty-four billion seven hundred and ninety-eight million thirty thousand and twenty.
21. Three billion four hundred and seventy-nine million and nineteen.
22. Eighty trillion sixty billion four hundred million seven hundred thousand and ninety-one.
23. Eight hundred and eleven million one hundred and twenty-three thousand three hundred and sixty-five.
24. Three hundred and for-

ty-seven billion sixteen thousand and eleven.

25. Three hundred and thirty-three billion three hundred and eleven million one hundred and twelve thousand two hundred and twenty-two.

26. Eighty-eight quadrillion sixty-six billion and forty-four thousand.

27. Fifty-five quadrillion one hundred million two hundred thousand and ten.

16.

1. . . . . 103.
2. . . . . 301.
3. . . . . 1,010.
4. . . . . 2,107.
5. . . . 20,030.
6. . . . 50,705.
7. . . . 300,050.
8. . . . 707,720.
9. . . . 1,000,370.
10. . . . 5,600,073.
11. . . 590,047,008.
12. . 3,670,000,302.
13. . 45,007,070,007.
14. 50,000,657,000,500.
15. 6,703,020,000,012.
16. . 77,010,019.
17. . 8,000,530,000.
18. . 49,000,000,360.
19. 86,000,010,100,000,060.

II. ADDITION.

19. Addition Table.

2 and 2 are 4.
3 and 2, or 2 and 3, are 5.
4 and 2, or 2 and 4, are 6.
5 and 2, or 2 and 5, are 7.
6 and 2, or 2 and 6, are 8.
7 and 2, or 2 and 7, are 9.
8 and 2, or 2 and 8, are 10.
9 and 2, or 2 and 9, are 11.
3 and 3 are 6.
4 and 3, or 3 and 4, are 7.
5 and 3, or 3 and 5, are 8.
6 and 3, or 3 and 6, are 9.
7 and 3, or 3 and 7, are 10.
8 and 3, or 3 and 8, are 11.
9 and 3, or 3 and 9, are 12.
4 and 4 are 8.
5 and 4, or 4 and 5, are 9.
6 and 4, or 4 and 6, are 10.
7 and 4, or 4 and 7, are 11.
8 and 4, or 4 and 8, are 12.
9 and 4, or 4 and 9, are 13.
5 and 5 are 10.
6 and 5, or 5 and 6, are 11.
7 and 5, or 5 and 7, are 12.
8 and 5, or 5 and 8, are 13.
9 and 5, or 5 and 9, are 14.
6 and 6 are 12.
7 and 6, or 6 and 7, are 13.
8 and 6, or 6 and 8, are 14.
9 and 6, or 6 and 9, are 15.
7 and 7 are 14.
8 and 7, or 7 and 8, are 15.
9 and 7, or 7 and 9, are 16.
8 and 8 are 16.
9 and 8, or 8 and 9, are 17.
9 and 9 are 18.

23.

1. 198 bushels.
2. 1959 pounds.
3. 4085 pounds.
4. 6220 pounds.
5. 78 strokes.
6. 365 days.

7. 10632 dollars.
8. 8684 dollars.
9. 29841 dollars.
10. 5050.
11. 66280 sq. miles.
12. 103505 sq. miles.
13. 372446 sq. miles.
14. 399400 sq. miles.
15. 941631 sq. miles.
16. 5850 years.
17. 4194 years.
18. 4004 years.
19. 2599 years, A.D. 1846.
20. 2730 years, A.D. 1846.
21. 2159 miles.
22. 1915 miles.

III. MULTIPLICATION.

26. MULTIPLICATION TABLE.

2 times 2 equal 4.
3 × 2, or 2 × 3, = 6.
4 × 2, or 2 × 4, = 8.
5 × 2, or 2 × 5, = 10.
6 × 2, or 2 × 6, = 12.
7 × 2, or 2 × 7, = 14.
8 × 2, or 2 × 8, = 16.
9 × 2, or 2 × 9, = 18.
3 times 3 equal 9.
4 × 3, or 3 × 4, = 12.
5 × 3, or 3 × 5. = 15.
6 × 3, or 3 × 6, = 18.
7 × 3, or 3 × 7, = 21.
8 × 3, or 3 × 8, = 24.
9 × 3, or 3 × 9, = 27.
4 times 4 equal 16.
5 × 4, or 4 × 5, = 20.
6 × 4, or 4 × 6, = 24.
7 × 4, or 4 × 7, = 28.
8 × 4, or 4 × 8, = 32.
9 × 4, or 4 × 9, = 36.
5 times 5 equal 25.
6 × 5, or 5 × 6, = 30.
7 × 5, or 5 × 7, = 35.
8 × 5, or 5 × 8, = 40.
9 × 5, or 5 × 9, = 45.
6 times 6 equal 36.
7 × 6, or 6 × 7, = 42.
8 × 6, or 6 × 8, = 48.
9 × 6, or 6 × 9, = 54.
7 times 7 equal 49.
8 × 7, or 7 × 8, = 56.
9 × 7, or 7 × 9, = 63.
8 times 8 equal 64.
9 × 8, or 8 × 9, = 72.
9 times 9 equal 81.

28.

1. 189 gallons.
2. 441 gallons.
3. 300 minutes.
4. 480 minutes.
5. 600 cents.
6. 400 cents.
7. 900 cents.
8. 2240 rods.
9. 1600 rods.
10. 17920 pounds.
11. 15680 pounds.
12. 360000 inhabitants.
13. 150000 persons.
14. 1050000 persons.
15. 6852 feet.
16. $200000 to Washington.
$100000 to J. Adams.
$200000 to Jefferson.
$200000 to Madison.
$200000 to Monroe.
$100000 to J. Q. Adams.
$200000 to Jackson.
$100000 to Van Buren.
$100000 to Harrison & T.

17. 11184.
18. 3375.
19. 12523.

31.

1. 2205 gallons.
2. 2835 gallons.
3. 6720 rods.
4. 1080 minutes.
5. 4200 cents.
6. 20160 rods.
7. 2100 dollars.
8. 1392 dollars.
9. 4704 dollars.

33.

1. 3807.
2. 3360.
3. 4266.
4. 8856.
5. 256256.
6. 1750000.
7. 40863150.
8. 657851220.

35.

1. 50 dollars.
2. 700 dollars.
3. 579600 dollars.
4. 20000 cents.
5. 5000 cents.
6. 1610.
7. 172800.
8. 18000.
9. 125000.
10. 20000.
11. 500000000.
12. 1020000.
13. 1000000.

37.

1. 210 dollars.
2. 120 days.
3. 160000 rods.
4. 80 days.
5. 37500.
6. 64800.
7. 11016000.
8. 490000.
9. 6250000.

39.

1. 58 pints.
2. 192 cents.
3. 60 cents.
4. 45000 grains.
5. 8730 cents.
6. 2700 dollars.
7. 4700 cents.
8. 71000 mills.
9. 5300 cents.
10. 53000 mills.
11. 520 dollars.
12. 460 bushels.
13. 1920 soldiers.
14. 2600 pounds.
15. 64 squares.
16. 96 squares.
17. 1640 trees.

45.

1. 1222 miles.
2. 8650 times.
3. 2775 dollars.
4. 1247 dollars.
5. 5529 dollars.
6. 4740 miles.
7. 246018 yards.
8. 3610 bushels.
9. 248832.
10. 3999996.
11. 407025.
12. 53283996.
13. 1854360.
14. 2468642.
15. 2468642.

16. 927007416.
17. 927007416.

48.

1. 3000 miles.
2. 96000000 miles.
3. 600000 dollars.
4. 800000 dollars.
5. 1800000 dollars.
6. 90000 dollars.
7. 1560000 dollars.
8. 1600000 dollars.
9. 4477000.
10. 4477000.
11. 1326000.
12. 1326000.
13. 19600.
14. 2560000.
15. 244200000.
16. 324700.
17. 384000000.
18. 979392960.

49.

1. 96 months.
2. 48 months.
3. 180000 pounds.
4. 480 panes.
5. 1485000.
6. 1728.
7. 616105.
8. 4570000.
9. 71000 dollars.
10. 15000 days.
11. 15000 men.
12. 700 cent-loaves.
13. 135 yards.
14. 312 days.
15. 1551 hours.
16. 4680 strokes.
17. 190080 bricks.
18. 395 dollars.
19. 900 dollars.
20. 1538 dollars.
21. 439 dollars.
22. 3800 dollars.
23. 370 dollars.
24. 264 scholars.
25. 264.
26. 1008 seats.
27. 9216 shingles.
28. 9216.
29. 1632000 miles.
30. 45696000 miles.
31. 195840000 miles.
32. 354144000 miles.
33. 595680000 miles.
34. 96000000 miles.
35. 105.
36. 223092870.
37. 3105 dollars.
38. 406 miles.
39. 70 miles.
40. 144.
41. 225.
42. 900.
43. 2500.
44. 10000.
45. 1728.
46. 3375.
47. 729.
48. 15625.
49. 81.
50. 625.
51. 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024.
52. 10, 100, 1000, 10000, 100000, 1000000, 10000000, 100000000, 1000000000, 10000000000.
53. 144000.
54. 1800000.
55. 50000.

IV. SUBTRACTION.

52. Subtraction Table.

2 − 2 = 0.	3 − 3 = 0.	4 − 4 = 0.	5 − 5 = 0.
3 − 2 = 1.	4 − 3 = 1.	5 − 4 = 1.	6 − 5 = 1.
4 − 2 = 2.	5 − 3 = 2.	6 − 4 = 2.	7 − 5 = 2.
5 − 2 = 3.	6 − 3 = 3.	7 − 4 = 3.	8 − 5 = 3.
6 − 2 = 4.	7 − 3 = 4.	8 − 4 = 4.	9 − 5 = 4.
7 − 2 = 5.	8 − 3 = 5.	9 − 4 = 5.	10 − 5 = 5.
8 − 2 = 6.	9 − 3 = 6.	10 − 4 = 6.	11 − 5 = 6.
9 − 2 = 7.	10 − 3 = 7.	11 − 4 = 7.	12 − 5 = 7.
10 − 2 = 8.	11 − 3 = 8.	12 − 4 = 8.	13 − 5 = 8.
11 − 2 = 9.	12 − 3 = 9.	13 − 4 = 9.	14 − 5 = 9.
6 − 6 = 0.	7 − 7 = 0.	8 − 8 = 0.	9 − 9 = 0.
7 − 6 = 1.	8 − 7 = 1.	9 − 8 = 1.	10 − 9 = 1.
8 − 6 = 2.	9 − 7 = 2.	10 − 8 = 2.	11 − 9 = 2.
9 − 6 = 3.	10 − 7 = 3.	11 − 8 = 3.	12 − 9 = 3.
10 − 6 = 4.	11 − 7 = 4.	12 − 8 = 4.	13 − 9 = 4.
11 − 6 = 5.	12 − 7 = 5.	13 − 8 = 5.	14 − 9 = 5.
12 − 6 = 6.	13 − 7 = 6.	14 − 8 = 6.	15 − 9 = 6.
13 − 6 = 7.	14 − 7 = 7.	15 − 8 = 7.	16 − 9 = 7.
14 − 6 = 8.	15 − 7 = 8.	16 − 8 = 8.	17 − 9 = 8.
15 − 6 = 9.	16 − 7 = 9.	17 − 8 = 9.	18 − 9 = 9.

55.

1. 13 cents.
2. 12 cents.
3. 12 years.
4. 21 years.
5. 51 boys.
6. 35 dollars.
7. 2000 dollars.
8. 675 dollars.
9. 1000 dollars.
10. 5000.
11. 10305.
12. 220.
13. 313233.

57.

1. 15 dollars.
2. A. D. 1706.
3. 67 years.
4. 226 years, in A. D. 1846.
5. 354 years, in A. D. 1846.
6. 70 years, in A. D. 1846
7. 8940 feet.
8. 900 miles.
9. 1691 sq. miles.
10. 38285 sq. miles.
11. 75.
12. 87.
13. 271.

14. 493.
15. 64.

60.

1. 79 feet.
2. 83 feet.
3. 27 feet.
4. 1656 years.
5. 2909 years.
6. 7824 inhabitants.
7. 11310 inhabitants.
8. 13690 inhabitants.
9. 25500 inhabitants.
10. 20675 inhabitants.
11. 27811 inhabitants.
12. 32189 inhabitants.
13. 14979 inhabitants.
14. 3104 inhabitants.
15. 17232 inhabitants.
16. 79301 inhabitants.
17. 191260 inhabitants.
18. 875301 miles.
19. 81254 miles.
20. 38000 miles per hour.
21. 1.
22. 292999.
23. 36996322.
24. 8844.
25. 1956.
26. 462365.
27. 1973 dollars.
28. 889.
29. 991.
30. 6999983.

V. DIVISION.

64.

1. 5 boys.
2. 3 apples.
3. 4 oranges.
4. 6 cents.
5. 6 apples.
6. 3 cents.
7. 5 barrels.
8. 8 dollars.
9. 7 dollars.
10. 6 shillings.
11. 6 pounds.
12. 9 cents.
13. 7 pounds.
14. 7 cents.
15. 9 pews.
16. 7 persons.
17. 9 dollars.
18. 8 nine-pences.
19. 8 classes.
20. 10 scholars.
21. 8 sections.
22. 8 soldiers.
23. 4 threes.
24. 3.
25. 5 threes
26. 3.
27. 6 times.
28. 3.
29. 7 times.
30. 3.
31. 5 times 4.
32. 4.
33. 6 parts.
34. 4.
35. 3 parts.
36. 10.
37. 7 parts.
38. 5.
39. 9 parts.
40. 5.
41. 8.
42. 6.

43. 8.
44. 7.
45. 9 parts.
46. 7.

65. Division Table.

2 ÷ 2 = 1.	3 ÷ 3 = 1.
4 ÷ 2 = 2.	6 ÷ 3 = 2.
6 ÷ 2 = 3.	9 ÷ 3 = 3.
8 ÷ 2 = 4.	12 ÷ 3 = 4.
10 ÷ 2 = 5.	15 ÷ 3 = 5.
12 ÷ 2 = 6.	18 ÷ 3 = 6.
14 ÷ 2 = 7.	21 ÷ 3 = 7.
16 ÷ 2 = 8.	24 ÷ 3 = 8.
18 ÷ 2 = 9.	27 ÷ 3 = 9.
4 ÷ 4 = 1.	5 ÷ 5 = 1.
8 ÷ 4 = 2.	10 ÷ 5 = 2.
12 ÷ 4 = 3.	15 ÷ 5 = 3.
16 ÷ 4 = 4.	20 ÷ 5 = 4.
20 ÷ 4 = 5.	25 ÷ 5 = 5.
24 ÷ 4 = 6.	30 ÷ 5 = 6.
28 ÷ 4 = 7.	35 ÷ 5 = 7.
32 ÷ 4 = 8.	40 ÷ 5 = 8.
36 ÷ 4 = 9.	45 ÷ 5 = 9.
6 ÷ 6 = 1.	7 ÷ 7 = 1.
12 ÷ 6 = 2.	14 ÷ 7 = 2.
18 ÷ 6 = 3.	21 ÷ 7 = 3.
24 ÷ 6 = 4.	28 ÷ 7 = 4.
30 ÷ 6 = 5.	35 ÷ 7 = 5.
36 ÷ 6 = 6.	42 ÷ 7 = 6.
42 ÷ 6 = 7.	49 ÷ 7 = 7.
48 ÷ 6 = 8.	56 ÷ 7 = 8.
54 ÷ 6 = 9.	63 ÷ 7 = 9.
8 ÷ 8 = 1.	9 ÷ 9 = 1.
16 ÷ 8 = 2.	18 ÷ 9 = 2.
24 ÷ 8 = 3.	27 ÷ 9 = 3.
32 ÷ 8 = 4.	36 ÷ 9 = 4.
40 ÷ 8 = 5.	45 ÷ 9 = 5.
48 ÷ 8 = 6.	54 ÷ 9 = 6.
56 ÷ 8 = 7.	63 ÷ 9 = 7.
64 ÷ 8 = 8.	72 ÷ 9 = 8.
72 ÷ 8 = 9.	81 ÷ 9 = 9.

67.

1. 100 desks.
2. 40 scholars.
3. 30 hats.
4. 20 dollars.
5. 20 yards.
6. 200 dollars.
7. 100 hours.
8. 100 dollars.
9. 1000 families.
10. 30000 dollars.

69.

1. 50 pairs.
2. 70 dollars.
3. 50 hours.
4. 10 miles.
5. 90 times.
6. 700 parts.
7. 400.
8. 500.
9. 700.
10. 9000.
11. 25.
12. 125.
13. 200.

72.

1. 22 bushels.
2. 11 weeks.
3. 21 dollars.
4. 42 wagons.
5. 41 hours.
6. 62 sheep.
7. 51 pairs.
8. 31 dollars.
9. 71 soldiers.
10. 52 weeks.
11. 91 times.
12. 411.
13. 612.
14. 911.

15. 111 parts.
16. 81.

76.

1. 144 bushels.
2. 256 sections.
3. 512 officers.
4. 1728 men.
5. 52 weeks.
6. 1836 sabbaths.
7. 63 acres.
8. 123 times.
9. 5003 times.
10. 1202.
11. 12817.
12. 4004.
13. 3735.
14. 1321.
15. 312.

78.

1. 102 dollars.
2. 104 dollars.
3. 252 dollars.
4. 25 dollars.
5. 160 families.
6. 165 yards.
7. 7032.
8. 308.
9. 648.
10. 6002.
11. 240900178.
12. 108002.
13. 3206.
14. 1001.
15. 2100.
16. 12300.
17. 200012.
18. 15015.
19. 204.
20. 192.
21. 8.
22. 9.

80.

1. 15 days.
2. 25 days.
3. 25 dollars.
4. 30 bushels.
5. 129 acres.
6. 15 horses.
7. 4 months.
8. 3 dollars.
9. 133 hogsheads.
10. 23 years.
11. 6 brothers.
12. 26 shares.
13. 33 acres.
14. 640 barrels.
15. 94 pounds.
16. 105 times.
17. 307 times.
18. 921 times.
19. 1007 times.
20. 105 times.
21. 1001.
22. 120 boys.
23. 25.
24. 2020.
25. 6551 inhabitants.
26. 1024 parts.
27. 64 times.
28. 64 parts.

81.

1. 81 and 3.
2. 19 years.
3. 88, 150, and 262.
4. 232 dollars, gain.
5. 107 dollars.
6. 40 apples.
7. 27.
8. 148 acres.
9. 290 dollars.
10. 3449 dollars.
11. 435 barrels.

12. 435 times.
13. 24 dollars.
14. 25 dollars.
15. 309 dollars.
16. 14 dollars.
17. 410 dollars, loss.
18. 52 weeks.
19. 20 dollars.
20. 3800 dollars.
21. 30 dollars.
22. 1584.
23. 4100.
24. 15 cents.
25. 1 cent.
26. 1250 quintals.
27. 256 barrels.
28. 25 days.
29. 52 weeks.
30. 12 times.
31. 36 days.
32. 15.
33. 45.
34. 12.
35. 4 daughters.
36. 2312.
37. 4800 dozen eggs.
38. 14 cents, lost.
39. 25 dollars.
40. 14 years.
41. 5 years.
42. 50 barrels.
43. 88 dollars.
44. 66.
45. 21.
46. 625.
47. 225.

VI. FRACTIONS.

85.

1. $2\frac{3}{5}$ pencils.
2. $3\frac{2}{5}$ pencils.
3. $6\frac{1}{4}$ oranges.
4. $4\frac{2}{5}$ cents.
5. $6\frac{5}{6}$ miles.
6. $9\frac{2}{7}$ dollars.
7. $11\frac{5}{8}$ slates.
8. $24\frac{4}{9}$ dollars.
9. $12\frac{5}{10}$ books.
10. $24\frac{7}{12}$ cents.
11. $14\frac{8}{15}$ shad.
12. $8\frac{6}{13}$ dollars.
13. $4\frac{5}{22}$ inkstands.
14. $7\frac{9}{25}$ bushels.
15. $12\frac{2}{29}$ cows.
16. $22\frac{25}{32}$ bushels.
17. $150\frac{5}{37}$ acres.
18. $5\frac{169}{320}$ yards.
19. $144\frac{20}{320}$ miles.
20. $16\frac{48}{105}$ feet.
21. $199\frac{36}{125}$ days.
22. $22\frac{96}{132}$ days.
23. $6\frac{54}{63}$ hours.
24. $29\frac{25}{75}$ dollars.
25. $328\frac{44}{100}$ boxes.
26. $24\frac{24}{99}$ bushels.
27. $25\frac{125}{175}$ days.
28. $14\frac{2}{52}$ dollars.
29. $10\frac{349}{365}$ years.
30. $15\frac{80}{168}$ miles.
31. $12\frac{181}{199}$ times.
32. $106\frac{7}{25}$, each part.
33. $204\frac{8}{14}$, quotient.
34. $8014\frac{1308}{12478}$, quotient.

88.

1. $\frac{1}{6}$, $\frac{2}{6}$, and $\frac{5}{6}$ of the pie.
2. $\frac{1}{3}$ of the orange.
3. $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$ of an acre.
4. $\frac{1}{7}$, $\frac{3}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$ of a piece.
5. $\frac{1}{8}$ of a barrel.
6. $\frac{5}{18}$, $\frac{7}{18}$, $\frac{11}{18}$, and $\frac{17}{18}$ of a ton.

7. $\frac{10}{27}$, $\frac{14}{27}$, $\frac{19}{27}$, and $\frac{25}{27}$ of a hogshead.
8. $\frac{16}{100}$, $\frac{29}{100}$, $\frac{67}{100}$, $\frac{89}{100}$, and $\frac{93}{100}$ of a share.
9. $\frac{12}{75}$, $\frac{24}{75}$, $\frac{36}{75}$, and $\frac{58}{75}$ of a bushel.
10. $\frac{7}{100}$, $\frac{23}{100}$, $\frac{37}{100}$, $\frac{47}{100}$, $\frac{67}{100}$, and $\frac{97}{100}$ of a dollar.
11. $\frac{11}{30}$, $\frac{17}{30}$, and $\frac{29}{30}$ of June.
12. $\frac{16}{31}$, $\frac{21}{31}$, and $\frac{27}{31}$ of July.
13. $\frac{13}{60}$, $\frac{43}{60}$, and $\frac{59}{60}$ of an hour.
14. $\frac{1}{24}$, $\frac{7}{24}$, $\frac{19}{24}$, and $\frac{23}{24}$ of a day.
15. $\frac{11}{320}$, $\frac{21}{320}$, $\frac{87}{320}$, $\frac{133}{320}$, and $\frac{319}{320}$ of a mile.
16. $\frac{3}{5}$ of 5 dollars.
17. $\frac{3}{25}$, $\frac{7}{25}$, $\frac{14}{25}$, and $\frac{21}{25}$ of 25 cents.
18. $\frac{16}{63}$, $\frac{31}{63}$, and $\frac{44}{63}$ of 63 gals.
19. $\frac{31}{365}$, $\frac{60}{365}$, $\frac{124}{365}$, $\frac{248}{365}$, and $\frac{316}{365}$ of 365 days.
20. $\frac{15}{52}$ of 52 weeks.
21. $\frac{11}{32}$, $\frac{15}{32}$, $\frac{25}{32}$, $\frac{32}{32}$ of a bushel.
22. $\frac{5}{8}$ of 8.
23. $\frac{2}{11}$, $\frac{9}{11}$, $\frac{12}{11}$, $\frac{14}{11}$, and $\frac{21}{11}$ of 11.
24. $\frac{5}{33}$, $\frac{7}{33}$, $\frac{16}{33}$, $\frac{25}{33}$, and $\frac{32}{33}$ of 33.
25. $2\frac{4}{15}$ times.
26. $\frac{8}{15}$ times.
27. $\frac{7}{24}$ of a time.
28. $\frac{11}{16}$ of 16.
29. $\frac{5}{12}$ of 12.
30. $\frac{21}{25}$, quotient.
31. $\frac{17}{25}$, quotient.
32. $\frac{4}{5}$ of an apple.
33. $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{8}$ of a barrel.
34. $\frac{7}{15}$ of a dollar.
35. $\frac{3}{8}$ of a bushel.
36. $\frac{1}{7}$ of a cord.
37. $\frac{16}{17}$, quotient.
38. $\frac{2}{11}$, quotient.
39. $\frac{17}{123}$, quotient.
40. $\frac{84}{1725}$, quotient.
41. $\frac{1728}{1837}$, quotient.
42. $\frac{37}{25}$, quotient.
43. $\frac{25}{36}$, quotient.
44. $\frac{81}{75}$, quotient.
45. $\frac{16}{9}$, quotient.
46. $\frac{7}{11}$, quotient.

90.

1. 84 cents.
2. 84.
3. 69 bushels
4. 69.
5. 300 dollars.
6. 300.
7. 125.
8. 198.
9. 112.
10. 320 rods.
11. 63 gallons.
12. 160 square rods.
13. 1440 minutes.
14. 365 days.
15. 420 dollars.
16. 100 cents.
17. 496.
18. 864.
19. 84 years.

93.

1. 50 cents.
2. $33\frac{1}{3}$ cents.
3. 20, $16\frac{2}{3}$, $12\frac{1}{2}$, 10, $8\frac{1}{3}$, and $6\frac{1}{4}$ cents.
4. 3 dollars.
5. 7, 8, 9, 10, 11, and 12.
6. 1, 2, 3, 4, 5, 9, and 12.
7. 93 acres.
8. 40 sq. rods.
9. 80 rods.
10. 40 rods.

11. 60 minutes.
12. 48 farthings.
13. 224 pounds.
14. $542\frac{1}{2}$ bushels.
15. 240, 120, 60, and 30 apples.
16. 400, 200, and $66\frac{8}{12}$ dolls.
17. $\frac{1}{4}$ of them; which is 8 qts.
18. $\frac{1}{16}$ of 64; which is 4.
19. $\frac{1}{4}$ of it.
20. 250 dollars.

94.

1. $\frac{7}{5}$ of 1.
2. $\frac{10}{3}$ of 1.
3. $\frac{11}{8}$ of 1.
4. $\frac{25}{16}$ of 1.
5. $\frac{5}{8}$ of 1, $\frac{1}{8}$ of 5, 5 divided by 8, and 5 such parts that 8 like them equal a unit, &c.
6. $\frac{175}{180}$.
7. $\frac{3}{2}$.
8. $\frac{175}{180}$.
9. $\frac{1}{5}$.
10. $\frac{5}{8}$, $\frac{5}{5}$, and $\frac{5}{14}$; $\frac{7}{11}$ and $\frac{7}{18}$; $\frac{9}{14}$ and $\frac{9}{10}$; $\frac{3}{2}$, $\frac{3}{3}$, and $\frac{3}{28}$.
11. $\frac{9}{14}$ and $\frac{5}{14}$; $\frac{2}{9}$, $\frac{10}{9}$, and $\frac{45}{9}$; $\frac{18}{7}$ and $\frac{11}{7}$; $\frac{1}{5}$ and $\frac{6}{5}$.
12. $\frac{3}{3}$ and $\frac{5}{5}$.
13. $\frac{20}{18}$, $\frac{18}{7}$, $\frac{10}{9}$, $\frac{25}{17}$, $\frac{45}{8}$, $\frac{84}{12}$, $\frac{3}{2}$, and $\frac{11}{7}$.
14. $\frac{84}{12}$.
15. $\frac{3}{28}$.
16. It expresses more parts of the same size.
17. It expresses larger parts.
18. It diminishes the value of the fraction.
19. It diminishes the value of the fraction.

97.

1. $9\frac{3}{8}$ dollars.
2. 202 dollars.
3. $9\frac{5}{6}$ pies.
4. $4\frac{23}{32}$ bushels.
5. $19\frac{7}{20}$ pounds.
6. $36\frac{5}{12}$ shillings.
7. $11\frac{14}{28}$ guineas.
8. 72 days.
9. $162\frac{17}{60}$ hours.
10. $120\frac{42}{365}$ years.
11. 12
12. 12.
13. $34\frac{2}{15}$.
14. $10\frac{37}{45}$.
15. $514\frac{15}{17}$.
16. $73\frac{11}{25}$.
17. $1\frac{8}{225}$.
18. 126.
19. 504
20. $7\frac{124}{125}$.
21. $1\frac{111}{444}$.

99.

1. $\frac{112}{16}$.
2. $\frac{103}{4}$.
3. $\frac{77}{6}$.
4. $\frac{86}{25}$.
5. $\frac{74}{7}$.
6. $\frac{122}{9}$.
7. $\frac{72}{8}$ of a dollar.
8. $\frac{128}{4}$ of a yard.
9. $\frac{371}{24}$ of a day.
10. $\frac{167}{20}$ of a pound.
11. $\frac{1063}{60}$ of an hour.
12. $\frac{390}{63}$ of a hogshead.
13. $\frac{3650}{365}$ of a year.
14. $\frac{4809}{11}$.
15. $\frac{3811}{365}$.
16. $\frac{2547}{100}$.
17. $\frac{2511}{60}$.

18. $\frac{140}{7}$.
19. $\frac{432}{12}$.
20. $\frac{75}{5}$, $\frac{90}{6}$.
21. $\frac{8}{2}$, $\frac{12}{3}$, $\frac{16}{4}$, $\frac{20}{5}$, and $\frac{24}{6}$.
22. $\frac{32}{2}$, $\frac{48}{3}$, $\frac{64}{4}$, and $\frac{80}{5}$.
23. $\frac{2}{2}$, $\frac{15}{15}$, and $\frac{75}{75}$.
24. $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$, and $\frac{7}{7}$.

102.

1. $9\frac{3}{5}$ dollars.
2. $3\frac{3}{4}$ dollars.
3. 10 dollars.
4. $\frac{11}{12}$ of a dollar.
5. $7\frac{3}{7}$ barrels.
6. $9\frac{3}{8}$ bushels.
7. $10\frac{1}{3}$ bushels.
8. $8\frac{1}{4}$ dollars.
9. $182\frac{1}{2}$ dollars.
10. $21\frac{9}{11}$ miles.
11. $2\frac{17}{19}$.
12. $3\frac{19}{24}$.
13. $3\frac{1}{100}$.
14. $1\frac{31}{365}$.
15. $2\frac{23}{50}$.
16. $14\frac{61}{76}$.
17. $1\frac{1}{365}$.
18. $\frac{159}{1728}$.

104.

1. $18\frac{6}{8}$ yards.
2. $412\frac{1}{2}$ feet.
3. 220 yards.
4. 100 cents.
5. $9\frac{93}{365}$ years.
6. $99\frac{3}{8}$ dollars.
7. 1575 gallons.
8. $3\frac{1}{4}$ dollars.
9. 135 dollars.
10. $67\frac{3}{8}$ dollars.
11. $58\frac{1}{12}$ dollars.
12. $242\frac{1}{12}$.
13. $201\frac{159}{541}$.
14. $7\frac{27}{49}$.
15. $166\frac{2}{3}$.
16. $12096\frac{1998}{3333}$.

107.

1. $\frac{1}{2}$, and 1 dollar.
2. $\frac{1}{3}$, $\frac{1}{2}$, and 1 dollar.
3. $\frac{1}{4}$, $\frac{1}{2}$, and 1 dollar.
4. $\frac{1}{4}$, $\frac{1}{2}$, and 1 dollar.
5. $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, and 1 dollar.
6. $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and 1 dollar.
7. $2\frac{1}{3}$, and 7 yards.
8. $\frac{23}{30}$, $1\frac{3}{20}$, $1\frac{8}{15}$, $1\frac{11}{12}$, $2\frac{3}{10}$, $3\frac{5}{6}$, $4\frac{3}{5}$, $5\frac{3}{4}$, $7\frac{2}{3}$, & 23 miles.
9. $5\frac{3}{8}$, $10\frac{3}{4}$, $14\frac{1}{3}$, $21\frac{1}{2}$, and 43 yards.
10. $\frac{4}{5}$.
11. $2\frac{5}{9}$.
12. $14\frac{3}{5}$.
13. 23.
14. 19.
15. 31.
16. 1582.
17. 197.
18. 2395.
19. 12.
20. 43.
21. $1250\frac{2}{5}$.
22. 52.
23. $1\frac{1}{8}$.
24. $2\frac{11}{12}$.
25. $1\frac{1}{24}$.

109.

1. $3\frac{1}{3}$.
2. $27\frac{1}{2}$.
3. $25\frac{1}{2}$.
4. $3\frac{1}{5}$.
5. $10\frac{2}{3}$.
6. 51.

115.

1. $\frac{3}{5}$ of a dollar.
2. $\frac{1}{4}$ of a bushel.
3. $\frac{3}{50}$ of a dollar.
4. $\frac{4}{11}$ of a ton.
5. $\frac{3}{25}$ of a dollar.
6. $4\frac{1}{3}$ dollars.
7. $8\frac{1}{3}$ shillings.
8. $7\frac{4}{5}$ bushels.
9. $8\frac{4}{5}$ bushels.
10. $1\frac{3}{8}$ dollars.
11. $5\frac{2}{11}$ miles.
12. $59\frac{3}{5}$ miles.
13. $5\frac{2}{11}$ barrels.
14. $6\frac{3}{5}$ dollars.
15. $\frac{11}{100}$ of a dollar.
16. $2\frac{3}{8}$ times.
17. $9\frac{2}{11}$.
18. $\frac{3}{7}$, quotient.
19. $5\frac{4}{17}$ times.
20. $\frac{3}{5}$.
21. $\frac{3}{4}$.
22. $4\frac{1}{7}$.
23. $\frac{7}{9}$, quotient.
24. $144\frac{2}{7}$, quotient.
25. $256\frac{1}{41}$, quotient.
26. $128\frac{1}{31}$, quotient.

116.

1. $\frac{1}{4}$ of a melon.
2. $\frac{1}{4}$.
3. $\frac{1}{8}$ of a pie, $\frac{1}{8}$.
4. $\frac{3}{8}$ of a dollar, $\frac{3}{8}$.
5. $\frac{1}{6}$ of a dollar.
6. $\frac{1}{10}$ of a pie, $\frac{1}{10}$.
7. $\frac{1}{6}$ of a dollar.
8. 2 parts, each $\frac{1}{2}$.
9. 4 parts, each $\frac{1}{4}$, $\frac{1}{4}$.
10. 8 parts, each $\frac{1}{8}$, $\frac{1}{8}$.
11. $\frac{1}{16}$ and $\frac{1}{32}$.
12. $\frac{3}{8}$ of a dollar.
13. $\frac{3}{16}$ of a dollar.
14. $\frac{3}{25}$ of a dollar.
15. $\frac{37}{48}$ of a dollar.
16. $1\frac{5}{12}$ bushels.
17. $\frac{7}{8}$ of a yard.
18. $\frac{17}{126}$ of a dollar.
19. $3\frac{1}{32}$ dollars.
20. $6\frac{3}{100}$ dollars.
21. $17\frac{13}{14}$ miles.
22. $3\frac{1}{21}$, quotient.
23. $\frac{2}{5}$ of 5.
24. $\frac{7}{15}$ of 5.
25. $\frac{1}{4}$ of 5.
26. $\frac{7}{12}$ of 12.
27. $\frac{3}{11}$ of 12.

117.

1. 29, quotient.
2. 24 times.
3. 125 casks.
4. $\frac{2}{27}$ of a mile.
5. $\frac{1}{6}$ of a barrel.
6. $5\frac{5}{28}$ times.
7. $\frac{5}{6}$, quotient.
8. $6\frac{1}{21}$, quotient.
9. $2\frac{5}{12}$ times.
10. $\frac{4}{33}$ of once.
11. $\frac{3}{10}$ of once.
12. $1\frac{4}{21}$ times.
13. $8\frac{1}{3}$, quotient.
14. $76\frac{1}{2}$, quotient.
15. $21\frac{1}{3}$, quotient.
16. $\frac{3}{34}$, quotient.
17. $1\frac{4}{57}$, quotient.
18. $43\frac{1}{9}$, quotient.
19. $\frac{3}{5}$ of 15.
20. $\frac{2}{7}$ of 18.
21. $\frac{1}{3}$ of 21.
22. 56.
23. 3 and 5.
24. 65.
25. 170, quotient.

26 126, quotient.
27. 12 times.
28. 3, quotient.

125.

The prime factors of
72 are 2^3, and 3^2.
88 are 2^3, and 11.
120 are 2^3, 3, and 5.
612 are 2^2, 3^2, and 17.
336 are 2^4, 3, and 7.
648 are 2^3, and 3^4.
930 are 2, 3, 5, and 31.
924 are 2^2, 3, 7, and 11.
936 are 2^3, 3^2, and 13.
450 are 2, 3^2, and 5^2.
360 are 2^3, 3^2, and 5.
966 are 2, 3, 7, and 23.
870 are 2, 3, 5, and 29.
684 are 2^2, 3^2, and 19.
396 are 2^2, 3^2, and 11.
432 are 2^4, and 3^3.
2480 are 2^4, 5, and 31.
8000 are 2^6, and 5^3.
10449 are 3^5, and 43.
10503 are 3^3, and 389.
24876 are 2^2, 3^2, and 691.

127.

1. $\frac{2}{3}$.
2. $\frac{3}{5}$, $\frac{2}{3}$, $\frac{2}{3}$, $\frac{5}{7}$, $\frac{3}{4}$, $\frac{3}{5}$, and $\frac{3}{4}$.
3. $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{5}$.
4. $\frac{4}{25}$, $\frac{5}{7}$, $\frac{1}{5}$, and $\frac{1}{3}$.
5. $\frac{1}{6}$, $\frac{1}{30}$, $\frac{41}{214}$, and $\frac{1}{12}$.
6. $\frac{17}{37}$.

130.

1. 6.
2. $\frac{5}{12}$.
3. 18.
4. $\frac{7}{19}$.
5. 16.
6. $\frac{8}{11}$.
7. $\frac{13}{14}$.

8. $\frac{1}{4}$.
9. $\frac{27}{64}$.
10. 384.
11. $\frac{1}{3}$.
12. 57.
13. $\frac{2}{5}$.
14. $\frac{1}{2}$.
15. $\frac{3}{17}$.
16. $\frac{1}{86}$.
17. $\frac{1}{20}$.

134.

1. $\frac{8}{12}$ and $\frac{9}{12}$.
2. $\frac{5}{15}$ and $\frac{6}{15}$.
3. $\frac{15}{20}$ and $\frac{4}{20}$.
4. $\frac{7}{21}$ and $\frac{9}{21}$.
5. $\frac{18}{24}$ and $\frac{15}{24}$.
6. $\frac{21}{28}$ and $\frac{8}{28}$.
7. $\frac{7}{35}$ and $\frac{25}{35}$.
8. $\frac{18}{30}$ and $\frac{25}{30}$.
9. $\frac{5}{30}$ and $\frac{9}{30}$.
10. $\frac{15}{40}$ and $\frac{28}{40}$.
11. $\frac{18}{60}$ and $\frac{35}{60}$.
12. $\frac{15}{36}$ and $\frac{10}{36}$.
13. $\frac{21}{30}$ and $\frac{4}{30}$.
14. $\frac{20}{72}$ and $\frac{15}{72}$.

140.

1. 24.
2. $\frac{15}{24}$ and $\frac{10}{24}$.
3. 56.
4. $\frac{21}{56}$ and $\frac{20}{56}$.
5. 45.
6. $\frac{10}{45}$ and $\frac{21}{45}$.
7. 90.
8. $\frac{24}{80}$ and $\frac{25}{80}$.
9. 35.
10. $\frac{14}{35}$ and $\frac{15}{35}$.
11. 210.
12. $\frac{105}{210}$, $\frac{70}{210}$, $\frac{42}{210}$, and $\frac{30}{210}$.
13. 210.
14. $\frac{63}{210}$, $\frac{75}{210}$, and $\frac{28}{210}$.
15. 2000.

16. $\frac{33}{2000}$ and $\frac{15}{2000}$.
17. 840.
18. $\frac{224}{420}$, $\frac{175}{420}$, and $\frac{144}{420}$.
19. $\frac{12}{15}$ and $\frac{5}{15}$.
20. $\frac{4}{15}$ and $\frac{6}{15}$.
21. $\frac{2}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$.
22. $\frac{4}{27}$ and $\frac{4}{27}$.
23. $\frac{75}{270}$, $\frac{80}{270}$, and $\frac{42}{270}$.
24. $\frac{276}{3000}$ and $\frac{175}{3000}$.

143.

1. $\frac{7}{16}$ of a dollar.
2. $\frac{3}{8}$ of a dollar.
3. $\frac{1}{4}$ of a mile.
4. $\frac{3}{16}$ of a dollar.
5. $\frac{21}{40}$ of a dollar.
6. Isaac, $\frac{5}{12}$ of a dollar.
7. 95 yards.
8. $16\frac{7}{8}$ dollars.
9. $\frac{5}{8}$ of a yard.
10. $15\frac{5}{8}$ yards.
11. $31\frac{1}{2}$ gallons.
12. $23\frac{5}{8}$ gallons.
13. $\frac{1}{4}$ of the time.
14. $\frac{5}{12}$ of the way level, and $\frac{1}{12}$ more up hill forth.
15. $\frac{7}{15}$.
16. $\frac{54}{91}$.
17. $\frac{17}{36}$ sum, & $\frac{1}{36}$ difference.
18. $18\frac{11}{20}$.
19. $4\frac{11}{60}$.
20. $28\frac{9}{14}$ sum, and $4\frac{3}{14}$ difference.
21. $\frac{5}{6}$.
22. $3\frac{1}{4}$.
23. $\frac{5}{12}$.
24. $10\frac{4}{15}$
25. 2.
26. $3\frac{1}{8}$.
27. $\frac{5}{7}$.
28. $\frac{25}{28}$.
29. $1\frac{17}{36}$.
30. $2\frac{1}{3}$.
31. $\frac{1}{12}$.

147.

1. $8\frac{1}{3}$ dollars.
2. $2\frac{1}{4}$ miles.
3. $3\frac{3}{7}$ dollars.
4. 4 dollars.
5. $7\frac{1}{2}$ dollars.
6. $11\frac{1}{4}$.
7. $13\frac{1}{8}$, product.
8. $21\frac{1}{3}$ dollars.
9. $10\frac{5}{6}$ dollars.
10. $586\frac{2}{3}$ rods.
11. $56\frac{2}{3}$, product.
12. $56\frac{2}{3}$, product.
15. $\frac{25}{28}$, product.

150.

1. $\frac{1}{3}$ of a bushel.
2. $\frac{3}{35}$ of a dollar.
3. 2.
4. $2\frac{1}{2}$ dollars.
5. $\frac{1}{3}$ of a dollar.
6. $\frac{21}{32}$ of a dollar.
7. $\frac{1}{7}$ of a dollar.
8. $13\frac{3}{4}$ dollars.
9. $\frac{21}{100}$ of a dollar.
10. 6 dollars.
11. $6\frac{11}{24}$ dollars.
12. 50 dollars.
13. $1\frac{7}{15}$ dollars.
14. 30 shillings.
15. $59\frac{1}{16}$ gallons.
16. $1482\frac{9}{14}$ dollars.
17. 1083 dollars.
18. $185\frac{5}{12}$ dollars.
19. $2\frac{21}{32}$ dollars.
20. $\frac{1}{4}$ of the ship.
21. $\frac{3}{4}$.
22. $\frac{9}{35}$.
23. $\frac{2}{11}$.

24. $\frac{2}{11}$.
25. $\frac{7}{88}$, product.
26. $\frac{1}{5}$, product.
27. $\frac{7}{20}$.
28. $3\frac{3}{4}$, product.
29. $15\frac{6}{11}$, product.
30. 98, product.
31. $\frac{2}{15}$.
32. $\frac{4}{5}$.
33. $\frac{2}{27}$.
34. $\frac{81}{625}$.

154.

1. 12 persons.
2. 16 dollars.
3. 16 days.
4. 16 days.
5. $5\frac{1}{3}$ weeks.
6. 3 bushels.
7. 24 acres.
8. 405 bottles.
9. $16\frac{2}{3}$ bushels.
10. $6\frac{6}{75}$ acres.
11. $1\frac{13}{50}$ dollars.
12. 16 dollars.
13. $69\frac{1}{3}$ hours.
14. 16 coats.
15. 200 bushels
16. $14\frac{2}{5}$ pairs.
17. $2\frac{14}{25}$ dollars.
18. $\frac{33}{128}$ of a dollar.
19. 960, quotient.
20. $31\frac{3}{14}$ times.
21. 16 times.
22. 42 times.
23. $\frac{18}{65}$, quotient.
24. $\frac{3}{7}$ of 7.
25. $\frac{3}{28}$ of 7.
26. $\frac{22}{3}$ of $\frac{3}{4}$.
27. $\frac{5}{7}$ of $2\frac{1}{3}$.
28. $\frac{4}{5}$ of $6\frac{1}{4}$.

157.

1. $\frac{9}{10}$ of a bushel.
2. $3\frac{1}{3}$ bushels.
3. $4\frac{2}{3}$ bushels.
4. 3 bushels.
5. 7 dozens.
6. $14\frac{1}{3}$ dozens.
7. 15 pounds.
8. $13\frac{1}{2}$ bushels.
9. $2\frac{2}{15}$ bushels.
10. 21 pounds.
11. $10\frac{5}{6}$ hats.
12. $9\frac{15}{28}$ yards.
13. $5\frac{11}{48}$ coats.
14. $7\frac{11}{25}$ horses.
15. $5\frac{1}{20}$ weeks.
16. $4\frac{1}{2}$ dollars.
17. $56\frac{1}{4}$ pounds.
18. $8\frac{3}{14}$ weeks.
19. $19\frac{1}{6}$ miles.
20. $2\frac{46}{67}$ barrels.
21. $2\frac{281}{404}$ hours.
22. $13\frac{127}{200}$ bushels.
23. $\frac{52}{75}$ of a dollar.
24. $\frac{52}{75}$ of once.
25. $23\frac{1}{3}$ yards.
26. $\frac{3}{70}$, quotient.
27. $23\frac{1}{3}$, quotient.
28. $4\frac{1}{2}$ pounds.
29. 5 times.
30. 10 times.
31. 8 times.
32. 2 times.
33. 8 times.
34. $\frac{2}{3}$ of a bushel.
35. $\frac{1}{24}$ of a yard.
36. $\frac{1}{24}$, quotient.
37. $\frac{7}{10}$ of 10.
38. $\frac{1}{24}$ of 3.
39. $\frac{1}{8}$ of 3.

40. $\frac{31}{60}$, quotient.
41. $\frac{31}{60}$ of 10.
42. $\frac{126}{325}$, quotient.
43. $\frac{126}{325}$ of $7\frac{2}{5}$.
44. $\frac{16}{35}$, quotient.
45. $\frac{16}{35}$ of a bushel.
46. $\frac{16}{35}$ of $\frac{7}{8}$.
47. $\frac{1}{2}$ of once.
48. $\frac{1}{2}$ of a barrel.
49. $\frac{1}{2}$ of $7\frac{1}{2}$.

VII. DECIMAL FRACTIONS.

165.

1. Five, and one hundred and eleven thousandths.
2. Three, and twelve hundredths.
3. Two, and six tenths.
4. Two tenths.
5. Twenty-five hundredths.
6. Seventy-five hundredths.
7. One hundred and twenty-five thousandths.
8. Seventeen, and three tenths.
9. One hundred and forty-four, and sixteen hundredths.
10. Three thousand four hundred and fifty-six, and four tenths.
11. Two hundred and fifty-two, and five tenths.
12. Twenty-five, and twenty-five hundredths.
13. Two, and five hundred and twenty-five thousandths.
14. Two thousand five hundred and twenty-five ten-thousandths.
15. Forty, and five tenths.
16. Four, and five hundredths.
17. Four hundred and five thousandths.
18. Three hundred and six, and one tenth.
19. Thirty, and sixty-one hundredths.
20. Three, and sixty-one thousandths.
21. Five hundredths.
22. Five thousandths.
23. Five ten-thousandths.
24. Five hundred - thousandths.
25. Five millionths.
26. Seven thousandths.
27. Seven hundred - thousandths.
28. Seventy-two thousandths.
29. Three, and four hundred and seven ten-thousandths.
30. Three, and four thousand and seven ten-thousandths.
31. Two, and forty thousand and three hundred-thousandths.
32. Two, and four hundred thousand three hundred and five millionths.
33. Five tenths.
34. Fifty hundredths.
35. Five hundred thousandths.
36. Five thousand ten-thousandths.

37. Fifty thousandths.
38. Eight, and nine hundred ten-thousandths.
39. Nine ten-millionths.
40. One, and eighty hundred millionths.

168.

1. 27.6.
2. 14.07.
3. 108.5.
4. 73.09.
5. 4.06.
6. 16.001.
7. .6.
8. .05.
9. .007.
10. .0002.
11. 13.23.
12. 1.043.
13. 17.0573.
14. .0807.
15. .40.
16. .420.
17. .300.
18. .080.
19. .02000.
20. .000205.
21. 17.409.
22. 6.065.
23. 7.0007.
24. .10809.
25. 26.000015.
26. 3.0101.
27. 4.00025.
28. 8.000604.
29. 1.0060005.
30. 2.30000.
31. 200 thousandths.
32. 250 hundredths.
33. $\frac{300}{1000}=.300$.
34. $\frac{1}{8}$.
35. $\frac{1}{4}$.
36. $\frac{4}{15}$.
37. .333.
38. .397.

172.

1. 1600 cents.
2. 16000 mills.
3. 1200 cents.
4. 12 dollars.
5. 750 mills.
6. 825 cents.
7. 5125 mills.
8. $512\frac{5}{10}$ cents.
9. 3 dolls., 37 cts., 5 mills.
10. 16.125 dollars.
11. 125.48 dollars.
12. 3750 cents.
13. 75.625 dollars.
14. .984 of a dollar.
15. 75 cents.
16. 125 mills.
17. 1250 mills.
18. $237\frac{5}{10}$ cents.
19. 12 dolls., 34 cts., 5 mills.
20. 8.5 times.
21. 6.25, quotient.
22. 1.836, quotient.
23. 17.28.
24. 12.76 times.
25. 1225 hundredths.

174.

1. $177.25.
2. $2.50.
3. $24.75.
4. $76.75.
5. $75.353.
6. 167.25 yards.
7. 22 yards.
8. 5 bushels and $16.
9. $65.625.
10. 26 sum.
11. 10.
12. 213.003.

13. 6.25.
14. 480, sum, and 10.015, difference.
15. 2.15, difference.

177.

1. 6.25 yards.
2. $23.
3. $1
4. $1.
5. $86.25.
6. $55.
7. $5.
8. $20.
9. $10.20.
10. $1.50.
11. 42.664, product.
12. 11.6, product.
13. 2.5.
14. $55.975.
15. $3.
16. $17.50.
17. $1.50.
18. $6.
19. $6.25.
20. $620.875.
21. 72, product.
22. 864.
23. 128, product.
24. 642.
25. 230, product.
26. 625.
27. .3, product.
28. .9.
29. $.384.
30. $.25.
31. .18, product.
32. .28.
33. .125, product.
34. $.18.
35. $2.06.
36. $196.50.
37. .2856.
38. 1.223227.
39. .00016.
40. .00000072.
41. .002012.
42. .000186.
43. .00001.
44. .25.
45. .0625.
46. .125.
47. .0625.

179.

1. 1.75 yards.
2. $.125.
3. $1.50.
4. .75 of a bushel.
5. $.125.
6. .6 of an acre.
7. .6.
8. .75.
9. .25.
10. .125.
11. .3125.
12. .25 of once.
13. 2.4 times.
14. .36, quotient.
15. .02, quotient.
16. .064, quotient.
17. .0625.
18. .05 times.
19. .05 of once.
20. .46875, quotient.
21. .625, quotient.

182.

1. 3.55 times.
2. .025, quotient.
3. 7.051 times.
4. .108, quotient.
5. .1217, quotient.
6. 8.474 tons.

7. .35 of an hour.
8. 11.5 years.
9. 21.6 months.
10. .030457, quotient.
11. .11925, quotient.
12. .864 of a ton.
13. .013964 times.
14. 246.4 hours.
15. 36.25 eagles.

185.

1. $.08\dot{3}$.
2. $.41\dot{6}$.
3. $.58\dot{3}$.
4. $.91\dot{6}$.
5. $.8\dot{3}$.
6. $.12\dot{7}$.
7. $\frac{8}{15}$.
8. $\frac{7}{15}$.
9. $\frac{161}{495}$.
10. $\frac{8}{33}$.
11. $.\dot{0}4\dot{5}$.
12. $.37\dot{4}$.

187.

1. $\$4444.44\dot{4}$.
2. $1.
3. $1.
4. $1.
5. $\$83.33\dot{3}$, or $\$83\frac{1}{3}$.
6. $\$.1\dot{6}$, or $\$.16\frac{2}{3}$.
7. $\$4.44\dot{4}$, or $4\frac{4}{9}$ dollars.
8. $\$.8\dot{3}$, or $83\frac{1}{3}$ cents.
9. $\$.041\dot{6}$, or $4\frac{1}{6}$ cents.
10. $\$.08\dot{3}$, or $8\frac{1}{3}$ cents.

191.

1. 8 umbrellas.
2. 4 pairs.
3. 112 pounds.
4. 75 pounds.
5. 16 yards.
6. 25 yards.
7. 30 yards.
8. 160 yards.
9. 27.125 yards.
10. 3.7 yards.
11. $6.30.
12. $5.75.
13. $1.25.
14. $.125.
15. $125.78.
16. $.78.
17. $2.36.
18. 124 pounds.
19. $.065.
20. 560 times.
21. 40 times.
22. 39 times.
23. 6.25 times.
24. .75, quotient.
25. 25.63, quotient.
26. .006, quotient.
27. .1728, quotient.
28. 5.01, quotient.
29. .375 of 8.
30. .0375 of 8.
31. .25 of 2.4.
32. .5 of 10.35.
33. .3 times.
34. .2 times.
35. .01, quotient.
36. .00103, quotient.
37. .0004, quotient.
38. 470, quotient.
39. 10000, quotient.
40. .002 of 1.006.

VIII. COMPOUND NUMBERS.

210.

1. 34001 feet.
2. 6 m. 3 fur. 20 rods, 3 yds. 2 feet.
3. 8000 rods.
4. 50 miles.

211.

1. 439 nails.
2. 100 yds. 3 qrs.
3. $13.6875.
4. 301 yards.

212.

1. 256080 sq. rods.
2. 320 acres.
3. $2450.25.
4. 2 roods, 35 rods, 196.25 ft.

213.

1. 212688 cub. inches.
2. 1 yd. 20 feet, 432 inches.
3. 259200 cub. inches.
4. 3 tons, 5 ft.
5. $32.25.
6. 13 cords, 1.6 cord-feet.

214.

1. 188 quarts.
2. 5 bushels.
3. $3.
4. $2.

215.

1. 206 pints.
2. 54 gal. 1 pint.
3. $150.
4. 31 gal. 77 cub. inches.

216.

1. 45681 grains.
2. 7 lbs. 11 oz. 3 dwt. 9 grs.
3. $437.60.
4. 8 lb. 5 oz. 11 dwt. 16 grs.

217.

1. 9272 grains.
2. 1 lbs. 8℥, 4ʒ, 1℈, 16 grs.
3. 39836 grains.
4. 6 lbs. 10℥, 7ʒ, 2℈, 16 grs.

218.

1. 83213 oz.
2. 3 tons, 11 lbs. 11 oz. 8 drs.
3. $303.75.
4. 3 tons, 810 lbs.

219.

1. 386522430 seconds.
2. 365 days.
3. 504921600 times.
4. 5 years, 131 days, 1 hour.

220.

1. 162000 seconds.
2. 6 signs, 28° 10′ 8″.
3. 13260 seconds.
4. 1° 39′.

221.

1. 24507.
2. £10, 3s. 1d. 2qr.
3. $503.
4. $25

222.

1. [illegible]qr.

2. 12s. 4d.
3. $\frac{1}{144}$ of a rod.
4. 6 fur. 26 rods. 11 ft.
5. $\frac{5}{64}$ of a sq. rod.
6. 3 roods, 13 rods, 10 yds. 108 inches.
7. $\frac{4}{5}$ of an ounce.
8. 8 oz. 11 dwt. $10\frac{2}{3}$ grs.
9. $\frac{7}{16}$ of a grain.
10. 5ʒ, $5\frac{4}{11}$ grains.
11. $\frac{4}{5}$ of a dram.
12. 138 lbs. 14 oz. $3\frac{3}{5}$ drs.
13. $\frac{2}{3}$ of a quart.
14. 20 quarts.
15. $\frac{32}{231}$ of a gill.
16. 1 qt. 1 pt. $1\frac{1}{3}$ gills.
17. 2 cub. inches.
18. 1152 cub. inches.
19. $\frac{2}{5}$ day.
20. 21 h. 36 m.
21. 190″.
22. 10 s. 15°.

225.

1. £$\frac{1}{400}$.
2. £$\frac{3}{40}$.
3. $\frac{1}{512}$ miles.
4. $\frac{3}{8}$ of a mile.
5. $\frac{1}{2640}$ of an acre.
6. $\frac{16133}{19360}$ acres.
7. $\frac{5}{168}$ lb.
8. $\frac{3}{5}$ lb.
9. $\frac{7}{46080}$ lb.
10. $\frac{1}{30}$ʒ.
11. $\frac{23}{48}$ʒ.
12. $\frac{3}{8000}$ ton.
13. $\frac{64051}{128000}$ of a ton.
14. $\frac{1}{96}$ of a bushel.
15. $\frac{7}{8}$ bushel.
16. $\frac{2}{141}$ gal.
17. $\frac{13}{32}$ gallon.
18. $\frac{1}{324}$ of a yard.
19. $\frac{1}{576}$ cub. yd.

27

20. $\frac{4}{1461}$ year.
21. $\frac{17}{360}$ day.
22. $\frac{1}{30}$ deg.
23. $\frac{32481}{864000}$ circle.

227.

1. 2 pk. 1 gall.
2. 2 fur. 20 rods.
3. 2 ft. 6 in.
4. 1 rood, 20 rods.
5. 24 cub. feet.
6. 2 qt. 1 pt. $1\frac{1}{3}$ gill.
7. 7ʒ.
8. 888 lbs. 14 oz. $3\frac{3}{5}$ dr.
9. 273 d. 22 h. 30 m.
10. 12° 12′ 12″+.
11. 12s. 6d.

229.

1. .421875 of a bushel.
2. .423+ mile.
3. .49+ yard.
4. .839+ sq. rod.
5. .2016 ton.
6. .46875 gall.
7. .0875 lb.
8. .75 of a ton.
9. .08487+ year.
10. .0416 year.
11. £.528125.

232.

1. 5s.
2. £.325
3. 12s. 6d.
4. £.375.
5. 17s. 6d.
6. £.224 —.
7. 6s. 5d. 2qr.
8. £.267—.
9. 2s. 4d. 3qr.
10. £.019 —.

11. 1s. 6d.
12. £.976.
13. 9s. 7d. 1qr.
14. £.649.
15. 3s. 9d.
16. £.061+.
17. 13s. 4d.
18. £.33$\dot{3}$.
19. £12, 3s. 7d. 1qr.
20. £125.5625.

236.

1. 111 y. 1 qr. 1 n.
2. 61 m. 7 fur. 16 rods, 5 yds.
3. 50 m. 1 fur. 32 rods.
4. 148 acres, 32 rods.
5. 3 tons, 36 ft. 1209 inches.
6. 112 bush. 3 pks.
7. 197 gallons.
8. 2 oz. 6 dwt. 21 grs.
9. 10℥, 1ʒ, 1℈, 10 grs.
10. 2 tons, 1184 lbs.
11. 4 tons, 696 lbs. 12 oz.
12. £36, 3s. 9d.
13. 38 y. 4 days, 20 hours.

239.

1. 15 yds. 1 qr. 1 n.
2. 4 c. 4 c. ft. 6 cub. ft.
3. 59 bush. 1 pk.
4. 76 gall. 3 qt.
5. 1 dwt. 3 grs.
6. £8, 6s. 2d.
7. 2 h. 55 m. 15 sec
8. 4 h. 16 min.
9. 97° 30′. 12° 34′.
10. 55° 59′ 58″.

241.

1. 1 lb. 3 oz. 12 dwt. 12 grs.
2. 2314 m. 5 fur. 20 rods.
3. 969 yds. 1 nail.
4. 32 acres, 3 roods, 25 rods.
5. 34 yds. 18 cub. ft.
6. 85 gal. 1 qt. 2 gills.
7. 371 bush. 1 pk.
8. 5℥, 6ʒ.
9. 1 ton, 940 lbs.
10. £5, 12s. 6d.
11. 243 days, 12 hours.
12. 226° 30′.
13. 4 lb. 7 oz. 6 dwt. 6 grs.
14. 606 m. 6 fur. 15 rods.
15. 2196 yds.
16. 9 m. 27 acres, 2 roods.
17. 8902 tons, 48 ft. 256 in.
18. 1266 gals. 1 qt. 1 pt 1 gill.
19. £1017, 3s. 6d.
20. 19 tons, 1501 lbs. 14 oz.
21. 558 y. 58 d. 5 h.

243.

1. 17 dwt. 8 gr.
2. 5 fur. 27 rods.
3. 5 y. 2 qr.
4. 8 acres, 1 rood, 30 rods.
5. 1 cord, 2 cd. ft. 4 cub. ft.
6. 2 qts. 1 pt. 3 gills.
7. 2 bush. 1 pk.
8. 3ʒ, 9⅓ grs.
9. 2 oz. 1 dr.
10. 6s. 5d. 3qrs.
11. 6 min. 36 sec.
12. 12° 12′ 12.2″+.

245.

1. 2 d. 9 h. 18 min.
2. 1 acre, 16 rods.
3. 5 yds. ⅘ of a nail.
4. 11988.48 cub. inches.
5. 2 bush. 2 qts.
6. 10 qts.
7. 7 oz. 1 dwt. 20 grs.
8. $6.80.

9. 1 ton, 456 lbs. 3 oz.
10. £1, 4s. 6d. $2\frac{2}{3}$qr.
11. No difference.
12. 44° 8′ 20″.

247.

1. 8 years, 4 m. 1 day.
2. 3 y. 2 m. 1 day.
3. 5 y. 25 days.
4. 33 y. 3 m. 4 days.
5. Sept. 21st, 1837.
6. Dec. 4th, 1839.
7. 2 y. 7 m. 4 d.
8. 1 y. 1 m. 24 d.

249.

1. $972.66—.
2. £2189, 13s. 6d—.
3. 10 ft. 4.592 inches.
4. $$29.56\frac{1}{4}$.
5. $$81.76\frac{7}{8}$.
6. £90, 17s. 9d. 3qr.
7. £67, 14s. 6d.
8. £1, 2s. 7d. $3\frac{1}{2}$qr.
9. £3, 1s. 6d. 3qr—.

251.

1. 40 sq. in.
2. $30\frac{1}{4}$ sq. yd.
3. $247\frac{1}{2}$ sq. ft.
4. $27\frac{1}{3}$ sq. yds.
5. $24\frac{3}{4}$ sq. rods.
6. 1 acre.
7. 26.625 ft.
8. 16 cub. ft.
9. 12 boards.
10. 2304 cub. in.
11. 27648 cub. in.
12. 27648 cub. in.
13. 64.8 cub. in.
14. 202.5 cub. ft.
15. 105 cub. ft.
16. $71.71\frac{7}{8}$ cub. ft.
17. 84 cub. yds.
18. 46.2 cub. yds.
19. $2868.9\frac{1}{16}$ cub. ft.
20. 8 cord ft.
21. 7 cords.

253.

1. 11 c. $4\frac{1}{2}$ cd. ft.
2. $$11.81\frac{1}{4}$.
3. $25.
4. $$28.43\frac{3}{4}$.
5. $6\frac{27}{32}$ acres.
6. $71\frac{37}{108}$ sq. yds.
7. $19\frac{14}{27}$ yds.
8. 400 days.
9. 9888 times.
10. 55 times.
11. $$13.33\frac{1}{3}$.
12. $180.
13. $18\frac{1}{14}$ loads.
14. 168 pills.
15. $205\frac{24}{73}$ crowns.
16. 624 yds.
17. £.75.
18. $\frac{27}{35}$ of a yard.
19. $\frac{63}{143}$ of £1, 15s. 9d.

255.

1. (See 205.)
2. (See 205.)
3. (See 205.)
4. $$113.33\frac{1}{3}$.
5. £5, 14s. 2d. 1qr.
6. $2000.
7. $250, gain.
8. $$2582.08\frac{1}{3}$.
9. $$54.68\frac{3}{4}$.
10. 3000 yards.
11. $516\frac{2}{3}$ yds.
12. New York,

13. \$92.88$\frac{8}{9}$, in Sterling.
\$83.60, in Ca. cur.
\$69.66$\frac{2}{3}$, in N. E. cur.
\$52.25, in N. Y. cur.
\$55.73$\frac{1}{3}$, in Penn. cur.
\$89.57$\frac{1}{7}$, in Ga. cur.
14. 1s. 1d. 2qr., Sterling.
1s. 3d., Ca. cur.
1s. 6d., N. E. cur.
2s., N. Y. cur.
1s. 10d. 2qr., Pa. cur.
1s. 2d., Ga. cur.
15. \$.33$\frac{1}{3}$, Sterling.
\$.30, Ca. cur.
\$.25, N. E. cur.
\$.18$\frac{3}{4}$, N. Y. cur.
\$.20, Pa. cur.
\$.32$\frac{1}{7}$, Ga. cur.

258.

1. \$203.
2. 151 lbs.
3. \$24.
4. 24 times.
5. \$432.
6. 74 bushels.
7. \$9 dollars.
8. \$15.
9. 24 pairs.
10. \$20.
11. 100 pairs.
12. 25 hats.
13. \$360.
14. 7 yards.
15. \$210.
16. 114 bushels.
17. 73 days.
18. \$44.

260.

1. \$13.
2. \$13.50.
3. \$63.
4. \$7.33$\frac{1}{3}$.
5. \$3.95+.
6. \$16.50.
7. 2070, product.
8. \$126.
9. \$5.50.
10. \$130.
11. \$40.50.

263.

1. $.041\dot{6}$.
2. $\frac{5}{24}$.
3. $.291\dot{6}$, $.458\dot{3}$, and $.541\dot{6}$.
4. $\frac{7}{24}$, $\frac{11}{24}$, and $\frac{13}{24}$.
5. .0625.
6. $\frac{3}{16}$.
7. .4375, .5625, .6875.
8. $\frac{13}{16}$, and $\frac{15}{16}$.
9. $.08\dot{3}$.
10. $\frac{7}{12}$.
11. $.91\dot{6}$.
12. $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$.
13. $.1\dot{6}$, $.8\dot{3}$.
14. .25, and .75.
15. $\frac{1}{3}$, and $\frac{2}{3}$.
16. $.41\dot{6}$ ft.
17. .3125 lbs.
18. 7 ounces.
19. $.291\dot{6}$ dwt.
20. 5 furlongs.

IX. PROPORTION.

265.

1. $\frac{4}{7}$.
2. $\frac{12}{5}$.
3. $\frac{1}{16}$.

4. $\frac{7}{20}$.
5. $\frac{8}{1}$.
6. $\frac{20}{27}$.
7. $\frac{24}{49}$.
8. $\frac{1}{3}$.
9. $\frac{2}{3}$.
10. $\frac{1}{10}$.
11. $\frac{5}{7}$.
12. $\frac{7}{16}$.
13. .45.
14. $\frac{10}{11}$.

267.

1. $\frac{1}{2}$ of \$24 = \$12.
2. $\frac{1}{3}$ of \$36 = \$12.
3. $\frac{3}{4}$ of \$4.20 = \$3.15.
4. $\frac{2}{3}$ of \$7.50 = \$5.
5. $\frac{23}{5}$ of \$.75 = \$3.45.
6. $\frac{5}{6}$ of 18 bu. = 15 bushels.
7. $\frac{60}{25}$ or $\frac{12}{5}$ of \$2.25 = \$5.40.
8. $\frac{12}{5}$ of \$87.50 = \$210.
9. $\frac{186}{54}$ or $\frac{31}{9}$ of 9 barrels = 31 barrels.
10. $\frac{3000}{480}$ or $\frac{25}{4}$ of 5 days = $31\frac{1}{4}$ days.
11. $\frac{75}{30}$ or $\frac{5}{2}$ of \$200 = \$500.
12. $\frac{8}{3}$ of \$$\frac{7}{8}$ = \$2.33$\frac{1}{3}$.

271.

1. $\frac{5\frac{2}{7}}{37}$ or $\frac{1}{7}$ of \$185 = \$$26\frac{3}{7}$.
2. $\frac{6}{\frac{25}{36}}$ or $\frac{216}{25}$ of \$12.50 = \$108.
3. $\frac{25}{4\frac{2}{7}}$ or $\frac{35}{6}$ of \$$\frac{25}{7}$ = \$$20\frac{5}{6}$.
4. $\frac{5\frac{2}{5}}{12}$ or $\frac{9}{20}$ of \$$\frac{50}{3}$ = \$7.50.

5. $\frac{1}{\frac{7}{8}}$ or $\frac{8}{7}$ of \$$\frac{5}{9}$ = \$.635—.
6. $\frac{7\frac{1}{2}}{1\frac{7}{8}}$ or 4 times $\frac{11}{7}$ w. = $6\frac{2}{7}$ weeks.
7. $\frac{8\frac{4}{9}}{12\frac{2}{3}}$ or $\frac{2}{3}$ of $72\frac{3}{7}$ a. = $48\frac{2}{7}$ acres.
8. $\frac{5}{3\frac{3}{4}}$ or $\frac{4}{3}$ of \$10 = \$$13\frac{1}{3}$.
9. $\frac{43\frac{3}{4}}{18\frac{3}{4}}$ or $\frac{7}{3}$ of \$33.75 = \$78.75.
10. $\frac{23}{2\frac{5}{8}}$ or $\frac{184}{21}$ of \$42.00 = \$368.
11. $\frac{12\frac{1}{2}}{2}$ or $\frac{25}{4}$ of £$\frac{9}{20}$ = £2, 16s. 3d.

275.

1. $\frac{9}{12}$ or $\frac{3}{4}$ of 32 d. = 24 days.
2. $\frac{15}{24}$ or $\frac{5}{8}$ of 72 m. = 45 men.
3. $\frac{6}{8}$ or $\frac{3}{4}$ of 12 w. = 9 weeks.
4. $\frac{200}{75}$ or $\frac{8}{3}$ of 10 q. = $26\frac{2}{3}$ quarts.
5. $\frac{4\frac{1}{2}}{5}$ or $\frac{9}{10}$ of £18 = £16, 4s.
6. $\frac{5}{4\frac{1}{2}}$ or $\frac{10}{9}$ of £36 = £40.
7. $\frac{5}{6}$ of £45 = £37, 10s.
8. $\frac{4\frac{1}{2}}{6}$ or $\frac{3}{4}$ of £36 = £27.
9. $\frac{6}{8}$ or $\frac{3}{4}$ of £72 = £54.

10. $\frac{8}{12}$ or $\frac{2}{3}$ of 20 rods = $13\frac{1}{3}$ rods.

11. $\frac{16}{16\times12}$ or $\frac{1}{12}$ of 9 in. = $\frac{3}{4}$ of an inch.

12. [illegible] or $\frac{3}{4}$ of $37\frac{1}{2}$ ft. = $28\frac{1}{8}$ feet.

13. $\frac{144}{16}$ or 9 times 18 in. = 13 ft. 6 in.

14. $\dfrac{1\frac{1}{2}}{\frac{3}{4}}$ or 2 times $12\frac{1}{2}$ y. = 25 yards.

15. $\dfrac{\frac{7}{8}}{1\frac{3}{4}}$ or $\frac{1}{2}$ of $91\frac{1}{2}$ y. = $45\frac{3}{4}$ yards.

16. $\dfrac{5\frac{1}{4}}{1\frac{1}{4}}$ or $\frac{21}{5}$ of 10 b. = 42 bushels.

17. $\dfrac{2\frac{3}{4}}{\frac{1}{12}}$ or 33 times 12 = 396 pounds.

18. $\frac{700}{1000}$ or $\frac{7}{10}$ of 13 w. = $9\frac{1}{10}$ weeks.

278.

1. $\frac{25}{15}$ or $\frac{5}{3}$ of \$48 = \$80.
2. $\frac{24}{16}$ or $\frac{3}{2}$ of \$176 = \$264.
3. $\frac{20}{12}$ or $\frac{5}{3}$ of 42 b. = 70 bushels.
4. $\frac{18}{45}$ or $\frac{2}{5}$ of \$225 = \$90.
5. $\frac{600}{75}$ or 8 times 9 lbs. = 72 lbs.
6. $\frac{56}{12}$ or $\frac{14}{3}$ of \$30 = \$140.
7. $\frac{35}{5}$ or 7 times \$375.75 = \$2630.25.
8. $\dfrac{52\frac{1}{2}}{10\frac{1}{2}}$ or 5 times 7 pairs = 35 pairs.
9. $\dfrac{25\frac{1}{2}}{4\frac{1}{4}}$ or 6 times 19 lb. = 114 lbs.
10. $\frac{21}{24}$ or $\frac{7}{8}$ of $\$1\frac{1}{2}$ = $\$1.31\frac{1}{4}$.

11. $\dfrac{25}{1\frac{3}{4}}$ or $\frac{100}{7}$ of \$.42 = \$6.

12. $\dfrac{10}{8\frac{3}{4}}$ or $\frac{8}{7}$ of \$4.20 = \$4.80.

13. $\dfrac{\frac{5}{6}}{\frac{8}{9}}$ or $\frac{15}{16}$ of \$6 = $\$5.62\frac{1}{2}$.

14. $\dfrac{12\frac{1}{2}}{\frac{3}{5}}$ or $\frac{125}{6}$ of $\$\frac{7}{8}$ = \$18.23—.

15. $\dfrac{\frac{3}{32}}{\frac{7}{16}}$ or $\frac{3}{14}$ of \$2520 = \$540.

16. $\dfrac{212}{5\frac{1}{3}}$ or $\frac{159}{4}$ of an ounce = 3 lb. 3 oz. 15 dwt.

17. $\frac{153}{6}$ or $\frac{51}{2}$ of $\frac{17}{3}$ ft. = $144\frac{1}{2}$ ft.

18. $\dfrac{51\frac{1}{4}}{68}$ or $\frac{205}{272}$ of \$791.18 = $\$596.29\frac{3}{8}$.

19. $\dfrac{31\frac{1}{2}}{12}$ or $\frac{21}{8}$ of \$30 = 78.75.

20. $\dfrac{21}{200\frac{1}{2}}$ or $\frac{42}{401}$ of \$100.25 = 10.50.

21. $\dfrac{5\frac{1}{4}}{1\frac{1}{2}}$ or $\frac{7}{2}$ of \$12.50 = \$43.75.

22. $4\frac{3}{4}$ times \$6 = \$28.50.

23. $\dfrac{12\frac{1}{2}}{17\frac{1}{2}}$ or $\frac{5}{7}$ of \$1.05 = \$.75.

24. $175\frac{1}{4}$ times \$.06 = $\$10.51\frac{1}{2}$.

280.

1. $\frac{4}{8}$ or $\frac{1}{2}$ of 20 d. = 10 days.
2. $\frac{21}{14}$ or $\frac{3}{2}$ of 6 m. = 9 men.

3. $\frac{12}{9}$ or $\frac{4}{3}$ of 6 d. = 8 days.
4. $\frac{1500}{2000}$ or $\frac{3}{4}$ of 150 m. = $112\frac{1}{2}$ miles.
5. $\frac{800}{300}$ or $\frac{8}{3}$ of 2 m. = $5\frac{1}{3}$ months.
6. $\frac{6}{21}$ or $\frac{2}{7}$ of 91 d. = 26 days.
7. $\frac{5}{8}$ of $11\frac{2}{3}$ = $7\frac{7}{24}$ days.
8. $\frac{12}{9\frac{1}{2}}$ or $\frac{24}{19}$ of $\frac{38}{5}$ d. = $9\frac{3}{5}$ days.
9. $\frac{\frac{5}{6}}{1\frac{1}{4}}$ or $\frac{2}{3}$ of $\frac{60}{7}$ d. = $5\frac{5}{7}$ days.
10. $\frac{3\frac{1}{4}}{2\frac{7}{8}}$ or $\frac{26}{23}$ of $11\frac{1}{2}$ w. = 13 weeks.
11. $\frac{625}{250}$ or $\frac{5}{2}$ of 40 b. = 100 bushels.
12. $\frac{4\frac{1}{2}}{6}$ or $\frac{3}{4}$ of 25 l. = $18\frac{3}{4}$ loaves.
13. $\frac{24}{64}$ or $\frac{3}{8}$ of 200 lbs. = 75 pounds.
14. $\frac{6}{9}$ or $\frac{2}{3}$ of 783 lbs. = 522 pounds.
15. $\frac{2\frac{3}{4}}{16\frac{1}{2}}$ or $\frac{1}{6}$ of 165 y. = $27\frac{1}{2}$ y.
16. $\frac{370}{500}$ or $\frac{37}{50}$ of $\$.33\frac{1}{3}$ = $\$.24\frac{2}{3}$.
17. $\frac{7}{9}$ of 43 b. = $33\frac{4}{9}$ bushels.
18. $\frac{\frac{1}{3}}{\frac{1}{6}}$ or 2 times 18 = 36 lbs.
19. $\frac{20}{1\frac{1}{2}}$ or $\frac{40}{3}$ of 13 = $173\frac{1}{3}$ bottles.
20. $\frac{231}{282}$ of $31\frac{1}{2}$ g. = $25\frac{151}{188}$ gallons.
21. $\frac{416}{252}$ of 10 = $16\frac{32}{63}$ spoons.
22. 2 times 7 y. = 14 yards.
23. $\frac{3}{4}$ of 9 y. = $6\frac{3}{4}$ yards.
24. $\frac{\frac{5}{8}}{\frac{3}{4}}$ or $\frac{5}{6}$ of 12 y. = 10 yards.
25. $\frac{15}{6}$ or $\frac{5}{2}$ of 20 b. = 50 boards.
26. $\frac{4}{2\frac{1}{2}}$ or $\frac{8}{5}$ of 40 r. = 64 rods.
27. $\frac{160}{8}$ of 1 rod = 20 rods.
28. $\frac{144}{9}$ of 1 in. = 16 inches.
29. $\frac{1\frac{1}{2}}{\frac{3}{4}}$ or 2 times $1\frac{7}{8}$ yards = $3\frac{3}{4}$ yards.
30. 8 times 3 m. = 24 months.
31. $\frac{1}{3}$ of 24 m. = 8 months.
32. $\frac{30}{12}$ or $\frac{5}{2}$ of 6 m. = 15 months.
33. $\frac{25}{75}$ or $\frac{1}{3}$ of 12 m. = 4 months.
34. $\frac{225}{675}$ or $\frac{1}{3}$ of 12 m. = 4 months.
35. $\frac{100}{150}$ or $\frac{2}{3}$ of 12 m. = 8 months.
36. $\frac{100}{33\frac{1}{3}}$ or 3 times 1 y. = 3 years.
37. $\frac{500}{75}$ or $\frac{20}{3}$ of 6 m. = 40 months.

284.

1. 20 men.
2. $19\frac{1}{3}$ bushels.
3. \$948.89 —.
4. 2808 quarters.
5. 80 days.
6. 120 days.
7. 18 men.
8. 168 tailors.
9. \$9.375.
10. 270 lbs.

11. 20 lbs.
12. 15 lbs.
13. $351.40.
14. 13⅓ lbs.
15. 432 barrels.
16. 648 tiles.
17. 43200 bricks.
18. 14400 shingles.
19. 4320 slates.
20. 160632 bricks.
21. 12½ lbs.
22. $150.
23. $180.
24. $.75.
25. 6 months.
26. $6.
27. $25.
28. $42.50.
29. $80.55.

286.

1. 2 tons, 1760 lbs.
2. 40 lbs.
3. 18 days' work.
4. 13⅛ lbs.
5. £133⅓, N. E.
6. 80 bushels.
7. 83⅓s. Ca.
8. $1.20 +.

290.

1. 45 bushels.
2. 16⅔ bushels.
3. 12 cords.
4. 40 yards.
5. 21 $\frac{9}{11}$ bushels.
6. 25 days.
7. 26 $\frac{1}{11}$ gallons, and 91½ lbs.
8. 8000 ft.
9. $.10.
10. $2000.
11. 5 dozen.
12. £131, 5s.

292.

1. $17.50, A's; $10.50, B's.
2. $150, A's; $120, B's.
3. $.75, George's; $.50, Charles'; $.25, John's.
4. $60.
5. $60, A's; 40, B's.
6. $300, C's; $200, D's.
7. $300, E's; $400, F's.
8. $100, A's; $60, B's; $140, C's.
9. $300, A's; $340, B's; $360, C's.
10. $420, Isaac's; $450, George's; $630, Nath's.
11. $600, White's; $284, Black's; $1768, Green's.
12. 100 tons, A's; 71.2 tons, B's; 18.8 tons, C's.
13. $13500, A's; $18000, B's; $6000, C's.
14. $36, A's; $24, B; $15, C's.
15. $96, A's; $144, B's; $240, C's.

294.

1. $18, A's; $15, B's.
2. $16, A's; $20, B's.
3. $22.50, A's; $20, B's; $30, C's; $28, D's.
4. $180, A's; $90, B's; $120, C's.
5. $288, E's; $324, F's; $108, G's.
6. $239.25 +, L's; $249.22 +, M's; $311.53 —, P's.

7. $401.70, R.'s; $370.50, S's.

8. 533\frac{1}{3}$, Mason's; 1741\frac{2}{3}$, Carpenter's; $225, Painter's

X. PERCENTAGE.

297.

1. 1233.12 lbs.
2. 22.4 lbs.
3. 156.24 lbs.
4. 266 lbs.
5. $133, house-rent; $175, stores; $105, clothes; $91, other purposes; $196, saved.
6. $397.50.
7. 30 m. 1st day; 36 m. 2d day; 42 m. 3d day; 12 m. 4th day.
8. 105.56 lbs.
9. $58.85
10. $7.50.
11. $100.98.
12. 108 in Arithmetic; 27 in Algebra; 24 in Geometry; 126 in Grammar; 45 in Geography; 18 in Philosophy; 12 in Astronomy.
13. $.125.

300.

1. $45.94 —.
2. $96.50.
3. 5.87\frac{1}{2}$.
4. $525.
5. $1543.745.
6. $15.99 —.

302.

1. $18.
2. $1080.
3. $42.
4. $9.375.
5. $75.
6. $600.

304.

1. $20.
2. $2000.
3. $213.75.
4. $6.375.
5. $26.72.
6. $7098.28.

305

1. $23.775.
2. $42.63.
3. $38.
4. 25.03\frac{1}{4}$.

309.

1. $52.80.
2. $5412.60.
3. 1464.09\frac{3}{8}$.
4. $330.75
5. $1976.
6. $14.40.
7. $200.

312.

1. $81.
2. $20.
3. $12.55.
4. $2.28 +.
5. $49.08 +.
6. $27.19 —.
7. $22.50.
8. $.35.

9. $1.03\frac{1}{3}$.
10. $1.09 —.
11. $1.71 —.
12. $1.21.
13. $37.50.
14. $15.62\frac{1}{2}$.
15. $4.79 +.
16. $3.77 +.
17. $5.29 +.
18. $5.96 +.
19. $63.
20. $18.91 —.
21. $15.
22. $1177.
23. $1365.97 +.
24. $369.72 +.
25. $886.98 —.

316.

1. $336.42.
2. $244.82.
3. $410.80.
4. $1044.30.
5. $940.50.
6. $1386.23.

319.

1. $334.99.
2. $902.42.
3. $507.68.
4. $1010.295.
5. 1240.15\frac{5}{8}$

322.

1. $1009.98.
2. $86.38.
3. $230.08.
4. $1381.27.
5 $15.22.
6. $139.45.
7. $298.50.
8. $3.82.

325.

1. $520.44.
2. $860.715.

328.

1. 15 days.
2. 1 y. 1 month.
3. 6 months.
4. 16 y. 8 months.
5. 16 y. 8 months
6. 20 years.

331.

1. 6$\frac{2}{3}$ per cent.
2. 6 per cent.
3. 35 per cent.
4. 10$\frac{2}{10}$ per cent. & $.612 +.
5. $\frac{1}{2}$ per cent.
6. $\frac{1}{2}$ per cent.
7. 6$\frac{1}{2}$ per cent.
8. 20 per cent.
9. 12$\frac{1}{2}$ per cent.
10. 2$\frac{1}{10}$ tenths per cent.
11. 11$\frac{1}{9}$ per cent.

334.

1. 6 per cent.
2. 8 per cent.
3. 6 per cent.
4. 8 per cent.
5. 7 per cent.

337.

1. $257.143 —.
2. $12500.
3. $840.
4. $3360.

340.

1. $332.80.
2. $1250.

3. 16.66\frac{2}{3}$.
4. $2000.
5. $500.
6. $6.25.

343.

1. $60.
2. $60.
3. $48.
4. $30.
5. $3.75.
6. $3.60.
7. $6.

346.

1. $1200.
2. $360

3. $609.45 —.
4. $800.

349.

1. 6$\frac{1}{2}$ months, or 6 m. 14.8 days.
2. 60 days, or 59.9 days.
3. 7$\frac{1}{2}$ months, or 7 m. 14 days.
4. 3 months, or 3 m. 1 day.
5. 2 months, or 1 m. 28 days.
6. 2 m. 20 days, or 2 m. 17 days.
7. April 26th.
8. Lose, $.02 +.
9. $1509.13 +, or $1509.16 +.

XI. ALLIGATION.

352.

1. $.76 —.
2. $.32 +.
3. 1.12\frac{1}{2}$.
4. 20.8 carats fine.
5. 10 ounces fine.

356.

1. 1 lb. at $.50; 5 lbs. at $.80.
2. 1 bush. at $.50; 2 bush. at $.75; 7 bush. at $1.
3. 2 gallons at $1.50; 1 gallon at $2; 2 gallons of water.
4. 1 lb. at 5 cents; 1 lb. at 7 cents; 4 lbs. at 11 cents; 1 lb. at 14 cents.
5. 1 oz. at 15 carats fine; 1 oz. at 20 carats fine; 1 oz. at 22 carats fine; 2 oz. at 24 carats fine.

259.

1. 28 gallons of water.
2. 1 barrel at $6; 2 barrels at $5; 7 barrels at $12.
3. 3 lbs. at $.50; 2 lbs. at $.62$\frac{1}{2}$; 7 lbs. at $1.
4. 14 oz. at 18 carats fine; 1 oz. at 19 carats fine; 1 oz. at 21 carats fine.

360.

1. 15 gallons of wine, 5 gallons of water.
2. 16 lbs. at 8 cents; 32 lbs. at 10 cents; 64 lbs. at 14 cents.
3. 4 oz. at 18$\frac{1}{2}$ carats fine; 2 oz. at 20 carats fine; 6 oz. at 23 carats fine.

XII. POWERS AND ROOTS.

368.

1. 32.
2. 27.
3. 243.
4. 256.
5. 1024.
6. 625.
7. 7776.
8. 2401.
9. 343.
10. 19.
11. 81.
12. 41.569 +.
13. 1.72 +.
14. 6.5 inches.
15. 5.5 feet.
16. 12.65 — rods.

370.

1. 35.
2. 210.
3. 30.
4. 84.
5. 210.
6. 143.
7. 108.
8. 1260.

372.

1. $\frac{2}{5}$.
2. .3.
3. $\frac{27}{343}$.
4. $\frac{81}{125}$.
5. $\frac{25}{64}$.
6. .8.
7. 3.2.
8. 2.43
9. .216.
10. 24.01.
11. .707 +.
12. .95 —.
13. .127 —.
14. .433 +.
15. $\frac{2}{5}$.
16. 4.33 +.
17. .1897 +.
18. .0063 +.
19. 16.5.
20. $8\frac{1}{3}$.
21. 3 inches.
22. $12\frac{1}{2}$.
23. $5\frac{1}{2}$ yards.
24. $2\frac{1}{2}$.

377.

1. 8.
2. 9.
3. 25.
4. 50.
5. 81.
6. 64.
7. 36.
8. 49 feet.
9. 2.8 feet
10. 12.5.
11. 216.
12. 34.3.
13. 5.12.
14. 72.9.
15. 12.2 +.
16. 2.3 —.
17. 1458.
18. 19 ft. 3 inches.
19. 3.7 — feet.

379.

1. 8.
2. 27.
3. 125.
4. 6.
5. 15
6. 14.
7. 210.
8. 25.
9. 72.
10. 168.

381.

1. $\frac{3}{5}$.
2. .3.
3. $\frac{1}{2}$.
4. $\frac{1}{4}$.
5. $\frac{16}{27}$.
6. $\frac{7}{25}$.
7. $\frac{1}{3}$.
8. $\frac{1}{6}$.
9. .584 +.
10. .15.
11. 12.9 +.
12. .69 +.
13. .55 +.
14. $2\frac{1}{2}$.
15. $12\frac{1}{2}$.
16. .77 —.
17. 2 ft. 1.8 + inches.
18. 8 feet.
19. 15 feet.
20. 16 feet.

XIII. SERIES.

385.

1. 53, largest extreme ; 725, sum.
2. 304 ft. largest extreme ; 1600 ft. sum.
3. 250, smallest extreme ; 19125, sum.

388.

1. 2, com. difference, and 1800, sum.
2. 32 ft. increase per second, and 2704 ft. whole distance.

391.

1. 12 terms ; 894, sum.
2. 12 seconds.
3. 125250.
4. \$771.18$\frac{3}{4}$.
5. 850.
6. 1250.
7. 11 terms.
8. $6\frac{1}{4}$ cents.

396.

1. 1, 5, 25, 125, 625, 3125, 15625.
2. 9, 18, 36, 72, 144.
3. 81, 27, 9, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$.
4. 640, 320, 160, 80, 40, 20, 10, 5.
5. 8, $5\frac{1}{3}$, $3\frac{5}{9}$, $2\frac{10}{27}$, $1\frac{47}{81}$, $1\frac{13}{243}$.
6. 81, 54, 36, 24, 16, $10\frac{2}{3}$, $7\frac{1}{9}$, $4\frac{20}{27}$, $3\frac{13}{81}$.

400.

1. 1215.
2. 7168.
3. 4.
4. 27.
5. 5000000000.
6. .0001728.
7. .00164025.

8. $\frac{3}{125}$.
9. $\frac{1}{81}$.

3. $1984.58 +.
4. $88.

403.

1. 7651.
2. 315.
3. 3905.
4. $3\frac{1}{2}$ —.
5. $128\frac{1}{16}$.
6. 15624.
7. 166.66.

406.

1. $\frac{3}{7}$.
2. $\frac{1}{3}$.
3. $\frac{1}{6}$.
4. $\frac{2}{11}$.
5. 106.383 —.
6. 27.

409.

1. $127.63 —.
2. $13.124 —.
3. $563.08 +.
4. $1171.66 +.
5. $827.50.

412.

1. $792.09 +.
2. $6.

416.

1. $3575.
2. $4226.25.
3. $3630.
4. $1755.

419.

1. $2326.76 +.
2. $850.
3. $3075.96 +.
4. $589.19 —.

422.

1. $697.53.
2. $1841.90.
3. $4282.31.
4. $3556.93.

425.

1. $1732.57.
2. $1589.57.
3. $104.61.

428.

1. $1666.66.
2. $4000.
3. $12000.
4. 11666.66\frac{2}{3}$.

XIV. MENSURATION.

431.

1. $3\frac{29}{32}$ acres.
2. 31.5 sq. ft.
3. $150.
4. $3.97 —.
5. $2.75.
6. $1508.75.
7. 22.91\frac{2}{3}$

433.

1. 20 ft.
2. 23.324 — ft.
3. 28.653 + ft.
4. 20.52 — ft.
5. 2.8284 ft.
6. 50.11 — ft.
7. 116.6 + yards.
8. 12.92 + rods.
9. 4.93 — ft.
10. 35.07 + ft.

435.

1. 12.5664 sq. ft.
2. 201.0624 sq. ft.

3. 1256.64 sq. ft.
4. 150.7968 sq. ft.
5. 4 times as large.
6. 3.4336 sq. ft.

437.

1. 4.4 tons.
2. 600 sq. ft.; 1000 cub. ft.
3. 4 times as much.
4. 8 times as much.
5. $8\frac{1}{3}$ tons.
6. 2100 bricks.

439.

1. 47 gallons.
2. 34.336 cub. ft.
3. 70.3888 sq. ft.
4. 215.424 gals.
5. 12.67112 cub. ft.
6. 314.16 — cub. ft.
7. 367.2 gallons.
8. 24675.84 gallons.

441.

1. 62500 cub. ft.
2. 2261.952 cub. ft.
3. $15.708.
4. 2 c. 6 ft. 1.43 cub. ft.
5. 75 cubic inches.

443.

1. $71266\frac{2}{3}$ cub. ft.
2. 29.7024 gallons.
3. 389.5584 sq. in.
4. 20.4204 cub ft.
5. 149.226 cub. ft.

445.

1. 13.2192 gallons.
2. $9.4248.
3. 485.27 + lbs.
4. 3.9168 gallons.

447.

1. 50.26968 gallons.
2. 48.4 + bushels.
3. 2.35 gallons.

XV. REVIEW.

448.

1. $\frac{19}{42}$.
2. $1\frac{9}{16}$.
3. $\frac{71}{210}$.
4. $\frac{1}{7}$.
5. $\frac{6}{25}$.
6. $\frac{5}{49}$.
7. $\frac{4}{5}$.
8. $\frac{7}{25}$.
9. $20\frac{5}{36}$.
10. $\frac{7}{16}$.
11. $\frac{11}{12}$.
12. $4\frac{5}{7}$.
13. $\frac{50}{81}$, $\frac{72}{81}$.
14. $\frac{10}{63}$.
15. $132.50.
16. $5.03 +.
17. $113.75.
18. $15725.
19. $.67 —.
20. 90.
21. $16\frac{391}{620}$.
22. $6.67 —.
23. $2.50.
24. $.34.
25. $47.60.

449.

1. $\frac{37}{80}$ day.
2. .4625.

3. $69.44.
4. 10 m. 4 fur. 31 rods.
5. 4.1 inches.
6. 2164 — lbs.
7. £1, 9s. 3qrs.
8. 9⅙ sq. ft.
9. 4 1/256 c.
10. $6.62 —.
11. $.22 +.
12. 14 pairs.
13. $6.25.
14. ⅙.
15. 1.04.
16. 11 h. 6 m. A. M.
17. 664⅛ cub. in.
18. $647.25.
19. $464.06¼.
20. $13.20.
21. 21780 cub. ft.
22. 6.4 rods.
23. 2 h. 26 m. 40 sec.
24. 2 m. 4 fur. 30 6/11 rods.
25. 15′.

450.

1. 90 ft.
2. $32.56 +.
3. 10.568 cwt.
4. 48 men.
5. 10.15 — days.
6. £5, 11s. 8d.
7. 430.5 — gallons.
8. $2407.50.
9. 12 men.
10. 25 days.
11. 8 yds. 3 na. —.
12. 183 m. 4 fur. 32 + r.
13. 10.8 days.
14. $95.33⅓.
15. 7 yds.
16. 2.085 — oxen.
17. 2880 men.
18. 4166⅔ slates.
19. 48.6 days.
20. 10.9 + oxen.
21. 17280 stones.
22. 12.44 + days.
23. 306.66 + days.
24. $.306 +.
25. $96.
26. A, $234; B, $138; C, $228.
27. $450.
28. 3 horses.
29. A, $2000; B, $2100.
30. 540 acres.
31. $5.04.
32. $.26⅔.

451.

1. $9540.
2. $156.25.
3. $5.50.
4. $.2322 +.
5. $6726.56¼.
6. $2813.25.
7. $6.56¼.
8. $9795.
9. $18.125.
10. $26.871.
11. $1425.
12. $55.92.
13. $157.85.
14. $158.97.
15. $337.75.
16. $3230.31¼.
17. $541.10.
18. $1979.
19. $629.59.
20. $20.
21. $761.69.
22. 7 y. 1 m. 21 d.
23. 2 y. 8 m.
24. 28 y. 6 m. 26 d

25. 5 per cent.
26. 7.8 per cent.
27. 16 per cent.
28. \$475.
29. \$1117.05 —.
30. \$304.88 —.
31. \$216.
32. \$120.27 +.
33. \$249.33$\frac{1}{3}$.
34. \$150.50.
35. \$8.01 —.
36. \$2063.65.
37. \$16.47.
38. \$8.37.
39. \$29.43.
40. 9 months.
41. July 1st.
42. 4$\frac{1}{3}$ months.
43. \$2262.94, or \$2263.45.
44. 16$\frac{1}{2}$ months, or 17 m. 2 d.

452.

1. \$.83$\frac{1}{3}$.
2. 2 lbs. @ 12$\frac{1}{2}$ cts.
   1 lb. @ 15 cts.
   5 lbs. @ 24 cts.
   8 lbs. @ 30 cts.
3. 8 lbs. @ 40 cts.
   8 lbs. @ 51 cts.
   16 lbs. @ 62$\frac{1}{2}$ cts.
4. 35 gals. @ 18$\frac{3}{4}$ cts.
   7 gals. @ 37$\frac{1}{2}$ cts.
   21 gals. @ 50 cts.

453.

1. 6.5.
2. 49 rods.
3. 120 rods.
4. $\frac{25}{16}$.
5. 32 years.
6. 32 feet.
7. 2304 feet.
8. 168.

454.

1. \$321.315.
2. \$6.
3. 11 years.
4. \$890.
5. \$515.625.
6. \$339.
7. \$1926.846 —.

455.

1. \$500.
2. \$112.50.
3. \$42.4116.
4. \$12.49 +.
5. 3.4641 cub. ft.
6. 6911.52 cub. ft.
7. 2.618 cub. ft.
8. 544 cub. in.
9. 144.12 + sq. yds.
10. 42000 bricks.
11. 52930. bricks.
12. $\frac{37}{27}$ times as much.

456.

1. 6 years. 3 years.
2. 9 years.
3. \$150.
4. 16$\frac{4}{11}$ m. after 3 o'clock.
5. 45 m. after 3 o'clock, A. M.
6. 30 years. 25 years.
7. John, 15 years; Henry, 8 years.
8. 1st, \$7031.25; 2d, \$5468.75.
9. 28$\frac{2}{7}$ days.
10. 3$\frac{9}{37}$ days.
11. £4, 14s. 10$\frac{1}{2}$d.
12. 16 ft.
13. A, 6$\frac{1}{2}$ miles; B, 3$\frac{1}{2}$ miles.

14. Chaise, $225; horse, $135.
15. A, $12100; B, $11600; C, $11300.
16. 12 calves, 6 sheep.
17. 7 oxen, 14 cows, 42 sheep.
18. 6 men, 3 boys.
19. $.37$\frac{1}{2}$, $.25.
20. 1st, 29$\frac{1}{2}$ years; 2d, 24 years; 3d, 14$\frac{3}{4}$ years.
21 2 days.
22. 1$\frac{1}{3}$ days.
23. 350 lemons.
24. 48 apples.
25. $.12 a doz.
26. $25.
27. 300 leaps.
28. 2d cup, 16 oz.; cover, 20 oz.
29. 2$\frac{7}{9}$ cts. apiece.
30. A, 11666\frac{2}{3}$; B, 7291\frac{2}{3}$. C, 4166\frac{2}{3}$.
31. 52$\frac{1}{2}$ days.